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# IFIP

INTERNATIONAL FEDERATION  
FOR INFORMATION PROCESSING

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12th IFIP Conference on

## SYSTEM MODELLING AND OPTIMIZATION

BUDAPEST, HUNGARY, SEPTEMBER 2-6, 1985

### ABSTRACTS

#### I.

- Organized by the  
John von Neumann Society for Computer Sciences
- with the sponsorship of the  
Hungarian Academy of Sciences
- co-sponsored by  
IFORS, International Federation of Operational  
Research Societies





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MATHEMATICAL MODELLING AND OPTIMIZATION OF PVC POWDER  
BLENDING PROCESS FOR DEVELOPMENT OF MULTI-LEVEL, OPTI-  
MIZED PROCESS CONTROL SYSTEM

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For development of optimized, multi-level process control system mathematical modelling of PVC powder blending process in high speed mixers was made. For mathematical modelling of PVC powder blending process basic parameters of elementary processes were determined. On base of process analysis and data collection from PVC powder blending in industrial high speed mixers with help of microcomputer determined most important factors from point of view of process control.

On base of energy balance of blending process and semi-empirical formulas of power consumption of mixers here suggested an mathematical model, which describes temperature changes and power consumption of high speed mixers during mixing process. There was made identification of parameters of mathematical model on base of experimental data collected with help of on-line process control microcomputer.

Formulated the optimization task: minimizing of consumed energy for production of PVC compound and maximizing of production capacity of mixers. Developed an optimization algorithm, which makes possible to control the blending process according to selected objective function.

Suggested a new, multi-level structure of process control system for PVC powder blending, which uses developed mathematical model and optimization algorithm. Developed an supervision control program, which realises the upper level of optimized, multi-level, on-line , real-time microprocessor based control system.

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System Identification. An application to cell osmotic  
water permeability in a kidney tubule

José Aguilera and Ernesto González

The mechanisms that underlie fluid transport across epithelial cell layers are of fundamental importance in the regulation of the volume and composition of body fluid compartments, even in the most primitive life forms. This is evidenced in the functioning of various epithelia.

This talk focuses on the role of mathematical analysis in this task. The osmotic water permeability coefficient of the luminal aspect of cell membrane of rabbit kidney isolated proximal straight tubule is characterized without having to solve explicitly the equation that describes the osmosis process. The variation of the internal tubule can be described by a system of the following form.

$$\frac{\partial^2 u}{\partial t \partial x} = F(u, a) ,$$

$$u(0, x) = u_0(x) ,$$

$$\frac{\partial u}{\partial t}(0, x) = 0$$

where  $a$  is the permeability to be determined. Using a finite number of measurements, the identification problem leads to the minimization of a function  $J(a)$  defined over a compact parameter interval  $A \subset \mathbb{R}$ . For an appropriate  $f: A \times \mathbb{R} \rightarrow \mathbb{R}$  with  $f(\cdot, 0) = J$ , with the standard tools of convex analysis we get the following results.

Theorem 1. If  $a_0$  minimizes  $p_0^* \in \partial f(a_0, \cdot)(0)$  then  $J(a_0) - \ell(p_0^*) = 0$ ,  $J(a_0) + f^*(a_0, \cdot)(p_0^*) = 0$ , where for any  $a \in A$   $\partial f(a, \cdot)$  and  $f^*(a, \cdot)$  stand for the subdifferential and the conjugate of  $f(a, \cdot)$  respectively and  $\ell(p^*) := \inf\{f^*(a, \cdot) : a \in A\}$ .

Theorem 2. Suppose that  $\partial f(a, \cdot)(0) \neq \emptyset$  ( $a \in A$ ). Let  $p^* \in \mathbb{R}$  and  $a_n \in A$  ( $n \in \mathbb{N}$ ) such that  $\lim f^*(a_n, p^*) = \ell(p^*)$ . Then  $p^*$  minimizes  $\ell$  if and only if  $a_n$  is a minimizing sequence for  $J$ . Also  $J(a_n) - f^*(a_n, \cdot)(p^*) \rightarrow 0$  if  $n \rightarrow \infty$ .

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AN ECONOMIC APPROACH FOR OPTIMUM LONG-TERM  
PLANT MIX CHOICE

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ABSTRACT

The general aim of power system planning is to insure the provision of a reliable supply of power at lowest possible cost with expected maximum percentage generating units capacities according to fuel type and system uncertainties. The factors which should be considered in any plan are, future load forecast, availability of technology, real cost of money (cash flow), cost and availability of fuel, energy supply adequacy and environmental effects. Thus the problem of long-term plant mix means how to analyse the economic alternatives of power system developments according to such factors.

The time required to obtain the sites, design and construct modern power plant varies from 9 years for nuclear, to 6 years for large oil/coal fired, and 3 years for small thermal and transmission and distribution systems. These long lead time requires that decisions should be committed by long-term planning periods.

This paper introduces an economic approach for power system long-term optimum mix generation using a deterministic linear programming model. As the system composes multiple fossile, nuclear, and hydro-electric plants lacing with its uncertainties, the proposed model incorporates:

- i- a mathematical discription of existing power system capacity,
- ii- estimates of future load demand from the system during the whole planning period,
- iii- estimates of variable costs and capacity requirements of existing system,
- iv- future investment opportunities in term of the system additional capacities by building new plants of different types.

The model objective is to get the optimum decision, at less capital investment, less operating cost and maximum revenue, to justify construction group of power plants of particular types and sizes at certain locations to supply the estimated load demand at the end of corresponding planning period (say 5 years) from a long-term plan (say 20 years).

The proposed model is implemented for Egyptian power system to give a complete plan for it to the end of this century (year 2000/1). The model final decision for such period is to build and operate the future power plants, recommended by the general strategy of Electric Authority in Egypt, with their recommended capacities with some exceptions on some types. The capital investments decisions are to invest first for large hydraulic, nuclear, second for coal-fired, third for oil-fired and finally gas turbine plants. It is, also, recommended that there should be enough balance to be invested to develop the existing transmission network to withstand estimated load demand during such period. The model depicts that since number of new plants are decided to be built, there would be no need for old fired plants which could be shut down or replaced during the planning period.

Therefore, the long-term LP model has succeeded to solve the geographical and technical planning problems for power system. The model can be extended to solve the system cash flow problems.

Thus, the use of LP in making investment decisions is an enormous advance on the earlier discount cash-flow methods which rank projects according to net present values or rate of return.

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**SHORT TERM FORECAST OF FINAL ENERGY CONSUMPTIONS**

**EMIDIO D'ANGELO**

**ITALIA**

This paper is concerned with the description of a system of economic and energy models. The purpose of the system is to forecast the final energy demands for Italy in the short term (2-3 years).

The system consists of a three models:

- 1) macroeconomic;
- 2) input-output;
- 3) energy demand (econometric).

The national macroeconomic model is of a keynesian type, the offer being determined by the demand. The exogenous variables of the model are the exchange rates, the world demand trend and the raw materials import prices (oil particularly). The outputs of the model are the GDP components, the value added for industry and for the other economic sectors, the employment levels, the domestic prices, the import and export prices and so forth. These variables are fed into the input-output model (EXPLOR).

This last model has been developed by Battelle Institute and adapted by us to the italian situation.

The main objective of the input-output model is to obtain information on the sectoral activities of the country. The sectoral activities are necessary to derive final energy demand on a sectoral basis.

The Energy Demand Model (EDM) forecast the final energy consumptions separately for thirteen economic sectors and nine energy products, using an econometric approach.

This model was originally developed to forecast final energy demand for medium term. It has been adapted by us as follows.

- 1) Transformed into a short term forecast model.
- 2) New variables were introduced such as a relative prices, lagged endogenous and exogenous variables.

The model was run, under different macroeconomic assumptions for the year 1985 and 1986.

A sensitivity analysis was done, by analysing the four scenarios result, respect to the macroeconomic exogenous variables and respect to the production and sectoral prices levels.

This analysis was useful to check the models' system, particularly the econometric model.

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## REGULAR OPTIMIZATION OF PSEUDOBOOLEAN FUNCTIONS

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The classical problem of pseudoboolean optimization (see Saaty) after its "embedding" in  $R^n$ :

$$\mathcal{A}(X) \longrightarrow \min_{X \in D},$$

$D = \{X \in R^n \setminus x_j = 0 \vee 1\}$ ,  $\mathcal{A}(X) \in R^1$ , and an analogous problem with linear constraints are considered.

A notion of  $k$ -neighbourhood ( $k = \overline{1, n}$ ) of points of  $D$  is introduced and on its basis topological properties of the set of boolean variables are analyzed.

A notion of modality of  $\mathcal{A}$  on  $D$  is introduced. Classes of different-valued monotone, weakly non-monotone, non-monotone unimodal on  $D$  functions; locally monotone, locally weakly non-monotone, non-monotone polymodal on  $D$  functions; analogous classes of functions having constancy sets are determined. The stated classes cover the set of pseudoboolean functions completely.

On the basis of an analysis of properties of classes of functions exactregular algorithms of optimization are constructed. These algorithms on the average require computations of meanings of an optimized function in case of different-valued unimodal functions:

- monotone -  $(n+1)$ ,
- weakly non-monotone -  $(n^2)/2$ ,
- non-monotone - less than  $(2^n - n^2 + n)$ ;

in case of different-valued polymodal functions:

- locally monotone -  $Q(n+1) + \alpha$ , where  $Q$  is the general number of local minimums of  $\mathcal{A}$  on  $D$ ,  $\alpha = \text{const} \sim n$ ,
- locally weakly non-monotone -  $Qn^2 + \beta$ ,  $\beta = \text{const} \sim n$ ,

- non-monotone -  $Q(2^n - n^2 + n) + \gamma$ ,  $\gamma = \text{const} \sim n$ ;

for functions having constancy sets - less than  $2^n$  (the obtained analytical estimates depend on number and cardinality of existing constancy sets).

Comparison of efficiency of the proposed regular algorithms and randomized methods of pseudoboolean optimization (see Antamoshkin, Saraev) shows preference of the last methods in case of optimization of non-monotone polymodal pseudoboolean functions.

The regular algorithms which had been proposed for solving of some problems of conditional pseudoboolean optimization have essential advantage compared to the total sorting too.

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## DECOMPOSITION OF BLOCK-SEPARABLE PROGRAMMING PROBLEMS

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The paper is concerned with a problem of convex block-separable programming such as

$$(1) x^* \in \arg \min \left\{ \sum_{t=1}^N f_t(x_t) : \sum_{t=1}^N g_t(x_t) \leq 0, x_t \in Q_t \right\}$$

where at every  $t = 1, \dots, N$  the vector  $x_t$  lies in the  $n$ -dimensional Euclidean space  $R^n$ , the functions  $f_t(x_t)$  are convex,  $g_t(x_t) = g_{1,t}(x_t), \dots, g_{m,t}(x_t)$  is an  $m$ -dimensional vector whose every component  $g_{i,t}(x_t), i = 1, \dots, m$  is a convex function in  $R^n$ ,  $Q_t \subset R^n$ . Any solution of the problem (1), a vector  $x_t^*$ , contains  $n \times N$  components and lies in the Cartesian product of  $N$  spaces  $R^n \times \dots \times R^n$ .

The objective of the paper is to decompose the original high-dimensional problem (1) into a number of lower-dimensional problems which are to be solved independently from one another.

Two iterative algorithms, dual and primal, for finding a solution are proposed. The former iterative algorithm is described by recurrent relationships of the following form: if  $p^n, x_t^n, t = 1, \dots, N$  is the desired approximation, then

$$(2) \begin{cases} \bar{p}^n = \Pi_+(p^n + \kappa \sum_{t=1}^N g_t(x_t^n)) \\ x_t^{n+1} \in \arg \min \left\{ \frac{1}{2} |x_t - x_t^n|^2 + \kappa (f_t(x_t) + (\bar{p}^n, g_t(x_t))) : x_t \in Q_t \right\} \\ p^{n+1} = \Pi_+(p^n + \kappa \sum_{t=1}^N (g_t(x_t^{n+1}))) \end{cases}$$

where  $\Pi_+(a)$  is the operator of mapping a vector  $a$  on the positive orthant, or  $(\Pi_+(a))_i = \max(0, a_i), i = 1, \dots, m$ . The first relationship in equation (2) makes it possible to compute an extrapolated, or forecast vector of prices  $\bar{p}^n$  whereby an iterative step to the point  $x_t^{n+1}, p^{n+1}$  is made.

The later iterative process is described by recurrent formulae

$$(3) \begin{cases} \bar{x}_t^n \in \arg \min \left\{ \frac{1}{2} |x_t - \bar{x}_t^n|^2 + \kappa (f_t^n(x_t) + (p^n, g_t(x_t))) : x_t \in Q_t \right\} \\ p^{n+1} = \Pi_+ \left( p^n + \kappa \sum_t^N g_t(\bar{x}_t^n) \right) \\ x_t^{n+1} \in \arg \min \left\{ \frac{1}{2} |x_t - x_t^{n+1}|^2 + \kappa (f_t^{n+1}(x_t) + (p^{n+1}, g_t(x_t))) : x_t \in Q_t \right\}. \end{cases}$$

Solution of the first extremal subproblem in equation (3) yields a forecast value of primal variables  $x_t^n$ . The two subsequent relationships describe transition of the process to the point  $x_t^{n+1}$ ,  $p^{n+1}$ .

When the problem (1) has a nonempty set of saddle points  $X_t^* \times P^* \neq \emptyset$ , the functions  $f_t(x_t)$ ,  $g_t(x_t)$ ,  $t=1, \dots, N$  are convex, every component of the vector  $g_t(x_t)$  is satisfying the Lipschitz condition with a constant  $|g|$  (or  $|(g_t(x_t) - g_t(y_t))| \leq |g| \cdot |x_t - y_t|$ ), the sets  $Q_t$  are convex, closed and the parameter  $\kappa$  is such that  $\kappa < \frac{1}{\sqrt{2} |g| N}$ ; then the sequences described by formulae (2) and (3) (are shown to) converge to the saddle point of the Lagrangean in the problem (1), or  $x_t^n \rightarrow x_t^* \in X_t^*$ ,  $p^n \rightarrow p^* \in P^*$  as  $n \rightarrow \infty$ .

These conditions do not imply differentiability and smoothness of the functions  $f_t(x_t)$  and  $g_t(x_t)$ . Consequently, equations (2) and (3) are methods of nondifferentiable optimization.

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MODELLING AND OPTIMIZATION OF DIALOGUE CONTROL SYSTEMS  
UNDER UNCERTAINTY CONDITIONS

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In /1/ the following problem of determination of module contents for dialogue control systems (DCS) is discussed: there are  $m$  types of objects under control (OC) using DCS. For every type of  $OC_i$ ,  $i = 1, \dots, m$  the requirements presented to DCS are known.  $\theta_i$  is the real  $n_i$ -dimensional vector,  $i = 1, \dots, m$ . Since the requirement system (to which the DCS must satisfy) providing the maintenance of the perspective OC is not always known and depends on chance factors /2/, the sequence  $\theta = \{\theta_i\}_{i=1, \dots, m}$  represents that of chance quantities. Let us assume that distribution function  $x(\theta)$  is not quite known, but a set  $X$  of distribution functions is known belonging to it.

The process of chance of the rational variant of DCS is following. At the first step one considers the requirement system  $\theta^1 \in \theta$ , for which a DCS variant  $S^1$  is selected and his satisfying the requirements of set  $\theta$  is checked. If  $S^1$  variant is not accepted then a variant  $S^2$  is selected and so on. If the chance of DCS variants is finished of  $\kappa$ -th step then we will say that the check is performed by  $\kappa$  steps.

Let us denote by  $d(\theta; \theta^1, \dots, \theta^\kappa)$  and  $y(\theta; \theta^1, \dots, \theta^\kappa)$  nonrandomized and randomized decision functions respectively. Let  $H(x, y)$  be a vector function defined on  $X \times Y$ , where  $Y$  is the set of randomized decision functions representing the losses due to expressing volume, failure-free performance, accuracy of transformation and information transfer, speed of response and so on /3/. Then degree of pre-

ference for choice  $y \in Y$  with known  $x \in X$  can be expressed as follows:

$$H_j(x, y) \geq H_j(x, y') \quad \text{for all } y' \in Y.$$

Here is assumed that  $H_j(x, y)$  functions to be minimized are taken with minus sign and  $H_j(x, y)$  function are quantifiable. Let's define a vector function

$$H(\xi, \eta) = \int_X \int_Y H(x, y) d\xi d\eta$$

where  $\xi, \eta$  are probability distributions on  $X$  and  $Y$  respectively, and set  $V(\xi, \eta) = \{\xi', \eta' \mid H(\xi', \eta') \leq H(\xi, \eta)\}$ ,

$$V^1(\xi) = \bigcap_{\eta} V(\xi, \eta), \quad V^1 = \bigcup_{\xi} V^1(\xi), \quad V^2(\eta) = \bigcap_{\xi} V(\xi, \eta), \quad V^2 = \bigcup_{\eta} V^2(\eta),$$

$$U^2(\eta) = \bigcup_{\xi} U(\xi, \eta), \quad U^2 = \bigcap_{\eta} U^2(\eta), \quad U^1(\xi) = \bigcup_{\eta} U(\xi, \eta), \quad U^1 = \bigcap_{\xi} U^1(\xi).$$

Let's define following relations:

$$(\xi, \xi') \in \rho \rightarrow V^1(\xi') \subseteq V^1(\xi), \quad (\eta, \eta') \in \sigma \rightarrow V^2(\eta') \subseteq V^2(\eta).$$

The strategy  $\xi^*$  is called maximal if

$$V^1(\xi) \subseteq V^1(\xi^*) \quad \text{for every } \xi \in \Omega.$$

The strategy  $\xi^*$  is called optimal if

$$\xi^* \in \{\xi^* \in \Omega \mid V^1(\xi^*) = V^1\} \quad (1)$$

We get the conditions of existence of strategy  $\xi^*$  satisfying (1).

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An algorithm for getting a minimum cut-set of a graph

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Let  $G=(X,E)$  be an undirected connected non-complete graph without holes and multiple edges. For  $Y \subset X$ , the section graph  $G(Y)$  is the subgraph  $(Y,E(Y))$ , where  $E(Y) = \{ (x,y) \in E : x \in Y, y \in Y \}$ .  $D \subset X$  is a cut-set of  $G$ , if  $G(X \setminus D)$  is not connected and  $D$  is a minimum cut-set, if none of its subsets is a cut-set of  $G$ . For  $D \subset X$ , the nodes  $x,y \in X$  are separated by  $D$ , if they belong to different components of  $G(X \setminus D)$ .

For an arbitrary  $x \in X$ , the level structure rooted at  $x$  is the partitioning  $RLS(x) = \{ L_0(x), L_1(x), \dots, L_k(x) \}$  of  $X$  satisfying:  $L_0(x) = \{x\}$ ,  $L_1(x) = N(L_0(x))$ ,  
 $L_i(x) = N(L_{i-1}(x)) \setminus L_{i-2}(x)$ , ( $i = 2, 3, \dots, k$ )  
 $L_{k+1}(x) = \{\emptyset\}$ ,

where  $N(\cdot)$  denotes the relevant neighbour set.

Theorem. For arbitrary  $x,y \in X$  with  $(x,y) \notin E$ , there exists a minimum cut-set  $D(x,y)$  of  $G$  by which  $x$  and  $y$  are separated.

For proof, we present a construction by which the minimum cut-set  $D(x,y)$  is built up, and we have the partitioning  $X = X_1 \cup D(x,y) \cup X_2 \cup X_3$  such that  $G(X_1)$  and  $G(X_2)$  are connected and  $x \in X_1$ ,  $y \in X_2$ , while  $X_3$  is not necessarily

a connected subgraph of  $G$ . This construction serves as an algorithm for getting  $D(x,y)$  and its skeleton can be summarized as follows:

Set  $D(x,y) \leftarrow \emptyset$  ;  $i \leftarrow 1$ ;

For the current  $i$ , do the following:

- Generate  $L_i(x)$  and the nodes in  $(L_i(x) \setminus D(x,y)) \cap L_{i-1}(y)$  are put into  $D(x,y)$ , and remove these nodes from the relevant levels.
- By generating  $L_i(y)$ , the nodes belonging to the set  $L_i(x) \cap (L_i(y) \setminus D(x,y))$  are put into  $D(x,y)$ , while they are removed from the relevant levels.
- Repeat the above steps with  $i \leftarrow i+1$  until the current level pair is found to be empty.

Note that the minimum cut-set obtained in such a way contains the middle set of  $x$  and  $y$  defined as

$$M(x,y) = \left\{ z \in R(x,y) : d(x,z) = \left\lfloor \frac{d(x,y)}{2} \right\rfloor \right\},$$

where

$$R(x,y) = \{ h \in X : d(x,h) + d(h,y) = d(x,y) \};$$

$d(p,q)$  denotes the distance between  $p$  and  $q$ .

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Some new results in stochastic control and computational problems

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The problem of control and estimation of a Markovian /state/ process by use of observations on a second related process has been a focus of research for many years. In this paper the general formulation of this problem will be discussed for linear systems with constant coefficients.

We establish the optimality of certain adaptive control laws for partially observed stochastic systems with a finite set of unknown parameters and with loss functions in the form of quadratic functionals of the state and control values.

To the best of our knowledge, the Kalman filtering and the matrix Riccati equations and Riccati differential equations are a key theoretical and numerical tool in optimal control problems associated with linear systems having state space descriptions and quadratic performance indices.

The main results are the following: first we present an explicit formula for the optimal control when the parameters are known, using the solution of the Riccati equations, then the loss function will be derived. Second it is shown that most of the discrete time ARMAX models can be derived from continuous time AR models, which can be handled, as sufficient statistics exist. Third we present some results in parameter estimation, when exact distributions can be given for sufficient statistics and from these results the strong consistency and limit behavior of the adaptive control loop in the system identification part follows.

The role of ARMAX models was underlined e.g. by the IFAC World Congress in Budapest /1984/.

We shall consider the following two stochastic control problem, the first is given by the system

$$(1) \quad \begin{aligned} dx(t) &= [Ax(t) + Bx(t)] dt + b^{1/2} dw_1(t), \\ dy(t) &= Hx(t) + d\xi(t), \end{aligned}$$

and the second is given by the system

$$(2) \quad \begin{aligned} dx(t) &= [Ax(t) + Bu(t)] dt + b^{1/2} dw_1(t), \\ y(t) &= Hx(t) + \xi(t), \end{aligned}$$

where A, B, b, H are constant matrices, while  $\xi(t)$  is a linear process

$$d\xi(t) = \tilde{A}\xi(t)dt + b_2^{1/2}dw_2(t).$$

The processes  $w_1(t), w_2(t)$  are independent standard Wiener processes. The cost functional is given in the form /the linear regulator case/:

$$V(u, T) = E \left\{ \int_0^T x^*(t) R x(t) dt + \int_0^T u^*(t) \Lambda u(t) dt \right\}$$

We prove that in both cases (1), (2) one can get explicit solutions for the optimal admissible control  $u^*(t)$  with the help of separation principle and explicit solution of Riccati equations. These results can be successfully applied for regulation problems of ARMAX models.

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OPTIMAL DESIGN OF A REMOTE HEATING NETWORK

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In this paper the authors are concerned with a network design problem arising in the layout of the remote heating network of an urban district.

The problem can be modeled by a graph  $G=(V,E)$  whose  $n$  vertices  $v_1, v_2, \dots, v_n$  are grouped into a source  $s$ , a set  $U$  of demand points and a set  $I$  of intermediate vertices and whose  $m$  edges  $e_1, e_2, \dots, e_m$  represent the possible pipe connections between vertices. To each demand node  $v_i \in U$  a value  $r(v_i) > 0$  is associated representing its energy requirement. It is also assigned a finite set of commercially available pipes whose capacities are denoted by  $d_k$  ( $k=1, 2, \dots, q$ ), together with the overall cost  $c_{ij}(d_k)$  of a unit length of a pipe of capacity  $d_k$  placed along edge  $(v_i, v_j) \in E$ .

Among the different arborescences  $G'=(V', E')$  in  $G$  rooted at the source  $s$  and spanning all the demand nodes belonging to  $U$ , and the different pipe capacity assignments  $d_{ij}$ , associating to each edge  $(v_i, v_j) \in E'$  a capacity  $d_k$  ( $k=1, 2, \dots, q$ ), one is required to find the design of minimum cost for which there exists a corresponding feasible flow.

This combinatorial optimization problem is a particularly difficult one both from the theoretical point of view - it can be shown to be NP-hard - and the computational one - given the scale of the problems of practical interest.

The constraint structure of this problem is somewhat peculiar: indeed a flow qualifies for feasibility not only, when the requirements at the sinks are met but when also a set of relations

nonlinear in  $d_{ij}$ , expressing the thermal and dynamic specifications (pressure and velocity) of the flow are satisfied.

In this paper, we propose an approximation algorithm which is based on local search schemes. It consists of two phases: first a feasible design is generated using a randomized algorithm, which is subsequently improved by a search in a suitable neighborhood of the current design.

The accuracy of the solution can be evaluated a posteriori by a lower bound to the optimum obtained by Lagrangean relaxation techniques.

The computational results obtained for a large graph of 166 vertices, 64 demand points and 213 edges, representing a borough of Milano, are reported.

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COMPUTATIONAL COMPLEXITY OF SOME SEMIINFINITE PROGRAMMING  
METHODS

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The paper describes a new class of discretization methods for semiinfinite programming problems of the type:

$$\min_{x \in X} f(x), \quad X = \{x \in \mathbb{R}^n \mid c(x, t) \leq 0 \text{ for all } t \in C\} \quad (1)$$

where  $C \subseteq \mathbb{R}^T$  is the index set. Throughout the paper the following assumptions are used:

- (i) The objective function  $f: \mathbb{R}^n \rightarrow \mathbb{R}$  is convex;
- (ii) The constraint function  $c: \mathbb{R}^n \times C \rightarrow \mathbb{R}$  is convex with respect to  $x$  for each  $t \in C$ ;
- (iii) The constraint function  $c$  satisfies Lipschitz condition:  $|c(x, t) - c(x, s)| \leq L \|t - s\|$ ,  $x \in X$ ,  $t, s \in C$ ;
- (iv) Slater condition holds:  $c(\bar{x}, t) < 0$  for some  $\bar{x}$  and each  $t \in C$ ;
- (v) The index set  $C$  is compact.

Note that most of the semiinfinite programming problems arising from approximation theory, ordinary and partial differential equations satisfy these assumptions.

The underlying idea of the methods is the following: Consider the sequence of finite nets  $M_0 \subseteq M_1 \subseteq \dots \subseteq M_k \subseteq \dots \subseteq C$ , where the step  $h_j$  of the  $j$ -th net is  $h_0 / 2^j$ , and the corresponding sequence of finitely constrained problems:

$$\min_{x \in \bar{X}_k} f(x), \quad \bar{X}_k = \{x \in \mathbb{R}^n \mid c(x, t) \leq -\beta_k \text{ for all } t \in M_k\} \quad (2)$$

If  $\beta_0$  is suitably chosen and  $\beta_j = \beta_0 / 2^j$ , it is easy to show that the sequence of solutions to (2) is feasible and converges to the solution to (1). However, the cardinality of sets  $M_k$  grows exponentially with  $k$ . We describe the class of methods which use the sequence of subsets  $(C_j)$ ,  $C_j \subseteq M_j$  and solve the sequence of problems:

$$\min_{x \in X_k} f(x), \quad X_k = \{x \in \mathbb{R}^n \mid c(x, t) \leq -\beta_k \text{ for all } t \in C_k\} \quad (3)$$

It is possible to show that for each  $k$  solutions of (2) and (3) coincide. Moreover, the cardinality of  $C_k$  grows linearly with  $k$  if some additional assumptions are satisfied. It should be emphasized that the methods we describe do not require maximization over the index set  $C$ , unlike the other discretization methods. The only global information on the function  $c(x, t)$  we require is the Lipschitz constant  $L$  with respect to  $t$  (cf. assumption (fii)). The behaviour of the methods is illustrated on some numerical examples.

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On regularisation of semi-infinite linear  
programming problem

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U S S R

Consider a linear programming problem

$$L_{\infty} : \inf \{ (a_0, x) \mid (a_j, x) \leq b_j \quad (j=1, 2, \dots) \} = v$$

The set of conditions is infinite;  $x \in R^n$ .

Let us define a regularisation set of linear programming problems  $(L^t)_{\infty}$  with parameter  $t$  for the problem  $L_{\infty}$

$$(L^t)_{\infty} : \inf \left\{ \sum_{i=1}^{n+1} \lambda_i (a_0, te_i) \mid \sum_{i=1}^{n+1} \lambda_i = 1, \lambda_i \geq 0, \right. \\ \left. \sum_{i=1}^{n+1} \lambda_i [(a_j, te_i) - b_j] \leq 0 \quad (i=1, \dots, n+1; j=1, 2, \dots) \right\} = vt$$

where  $e_i \in R^n$  and  $\text{cone} \{e_1, \dots, e_{n+1}\} = R^n$ .

Consider the following subproblems for  $(L^t)_{\infty}$

$$(L^t)_s : \inf \left\{ \sum_{i=1}^{n+1} (a_0, te_i) \lambda_i \mid \sum_{i=1}^{n+1} \lambda_i = 1, \lambda_i \geq 0, \right. \\ \left. \sum_{i=1}^{n+1} \lambda_i [(a_j, te_i) - b_j] \leq 0 \quad (i=1, \dots, n+1; j=1, \dots, s) \right\} = (vt)_s.$$

Let  $(L^t)_s^*$  be the dual problem for  $(L^t)_s$  and

$M_{ts}^*$  be the optimal set for  $(L^t)_s^*$ .

Theorem 1. If the system of conditions of the problem

$L_{\infty}$  is consistent, then for every sequence  $\{t_{\kappa}\} \rightarrow +\infty$  there exists a sequence  $\{\tilde{s}_{\kappa}\}$  such, that  $\{s_{\kappa} > \tilde{s}_{\kappa}\}$  implies

$$\lim_{\kappa} (U^{t_{\kappa}})_{s_{\kappa}} = U.$$

Theorem 2. Let  $U < +\infty$  and  $(a_j, p) < \theta_j - \varepsilon$  ( $j=1,2,\dots$ ) for some  $p \in R^n$ ,  $\varepsilon > 0$ . Let  $t_{\kappa}, s_{\kappa}$  be chosen in accordance with Theorem 1. If  $\kappa_0$  is sufficiently large, then for  $\kappa \geq \kappa_0$  the sets  $\mathcal{M}_{t_{\kappa} s_{\kappa}}^*$  are nonempty and uniformly bounded.

Let us mention that  $\mathcal{M}_{t_{\kappa} s_{\kappa}}^* \subset R^{G_{\kappa}+1}$ ,  $\{s_{\kappa}\} \rightarrow \infty$ .

Theorem 1 will no longer be true, if the sequence  $\{s_{\kappa}\}$  is to be chosen before the sequence  $\{t_{\kappa}\}$  is chosen.

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An enumerative approach to 0 - 1 linear programming based on an implicit ordering of solutions.

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### Introduction

Since the beginning of 70s, linear relaxations have been applied to Integer Programming in order to reduce the size of the problem by some kind of data preprocessing.

In particular, Lagrangean relaxations have been extensively used - for example in Knapsack (KP) and Travelling Salesman problems - to get some criteria useful to fix the value of some variables in an optimal solution before starting the main algorithm. From a theoretical point of view, this approach has been proved to be meaningful for KP by a probabilistic analysis.

The aim of this paper is to design an enumerative algorithm which fully exploits Lagrangean relaxations by producing an implicit ordering of solutions.

Due to the exponential number of solutions, explicit sorting cannot be applied; hence, we have devised an algorithm to solve the following problem:

" Given an integer function  $f(x)$ ,  $x$  in  $IB^n$  ( the set of 0 - 1 n - vectors) such that, for any  $i$ ,  $f(x \mid x_i = 0) \geq f(x \mid x_i = 1)$ , recursively produce the ordered sequence of function values  $f^1 \geq f^2 \geq \dots$ , and a corresponding sequence  $x^1, x^2, \dots$ , such that  $f^k = f(x^k)$ ."

In the paper we specialize this algorithm to the case where  $f(x)$  is a Lagrangian upper bound to the optimal value.

### The method

Let (P) be the following 0 - 1 problem :

$$\max cx, Ax \leq b, x \text{ in } IB^n$$

where  $c$  is an integer  $n$  - vector,  $b$  is an integer  $m$  - vector and  $A$  is an integer  $m \times n$  matrix.

Given a  $m$  - vector  $u \geq 0$  we consider the Lagrangean function

$$L(u, x) = (c - uA)x + ub$$

Now, for a fixed multiplier  $u$ , by performing the suggested special ordering algorithm, we produce the sequence  $L^1, L^2, \dots$ , of non decreasing values of the Lagrangean function  $L$ , and a corresponding sequence  $x^1, x^2, \dots$ .

Let  $\bar{x}^k$  be the best feasible solution in the sequence  $x^0, x^1, \dots, x^k$ , where  $x^0$  is a starting feasible solution.

We show that  $c\bar{x}^k \geq L^k \implies \bar{x}^k$  is an optimal solution to (P).

Thus, at any stage, the method produces a lower and an upper bound to (P) and it has the special property that an optimal solution belonging to the generated sequence is eventually recognized.

Obviously, its practical use depends on the availability of a "good" starting feasible solution  $x^0$  and a "good" multiplier  $u$ . In the paper we discuss how to provide such  $x^0$  and  $u$  for a broad class of 0 - 1 linear programs.

Finally, since the complexity of the method is strictly related to the sorting phase, we present a complexity analysis of the suggested special ordering algorithm.

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## HEAVY VIABLE TRAJECTORIES OF CONTROLLED SYSTEMS

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### Introduction

When we study the evolution of macrosystems which arise in economics and the social sciences as well as in biological evolution, we should take into account not only :

- (1) our ignorance of the future environment of the system

but also :

- (2) the absence of determinism (including the impossibility of a comprehensive description of the dynamics of the system)
- (3) our ignorance of the laws relating certain controls to the states of this system
- (4) the variety of dynamics available to the system.

We propose to translate these requirements into mathematics by means of differential inclusions, which describe how the velocity depends in a multi-valued way upon the current state of the system. Another feature of such macrosystems is that the state of the system must obey given restrictions known as viability constraints, which determine the viability domain ; viable trajectories are those lying entirely within the viability domain. Finding viable trajectories of a differential inclusion provides a mechanism of selection of trajectories which, contrary to optimal control theory, does not assume implicitly

- (1) the existence of a decision maker operating the controls of the system (there may be more than one decisionmaker in a game-theoretical setting)
- (2) the availability of information (deterministic or stochastic) on the future of the system ; this is necessary to define the costs associated with the trajectories
- (3) that decisions (even if they are conditional) are taken once and for all at the initial time.

Finally the third feature shared by those macrosystems is the high inertia of the controls which change only when the viability of the system is at stake. Associated trajectories are called heavy viable trajectories: they minimize at each instant the norm of the velocity of the control. We shall provide a formal definition of heavy viable trajectories, which requires an adequate concept of derivative of the set-valued feedback map. We show that as long as the state of the system lies in the interior of the viability domain, any regulating control will work. Therefore, along a heavy trajectory, the system can maintain the control inherited from the past. (The regulatory control remains constant even though the state may evolve quite rapidly).

What happens when the state reaches the boundary of the viability domain? If the chosen velocity is "inward" in the sense that it pushes the trajectory back into the domain, then we can still keep the same regulatory control.

However, if the chosen velocity is "outward", we are in a period of crisis and must find, as slowly as possible, another regulatory control such that the new associated velocity pushes the trajectory back into the viability domain.

## Manpower Calculation for Aircraft Ground Service

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Abstract - After landing at an airport aircraft arrives at apron, parks and shuts down engines. At that moment the process of aircraft ground handling starts and lasts until the moment when the aircraft leaves its parking position. Most of the time this process includes the activity of unloading and loading cargo and passenger baggage from/to aircraft. This activity is performed by a team of workers which consists of specific number of workers. The number of workers in a team depends on the quantity of cargo and passenger baggage which have to be loaded or unloaded and on the time interval available for execution of this activity. Depending on the number of workers in the team loading and unloading specific quantity of cargo and baggage may be executed during longer or shorter time period but not longer than available time interval.

Workers which during one day execute loading and unloading activity to/from aircraft are grouped in specific number of different shifts. The number of shifts, shifts' starting time and the number of workers in each shift during considered day is determined depending on the distribution of times in which the request for service of an aircraft appears and the quantity of cargo and baggage that may be served to/from aircraft during the day.

If one assumes that each aircraft requests two kind of service (loading and unloading) then the problem which arises when the activity of loading and unloading is considered can be defined in the following way:

For a given set of  $N$  aircraft  $x_i \in X, i=1,2,\dots,N$  which in a given time period (e.g. during one day) request in the moment  $t_i \in T, i=1,2,\dots,2N$  loading and unloading specific quantity of cargo and baggage  $q_i \in Q, i=1,2,\dots,2N$ , determine the number of workers in team  $y_i \in Y, i=1,2,\dots,2N$  which execute those activities such that the total number of workers necessary to serve a set of aircraft  $X$  is minimum and that the minimum number of workers are also available in minimum number of shifts. Constraints which appear are that the time spent in one shift is not greater than the prescribed time in one shift and that there are no departure delays of aircraft due to those activities.

The above mentioned problem is combinatorial by its nature. The proposed solution offered in this paper is based on dynamic programming. The problem solution is illustrated by a numerical example.

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## ACCELERATED VARIABLE METRIC METHODS

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Variable metric methods (VMM) for minimization of a nonlinear function  $F=F(x), x \in R^n$ , possessing a continuous gradient  $g=g(x)$  are based upon the iterative process  $x_{k+1}=x_k-s_k H_k g_k$ , where  $s_k$ -the stepsize is determined by approximate line search,  $H_k$ -is an  $n \times n$  matrix, which is updated using information obtained in the last iterative step:

$$H_{k+1} = (H_k - \frac{H_k y_k y_k^T H_k}{y_k^T H_k y_k} + \Phi y_k^T H_k y_k v_k v_k^T) \Gamma + \frac{d_k d_k^T}{d_k^T y_k} \quad (1)$$

where  $d_k = x_{k+1} - x_k, y_k = g_{k+1} - g_k, v_k = \frac{d_k}{d_k^T y_k} - \frac{H_k y_k}{y_k^T H_k y_k}, \Phi \in [0, 1], \Gamma \in R$ .

In order to improve the efficiency of VMM we propose a modified form of (1):

$$H_{k+1} = (H_k - \frac{H_k y_k y_k^T H_k}{y_k^T H_k y_k} + \Phi y_k^T H_k y_k v_k v_k^T) \Gamma + T \frac{d_k d_k^T}{d_k^T y_k} \quad (2)$$

where scalar  $T$  is intended to take into account non quadratic behaviour of the objective function  $F$  and is determined in the process of line search. Note, that modified forms of (2) with  $\Gamma = 1, \Phi = 1$  or  $\Phi = 0$  were proposed by M.C. Biggs. Numerical results showed that sometimes the correction parameter  $T$  decreases the efficiency of VMM. We prove that improper use of the correction  $T$  increases the condition number  $K(R_k) = K(H_k^{-1/2} H_{k+1} H_k^{-1/2})$  and according to Luenberger's theorem decreases the rate of global convergence of VMM. We state, that the correction  $T$  may be applied to the entire class (2), but should be used only in iterations when it decreases the  $K(R_k)$ .

We assume that  $H_k$  is symmetric, positive definite  $n \times n$  matrix and prove the following statements:

T1. Let  $H_{k+1}^1$  be defined by (1),  $H_{k+1}^2$  - by (2),  $T > 0$ .

Then  $K(H_k^{-1/2} H_{k+1} H_k^{-1/2}) \geq K(H_k^{-1/2} H_{k+1} H_k^{-1/2})$  when the following inequalities hold:

$$1 \leq (\Gamma b(1 + \bar{\Phi} z)/c)^2 \quad \text{and} \quad 1 \leq T \leq (\Gamma b(1 + \bar{\Phi} z)/c)^2, \text{ or}$$

$$1 > (\Gamma b(1 + \bar{\Phi} z)/c)^2 \quad \text{and} \quad (\Gamma b(1 + \bar{\Phi} z)/c)^2 \leq T \leq 1.$$

where:  $a = y_k^T H_k y_k$ ,  $b = y_k^T d_k$ ,  $c = d_k^T H_k^{-1} d_k$ ,  $z = (ac - b^2)/b^2$ .

T2. Let  $H_{k+1}$  be defined by (2),  $0 < b < \min(a/T, cT)$ ,  $\Gamma = 1$ ,  $T > 0$ ,  $\bar{\Phi}_{OC} = \arg \min K(R_k)$ . Then  $\bar{\Phi}_{OC} = (Tc - b)(ac - b^2)^{-1}$ , and optimally conditioned  $K_{OC}$  is independent of T:

$$K_{OC} = b^{-2} (2ac - b^2 + 2\sqrt{a^2 c^2 - ab^2 c}).$$

T3. Let  $H_{k+1}$  be defined by (2),  $0 < b < \min(a\Gamma/T, Tc/\Gamma)$ ,  $T > 0$ . Then optimal  $\Gamma_{OC}$  and  $\bar{\Phi}_{OC}$  are defined as follows:

$$\bar{\Phi}_{OC} = \frac{(Tc - \Gamma b)b}{(ac - b^2)\Gamma} \quad \text{or} \quad \Gamma_{OC} = \frac{Tc}{(1 + \bar{\Phi} z)b}$$

Result T1 enables to apply the correction T only in iteration when T decreases the  $K(R_k)$  and, consequently, increases the rate of convergence of VMM. Results T2 and T3 enable to define the parameters  $\Gamma_{OC}$  and  $\bar{\Phi}_{OC}$  minimizing  $K(R_k)$  and ensuring the highest possible theoretical rate of convergence.

Nine VMM algorithms from the modified class (2) were numerically tested on ten hand selected test problems. Numerical results approved theoretical statements. Modified VMM implementing statement T1 appeared to be promising since in most cases they converged in fewer iterations and required less function evaluations.

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COMPUTATIONAL EXPERIENCES WITH CONSTRAINT IDENTIFICATION  
PROCEDURES FOR INTEGER PROGRAMMING

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Since the earlier works of Padberg, Grötschel and others, constraint identification procedures have proven its usefulness in solving special classes of combinatorial problems such as traveling salesman, matching and knapsack among others. On the other hand many other integer programming problems have explicit combinatorial features imbedded in its structure, which can be exploited from a constraint identification approach in a cutting plane fashion together with other branch and bound or heuristic methods. In this paper some cases of capacitated plant location, scheduling with resource constraints, and set partitioning problems are studied, and the results of a series of computational experiences are reported.

The formulation of scheduling problems with resource constraints includes typically constraints like:

$$\sum_{i \in I} r_{ik} x_{it} \leq b_k$$

where  $r_{ik}$  are the requirements of resource  $k$  to perform task  $i$ ,  $b_k$  is the amount of resource  $k$  available and  $x_{it}=1$  if task  $i$  is in process at time  $t$ , and  $x_{it}=0$  otherwise. Capacitated plant location problems could be formulated including a constraint like:

$$\sum_{j \in J} b_j y_j \geq D$$

where  $J$  is the set of potential locations of the plants,  $b_j$

is the capacity of plant  $j$  and  $D$  is the total amount to be supplied from the opened plants.

Both kind of constraints are knapsack type constraints for which efficient cutting planes can be computed, using results from the facetial structure of the knapsack polytope. Solutions to the LP relaxation provide information which allows the identification of the most violated cover of the knapsack constraint by the current LP solution. From this violated cover a lifting procedure (Padberg) allows the computation of a facet of the knapsack polytope which is a valid constraint for the original problem. This paper studies computationally the performance of such procedures imbedded in a branch and bound algorithm when applied to these problems.

On the other hand for set partitioning problems, Balas has shown that inequalities of the form:

$$\sum_{j \in V} x_j \leq 1$$

are valid for the set partitioning polytope when  $V$  is the set of nodes of a complete subgraph of the strong intersection graph of the set partitioning problem. This paper studies also computationally the behaviour of a procedure which identifies inequalities of this type violated by the current LP solution to the set partitioning problem.

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Optimal Control of Storage Power Plant Systems

(W. Bauer, E. Lindner, H. J. Wacker)

In the paper we discuss the following problem:

Maximize the rated energy production of a serially connected storage power plant system within a given interval of time.

This problem may be modelled as a problem of optimal control.

For  $n = 2$  one gets (in the simplest case) the following model (M)

$$(M1) \quad E(T) = g \cdot \rho \int_0^T a(t) \sum_{i=1}^2 [f_i(V_i(t)) Q_i(t) \eta_i(V_i(t), Q_i(t))] dt = \text{Min!}$$

$$(M2) \quad \dot{V}_1(t) = Z_1(t) - Q_1(t), \quad \dot{V}_2(t) = Z_2(t) + Q_1(t) - Q_2(t) \quad \text{for } t \text{ in } ]0, T[$$

$$V_1(0) = V_1(T) = V_{1\text{max}}, \quad V_2(0) = V_2(T) = V_{2\text{max}}$$

$$(M3) \quad V_{i\text{min}} \leq V_i(t) \leq V_{i\text{max}}; \quad 0 \leq Q_i(t) \leq Q_{i\text{max}} \quad \text{for } i=1,2 \text{ and } t \text{ in } ]0, T[$$

Here the discharge  $Q(t)$ -index  $i$  omitted - is used for control, the state variable being the volume  $V(t)$  of the reservoir. The head is described by the reservoir capacity function  $f(V)$ ;

Additionally we use the notations

$\eta(V, Q)$ : efficiency,  $a(t)$ : tariff function,  $Z(t)$ : influx to the reservoir.

The constraints are linear, the objective is nonlinear and - in case of at least two different tariff periods - nonconvex.

There are quite a few different methods by which (M) may be solved.

Here we first concentrate our interest on variational techniques.

These techniques are applicable to a somehow simplified model where we neglect the dependency of the efficiencies on the control  $Q$ .

The structure of the optimal solution for the last reservoir (in (M): index 2) is already given in a paper by Gfrerer [1].

Based on this result we analyse the structure of the optimal solution of the system for a two period tariff situation. As a result we get a finite dimensional model which is equivalent to (M). The resulting low dimensional model may be solved then numerically by a homotopy method, see for instance [2], [3].

As a practical application of a type (M) model we present the optimization of the Gosau storage power plant system. The serially connected system consists of two resp. three reservoirs of different capacities. Besides (M3) we have to respect additional constraints, (M2) becomes nonlinear for  $i = 1$  because of seepage losses depending on  $V(t)$ . Further  $f_1$  depends both on  $V_1$  and  $V_2$ .

The result of our optimization was already set into practise leading to considerable improvements.

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INVESTIGATION OF HYPERTOXIC CHRONIC FORMS  
OF THE DISEASE WITHIN THE FRAMEWORK  
OF MATHEMATICAL MODEL

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The results of investigation of existence and stability of nontrivial stationary solution of the infectious disease mathematical model are presented. The model is a system of four ordinary differential equations with delayed argument. This solution is interpreted as a chronic form of the disease course with strong damage of organ-target, and it is called hypertoxic chronic form of the disease.

It is shown that the increasing pathogeneity of latent form microbes initiated by external factors (stress, other infection joined, etc.) can be the cause of such form of the disease.

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SELF TUNING INNOVATION REPRESENTATION BASED ON BALANCED REALIZATION ALGORITHM WITH APPLICATION

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The aim of this paper is to present an algorithm with real-time adaptation gain, i.e., with exponential forgetting factor updated in every step. Based on the results of Desai-Pal(1982) a realization algorithm was developed for multivariable discrete stationary Gaussian processes (Bencsik-Fehér, 1985) which algorithm can be outlined as follows. Given the covariance sequence  $\Lambda(i) = E y(t+i) y^T(t)$ ,  $i \geq 0$ , one can form a Hankel matrix

$$H_h = \begin{bmatrix} \Lambda(1) & \Lambda(2) & \dots \\ \Lambda(2) & \Lambda(3) & \dots \\ \dots & \dots & \dots \end{bmatrix}$$

Applying Ho-Kalman type algorithms to find matrices H, F, G such that  $\Lambda(i) = H F^{i-1} G$ ,  $i \geq 1$ , G is derived as follows. The innovation representation has the form

$$\begin{aligned} \hat{x}(t+1) &= F \hat{x}(t) + K \epsilon(t) \\ \hat{x}(t) &= H \hat{x}(t) + \epsilon(t) \end{aligned}$$

and from this IR the following relations can be got in a straightforward manner:

$$\begin{aligned} E \hat{x} \hat{x}^T &= F E \hat{x} \hat{x}^T F^T + K E \epsilon \epsilon^T K^T \\ E y y^T &= H E \hat{x} \hat{x}^T H^T + E \epsilon \epsilon^T \\ G &= E x(t+1) y^T(t) = F E x x^T H^T + K E \epsilon \epsilon^T \end{aligned}$$

and they are iteratively solvable for  $E \hat{x} \hat{x}^T$ ,  $E \epsilon \epsilon^T$  and for K.

Then a backward IR is formed:

$$\begin{aligned} \hat{z}(t-1) &= F^T \hat{z}(t) + H^T \epsilon(t) \\ y(t) &= K^T \hat{z}(t) + \epsilon(t) \end{aligned}$$

where  $E \hat{z} \hat{z}^T$  is computable, as well. A transformation discussed in the paper of Desai-Pal(1982) is applied to determine  $E \hat{x} \hat{x}^T$  of the balanced IR. A subsystem of this balanced IR gives a reasonable reduced order IR.

In the case of short pattern with timevarying parameters simulation results have shown that the Hankel matrix is invertable in the overdetermined order case since  $\det H_h$  vanishes exponentially. This is the full rank case of the Ho-Kalman algorithm:

$$F_0 = z H_h \quad H_h^{-1} = \begin{bmatrix} 0 & I & \dots \\ -A_r & -A_{r-1} & \dots & -A_1 \end{bmatrix}, \quad H_0 = [I \ 0 \ \dots \ 0]$$

$$K_0 = \begin{bmatrix} M_0 \\ \vdots \\ M_{r-1} \end{bmatrix}, M_i = HF^i K, G_0 = \begin{bmatrix} \Lambda(1) \\ \vdots \\ \Lambda(r) \end{bmatrix}, \hat{x}_0(t+1) = \begin{bmatrix} \hat{y}(t+1|t) \\ \vdots \\ \hat{y}(t+r|t) \end{bmatrix}, z_{H_h} = \begin{bmatrix} \Lambda(2) \ \Lambda(3) \dots \\ \vdots \\ \Lambda(3) \ \Lambda(4) \dots \end{bmatrix}$$

which is the standard observable representation /SOR/ or the realization of systems with timevarying parameters. Performing the transformation of Desai-Pal(1982) we get the balanced IR and the dimension of the final SOR is chosen by investigating the eigenvalues of  $E \hat{x} \hat{x}^T$  of the balanced IR.

Now the  $F_0, K_0$  matrices can be updated recursively:

$$F_0(t+1) = F_0(t) + \gamma(t) \left\{ \hat{x}_0(t+1) \hat{x}_0^T(t) \left[ E \hat{x}_0 \hat{x}_0^T \right]^{-1} - F_0(t) \right\}$$

$$K_0(t+1) = K_0(t) + \gamma(t) \left\{ \hat{x}_0(t+1) \varepsilon^T(t) \left[ E \varepsilon \varepsilon^T \right]^{-1} - K_0(t) \right\}$$

where  $\gamma(t) = 1/t$  in the stationary case

$$- E \hat{x}_0(t+1) \hat{x}_0^T(t) = E \begin{bmatrix} y_i(t+1) \\ y(t+r) \end{bmatrix} \hat{x}_0^T(t) \text{ and}$$

$$- E \hat{x}_0(t+1) \varepsilon^T(t) = E \begin{bmatrix} y_i(t+1) \\ y(t+r) \end{bmatrix} \varepsilon^T(t)$$

$$\text{since } x_{0, \text{measured}}(t+1) = \hat{x}_0(t+1) + \begin{bmatrix} \varepsilon(t+1) \\ \varepsilon(t+2) + M_0 \varepsilon(t+1) \\ \varepsilon(t+r) + \dots + M_{r-1} \varepsilon(t+1) \end{bmatrix} =$$

$$= \hat{x}_0(t+1) + \varepsilon_r(t+1), \text{ and } \varepsilon_r(t+1) \perp \hat{x}_0(t) \text{ and } \varepsilon_r(t+1) \perp \varepsilon(t).$$

It has to be mentioned that these estimations are conditional expected values, i.e. linear solutions of a nonlinear filtering problem / see Ljung-Söderström, 1984.Ch.2.3 /.

According to our proof the  $\gamma(t)$  adaptation gain can be updated by investigating the eigenvalues of  $H_h$  Hankel matrix instead of  $E \hat{x} \hat{x}^T$  of the balanced IR since a connection of them can be stated. In this way an updated exponential forgetting factor is computed / see Ljung-Söderström, 1984.Ch.5.6 /.

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SEQUENTIAL STRATEGIES FOR THE STOPPING OF MONTE  
CARLO ALGORITHMS IN GLOBAL OPTIMIZATION

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The global optimization problem deals with finding the global optimum of a real valued function  $f$  over some compact set  $K$ . A considerable number of computational approaches have been as yet considered and it is now recognized that an effective scheme consists of repeatedly performing a number of independent realizations of a randomized search procedure able to achieve "good" function values. The crucial point in algorithms based upon this Monte Carlo scheme is the criterion for stopping computations; in fact, as no useful analytical characterization of the global optimum is available, it is practically impossible to give exact answer to the question of how close to the global optimum is the best function value observed. Therefore a rigorous setting of a proper stopping criterion is possible only in a statistical framework.

In this paper a Bayesian decision theoretic approach is adopted, in connection with the so called Multistart

method, in which a local optimization routine, able to reach a local optimum, is started from points randomly and independently chosen in  $K$ . Roughly speaking, the approach enables to give strategies for stopping when it is expected that the search for new local optima, with a function value better than the best obtained so far, would be too costly. A cost function is introduced which combines the cost of a new local search with the utility corresponding to a unit increase in the maximum observed value. The local optimum values found are seen as realizations of a random variable whose distribution is unknown. In order to deal with the problem of optimal stopping, the Bayesian paradigm requires that a probability measure is defined over a space of probability distributions containing the unknown one. A simple nonparametric process is considered which provides in a manageable and general enough form such a probability measure after which derivation of sequential stopping rules of the  $k$ -stage look-ahead type is exhibited and their actual implementation discussed. Finally, the behavior of these stopping rules is investigated for some test problems.

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STOCHASTIC CONTROL IN URBAN TRAFFIC

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The paper deals with the problem of controlling an intersection in urban traffic. The availability of traffic sensors has given the possibility of implementing control strategies having the capability of adapting themselves to traffic flow fluctuations. Models have thus to be introduced possessing the feature of taking into account the information gathered by the sensors; such models should also be dynamic, stochastic and easily implementable. In fact, it is an obvious observation that not only deterministic facts, but also random fluctuations have great influence on traffic flows and that the probabilistic behaviour of these latter depends on time.

The following situation is considered: a traffic light controls a junction; sensors counting vehicle passages are located upstream the traffic light in entering arms and at the stop lines; the upstream sensors are located in correspondence with upstream traffic lights whose control is assumed to be fixed and known. The traffic evolution at the intersection is then modelled through a partially

observed Markov chain, whose components are the length of each queue, the departures from the stop line and the (unobserved) stochastic arrival rate. This enables to derive in a recursive form optimal filtering formulas for the queue lengths. Such formulas are then incorporated into a control policy of the  $\mu c$ -type, aimed at minimizing the average delay experienced by vehicles crossing the intersection. The performance of such control policy has been tested on a number of different situations considering both simulated and real data.

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DECISION SUPPORT SYSTEMS FOR TRAFFIC AND TRANSPORTATION  
MANAGEMENT

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Abstract

The increasing development of advanced information processing technologies and new high level programming languages, as well as last theoretical contributions in Artificial Intelligence have set new and interesting instances in the field of Decision Support Systems (DSS). But, although the availability at decreasing costs of many tools, as videographics systems, database systems, personal computers and local area network offer new chances in supporting managers for decisionmaking, a lot of work has to be done in order to implement and build up effective DSS for practical applications.

In this work the peculiarities and the basic structure of a DSS are defined and the differences with respect to management information systems, expert systems and management support systems are remarked.

This paper, among the components of a DSS, in particular deals with the formalization of decisional problems, the analysis of model-based methodologies, the use of knowledge-based modelling and the structure of the models-base management systems.

In order to come into practical applications in the field of traffic and transportation management some typical decision problems, arising at different level of intervention both in public and private organizations are illustrated.

Relatively to urban systems, integrated transportation planning is discussed and the centralized management of urban traffic via integrated simulation and optimization models is investigated.

With reference to resources allocation in a transit company, management activities pertaining to different functional areas can have different degrees of structure both in strategic planning management control and operational control.

So typical semi-structured problems such as bus network planning, bus allocation to lines and garages location, bus routing and scheduling are analysed and a heuristics approach to deal with large-scale and complex problems is presented.

Finally, relatively to these problems, the data base and models-base requirements for the design of a DSS at operational level are illustrated.

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Variation of the solution of differential inclusion and  
 maximum principle  
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Consider the differential inclusion

$$\dot{x} \in F(x) \quad (1)$$

in  $n$ -dimensional Euclidean space  $E^n$ . An absolutely continuous function  $x(t)$  is said to be solution of (1) on the time interval  $I = [t_0, t_1]$  if the inclusion  $\dot{x}(t) \in F(x(t))$  holds for almost all  $t \in I$ .

Let  $F(x)$  be nonempty compact set of  $E^n$  for all  $x \in E^n$  multivalued function  $F(x)$  be Lipschitzian, it's support function

$$c(F(x), \psi) = \max_{f \in F(x)} (f, \psi)$$

be continuously differentiable in  $x$  for all  $\psi \in E^n$  and gradient  $\frac{\partial c(F(x), \psi)}{\partial x}$  be Lipschitzian in  $\psi$  with the constant  $k(x)$  which is continuous in  $x$ .

Let  $x(t)$  be given solution of (1) on time interval  $I = [t_0, t_1]$ . Consider the convex closed cone

$$N(t) = \{ \psi \in E^n : (\dot{x}(t), \psi) = c(F(x(t)), \psi) \}$$

and fix some vector  $\delta x \in E^n$ . Define scalar function

$$c(\psi) = \begin{cases} +\infty & , \psi \notin N(t), \\ \left( \frac{\partial c(F(x(t)), \psi)}{\partial x}, \delta x \right) & , \psi \in N(t). \end{cases}$$

It is proved that the function  $c(\psi)$  is a support function of some convex closed set  $P(x(t), \delta x)$ . Consider the differential inclusion in variation

$$\dot{\delta x} \in P(x(t), \delta x). \quad (2)$$

If the inclusion (1) coincides <sup>with</sup> the ordinary differential equation then the differential inclusion in variation (2) coincides with classical system of equation in variations [1].

Theorem 1. Let  $\tau \in I$ ,  $\delta x(t)$  be given solution of the inclusion (2) with the initial value  $\delta x(\tau) = \delta x_\tau$  and  $\varepsilon > 0$ . Then there exists the solution  $y(t)$  of the differential inclusion (1) with the initial value

$$y(\tau) = x(\tau) + \varepsilon \delta x_\tau + o(\varepsilon)$$

which has the form

$$y(t) = x(t) + \varepsilon \delta x(t) + o(\varepsilon)$$

for all  $t \in I$ .

Using this theorem it is possible to construct the variations of the solution  $x(t)$  of the inclusion (1) and to prove necessary optimality conditions in the form of Pontryagin maximum principle [2]. Let us consider here this maximum principle for the time optimal control problem. This problem is to find the solution  $x(t)$  of the inclusion (1) which transfers the given initial point  $x_0$  to the given final point  $x_1$  for the minimal time.

Theorem 2 (maximum principle). Let  $x(t)$ ,  $t \in [t_0, t_1]$  be optimal solution of (1). Then there exists the nontrivial solution  $\psi(t)$  of the adjoint differential equation

$$\dot{\psi} = - \frac{\partial c(F(x(t)), \psi)}{\partial x}$$

such that the maximum condition

$$(\dot{x}(t), \psi(t)) = c(F(x(t)), \psi(t))$$

holds for almost all  $t \in [t_0, t_1]$  and the function  $c(F(x(t)), \psi(t))$  is constant and nonnegative.

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## THROUGHPUT OPTIMIZATION OF PACKET COMMUNICATION NETWORKS

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High-performance computer systems are developed as a composition of many functional modules. These systems create a network of independent modules which communicate asynchronously by unidirectional channels and a number of modules are active simultaneously. The asynchronous communication mode implicates the necessity to incorporate the buffers between each pair of connected modules. Data to be processed in the network are transmitted in the information packet form. This principle is used in very perspective data-flow computers and distributed computer systems.

The problem of the design, which is the subject of our considerations, lies in the design of right delay times in nodes or modules of network structure, in order that the network may be balanced without bottlenecks for the given workload. We say, that the network is dynamically balanced when all modules or nodes may be uninterruptedly active without blocking one another and their utilization approximates to the ideal value, i.e. 100%. In that situation we can say that the delay time in the nodes is optimal.

We developed the synthesis procedure for the design of the optimal throughput in packet communication networks. The initial data for the synthesis are a structure of the network, a probabilistic workload model and approximate real delay time values in the network.

Firstly, we concentrate upon basic parallel structures such as pipeline and parallel array, which may be connected to hybrid clusters or networks. We have derived the formulas for throughput, speed-up

coefficient, delay time in the module, input and output period of the items processed by modules of these basic structures. A node or a module of the network can absorb or generate one or more items (packets) by an activation. The formulas for a dynamically balanced network have been derived from formulas for basic structures. Secondly, we characterized a workload of the network by probabilistic parameters. The synthesis procedure consists of ten rules. In the procedure we use the derived formulas. According to the developed procedure we are able to design an optimal delay time in network nodes. It results in the network whose dynamics or throughput is specialized for the given workload.

The synthesis procedure was verified by simulation. With regards to the simulation results, we can say, that the synthesis is correct, because values of the performance parameters approximate to the ideal value.

Some ideas for future research in this field are introduced in conclusion remarks. Due to the generality of obtained results, this optimization is applicable, according to our opinion, for another technical systems too, e.g. discrete technological processes, traffic and transportation.

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## SCHEDULING MULTIPROCESSOR TASK UNDER RESOURCE CONSTRAINTS

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One of the assumptions commonly imposed in the machine scheduling theory is that each task is processed on at most one machine at a time. In fact, all polynomial-in-time algorithms as well as NP-completeness results for scheduling on machines (processors) were obtained on this assumption.

However, in recent years, together with the rapid development of microprocessor and especially multi-microprocessor systems, the above assumption has ceased to be justified in some important applications. There are, for example, self-testing multi-microprocessor systems in which one processor is used to test others or diagnostic systems in which testing signals stimulate the tested elements and their corresponding outputs are simultaneously analyzed. When formulating scheduling problems in such systems, one must take into account the fact that some tasks have to be processed on more than one processor at a time.

These problems create a new direction in the machine scheduling theory, in which preliminary results concerning the preemptive scheduling of tasks requiring one or two processors were obtained in [1], [2] for the schedule length criterion.

In this paper we study an extension of the model defined above by assuming that tasks require additional resources (e.g. channels, memory, I/O devices, etc.) for their processing. We show an impact of this assumption on the computatio-

nal complexity of algorithms for constructing minimum length schedules.

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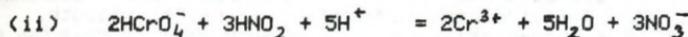
THE FEASIBILITY OF FEEDFORWARD MICRO-COMPUTER CONTROL IN  
MANAGEMENT OF INDUSTRIAL WASTE WATERS  
CONTAINING Cr 6 IMPURITIES

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Feedforward control techniques require accurate mathematical models providing the control performance, whereas feedback controllers can operate well without these. Feedforward control systems, however, offer advantages like a better performance whenever we can measure the disturbances and the mathematical models are accurate. A single feedforward controller cannot operate successfully. As it is well-known, the accumulation of errors result in a significant deviation from the prescribed values after a certain time. Additionally used simple feedback control loops, such as bang-bang or PID controllers can corrugate these failures.

In this lecture we will deal with the feedforward part of a synthesized (feedforward-feedback) system. Our purpose is to explain how microcomputers may be used for providing the feedforward control performance in management of industrial waste waters containing Cr 6 pollution.

Accurate mathematical models of continuous processes may be developed by means of the transport equations. These equations are partial differential equations expressing the laws of conservation of mass, energy and momentum. Taking into account the stoichiometric equations of Cr 6 redoxi process by means of nitrous acid and the precipitation of  $Cr_2O_3$  by caustic solution (where Q means either K or Na),



as well as the fact that the redoxi process obeys the ratio equation of Langmuir-Hinshelwood type

$$(v) \quad \frac{dc_1}{dt} = f(c_1, c_2, c_3) = \frac{k_1 c_1 c_2^2 + k_2 c_3 + k_3 c_1}{1 + k_4 c_2 + k_5 c_2^2}$$

the mathematical Problem to be solved may be expressed as the fixed-Point Problem Pertaining to the first and second initial and boundary value Problem of the Partial differentiation equation system

$$(vi) D \frac{\partial^2 c_i}{\partial x^2} - (v+W) \frac{\partial c_i}{\partial x} + a_i f(c_1, c_2, c_3) = \frac{\partial c_i}{\partial t} \quad (i=1-3)$$

$$(vii) c_i(x, 0) = c_{i0}(x), \quad c_3(0, t) = c_{30}(t), \quad c_{iX}(L, t) = 0 \\ c_1(L, t) = 0, \quad c_2(L, t) = 0$$

The solution of the above mentioned fixed Point Problem delivers us the setPoint values  $w$  and  $c_2(0, t)$ , i.e. the velocity and initial concentration of nitric acid to be added to the waste water containing Cr 6 impurities in order to Prevent Cr 6 Pollution.

Control may be maintained using PSV techniques. The storage capacity requirement will be in this case  $S_R =$

$$= b(m+1) \prod_{i=1}^n n_i F_i = 700 \text{ Kbytes, dividing each fast variable}$$

into 10 equidistant segments. Using Lagrangian interpolation, a feedforward adaptive control may be provided by low cost micro-computers and by means of a once and for all made database containing the setPoint values and the code numbers belonging to them.

#### NOMENCLATURE

$a_i$  : stoichiometric coefficients,  $b$  : number of bytes necessary for setPoint storage,  $c$  : concentration,  $D$  : diffusivity,  $k_i$  : reaction rate constants,  $L$  : reactor length,  $m$  : number of slow variables,  $n$  : division points of fast variables, PSV : PreComputed SetPoint Value; method Published in: L.Fay, Can.J.Chem.Eng. 62 (1984) 661-671

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OPTIMAL WATER QUALITY CONTROL BY TREATING THE EFFLUENT  
AT THE POLLUTION SOURCES AND BY FLOW REGULATION

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Research is concerned with applying control theory concepts to a problem of water quality in river. The problem to be addressed deals with improvements in the dissolved oxygen balance and chlorides balance by treating the effluent at the pollution sources and by flow regulation by means of multi-purpose reservoir. The last approach is considered for low flow periods. The task of controlling is formulated as an optimization problem for a distributed parameters system. A partial differential equations model for Biochemical Oxygen Demand, Dissolved Oxygen, chlorides and open channel flow is given. The open channel flow is modelled by two one-dimensional equations/ linearized equations of B. Saint-Venant/. The concentration of DO, BOD and chlorides in the stream is modelled by three one-dimensional partial differential equations of parabolic type.

A criterion functional is proposed in which the control can be found as the solution of an optimization problem. The methods of parametrical controlling for distributed parameters system are applied. Existence of an optimal solution for our optimal control problem is proved and necessary optimality conditions are derived. A numerical solution is found and numerical algorithm is applied to an example using historical data from the section of Vistula river. The section of river receives one major effluent discharge in the beginning and also the effluent discharges from the chemical factory on the section. Low flow augmentation

is considered by using the reservoir on the tributary. The problem is formulated within the framework of hierarchical water management system in the large industrial region. For the five partial equations describing the system finite difference method and the "double sweep" method is used. The minimization of a given performance index subject to the assumed constraints is achieved by applying the penalty shifted method combined with the conjugated gradient method.

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PERFORMANCE ANALYSIS AND OPTIMIZATION OF THE DATA LINK AND COMMUNICATION DEVICE CONTROL PROCEDURES IN DISTRIBUTED MICRO/MINICOMPUTER SYSTEMS

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We presents a number of analitical results of performance evaluation and optimization of communication control procedures (protocols) in computer networks. Some results focuses on the protocols of DECnet, which refers to a family of packet-switched network products developed by Digital Equipment Corporation (DEC).

This approach is developed in order to serve two purposes: first, to provide a tool for a fast and reliable performance evaluation of communication protocols, and second, to provide more insight into how the various parameters of protocol implementations and the data channel, communication device and computer characteristics influence the performance.

A queueing models are used for data link procedures which include to bite-counting in information field of the frame. The example is well known Digital Data Communications Message Protocol (DDCMP). Explicit expressions for the maximum through-put as well as the mean transfer time of frames (messages) and mean buffer holding time in the presence of transmissions errors, time-out duration and some other detail are found.

These procedures are compared with ISO link control procedure HDLC. We find out that DDCMP is frequently more effective than HDLC in the cases of real combinations of the parameters. This is due to transmission error within an DDCMP head of the frame is detected with the help of a special 16-bit head checking sequence. Thus, the control information in the head of the frame can be used by a receive node even if an error occure in the information field of the frame.

A general model of "Send Messages and Wait" communication procedures is concerned. This is a system with N finite queues where all the queued requests are served in one batch when the server is switched on (M/G/1/x system where x=(x1,...,xN), n=1,...,N). The service cost is independent of the number of batched requests. The requests which arrive

during the service phase are lost. Explicit expressions are derived for the steady state probabilities and performance measures. Communication procedures differ from one another in the switching server strategies used. The problem of finding the optimal strategy is solved by means of finite state markovian decision process.

Attention is paid to the DEC communication devices (hardware interface between the processor and the communication line) since computer network performance tend to be relatively sensitive to the choice of this particular component. Several types of PDP-11 and VAX-11 computers and communication devices such as DL-11, DUP-11, DZ-11 and CAMAC interfaces are considered. Analytical models are developed for all these devices. Explicit expressions for effective node throughput as a function of the number of communication devices and lines are derived. This analysis allow us to obtain optimum values of the number of communication lines and the time-out period for interfaces without hardware positive acknowledgements.

The object of our studies was to provide information which would help determine what resources (eg. processors, communication devices, communication lines) and protocol parameters are necessary to support arbitrary user distributed micro/minicomputer systems.

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APPROXIMATIONAL MULTI-CRITERIA SCHEDULING IN  
COMPUTER SYSTEMS

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Effectiveness of computer systems depends largely on the methods of scheduling requests for computer resources. Only automatic methods enable prompt reaction to a current situation. This gives rise to the problem of automatic ordering (sorting) of an arbitrary set of objects representable as points in the parameter space according to the structure of DM preferences (system manager ideas about request ordering, in this case).

A method of automatic multi-criteria sorting is proposed here relying on the aggregation of real (approximate) DM preferences according to the constructed theory of integration of partial sorting systems coordinated with the structure of DM preferences.

The triple  $\alpha_B = (B \subseteq A, I_B, \pi_B : B \rightarrow I_B : B_i \mapsto \{i\})$ , where  $I_B$  is strictly ordered index set is regarded as partial sorting of the original object set A. The system of partial sortings  $\mathcal{H} = (A, B \subseteq 2^A, \{\alpha_B\}_{B \in \mathcal{B}})$  is called coordinated with the choice function  $\alpha = (A, A \subseteq 2^A, C : A \rightarrow 2^A)$  reflecting the structure of DM preferences if  $\forall X \in A, B \in \mathcal{B}, X \subseteq B$  the choice with respect to sorting  $\alpha_B : C_B(X) \stackrel{\text{def}}{=} \pi_B^{-1}(\text{Min}(\pi_B(X))) \cap X$  coincides with that with respect to the choice function  $\alpha - C(X)$ . A sorting of the set A whose constraints on each  $B \in \mathcal{B}$  are equivalent to sorting  $\alpha_B$  is referred to as global for the system  $\mathcal{H}$  of partial sortings of the set A.

For a given system, sufficient conditions and existence criterion for global sorting have been obtained, and a solution has been found to the problem of uniqueness and construction of such a sorting. In particular, it was demonstrated that if families A and B meet some completeness condition, the partial sorting system coordinated with an arbitrary choice function  $\alpha$  can be continued down to the system of sorting of all the subsets of A coordinated with  $\alpha$  and given sortings.

To put it more exactly, set  $W \in A$  is called  $W^+$ -set of the family  $\mathcal{Y} \in 2^A$  if  $\forall X, Y \in \mathcal{Y} (X \cap Y \neq \emptyset \Rightarrow W \cap X \cap Y \neq \emptyset)$  and  $W \in \bigcup_{X, Y \in \mathcal{Y}} (X \cap Y)$ . The family  $\mathcal{X} \in 2^A$  is finitely  $W^+$ -compact in the family  $\mathcal{Y} \in 2^A$  if, for any finite subfamily  $\mathcal{Y}' \in \mathcal{Y}$ , there is  $W \in \mathcal{X}$  which is  $W^+$ -set of  $\mathcal{Y}'$ .

Theorem. Let  $\mathcal{A}$  be a system of partial sorting of the set coordinated with the choice function  $\alpha$ . The family  $\mathcal{A}$  is finitely  $W^+$ -compact in the family of all the sorts  $\mathcal{X}_{\mathcal{A}, \bar{y}} = \{B_i : B \in \mathcal{B}, i \in I_B\}$ , and, for each  $X \in \mathcal{A}$ , there will be  $B \in \mathcal{B} : X \subseteq B$ . Then, the global for  $\mathcal{A}$  sorting exists.

The following example of representation of families  $\mathcal{A}$  and  $\mathcal{B}$  was employed for the scheduling method: set  $A$  is decomposed into subsets  $A = \bigsqcup_{B \in \mathcal{B}} B$ , the elements of the family  $\mathcal{B}'$  are sorted according to the structure of DM preferences; a representative is selected in each sort  $B_i$ , and the representative set  $B^0$  is sorted as well. Thus,  $A = 2^B, \mathcal{B} = \mathcal{B}' \cup \{B^0\}$ .

Evidently, such families satisfy the conditions of the theorem. For the case of  $A \in R^n$ , it is suggested to take domains of small diameter as  $B \in \mathcal{B}'$ , and to sort them by means of substitution coefficients.

The above method was applied to automatic scheduling of computer task packages, and, since package versions rather than individual tasks were considered as sorted objects, one was able to take into consideration mutual influence of tasks at package processing. Effectiveness of the scheduling software DISMUCS was estimated over ten packages of multi-step tasks presenting different requirements for basic computer resources (central processor time, main memory space, I/O time). All the packages were many times run of the ES-1050 computer (main memory 512 Kbytes) under both standard and DISMUCS scheduling. For each run, a new distribution of input priorities of package tasks was given.

The DISMUCS program enables on the average 12.8% increase of CP loading, 10% improvement of main memory operation, and 14.2% reduction of task package processing time.

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APPLICATION OF BINARY PROGRAMMING FOR TURBOJET ENGINE  
 FAULT DETECTION

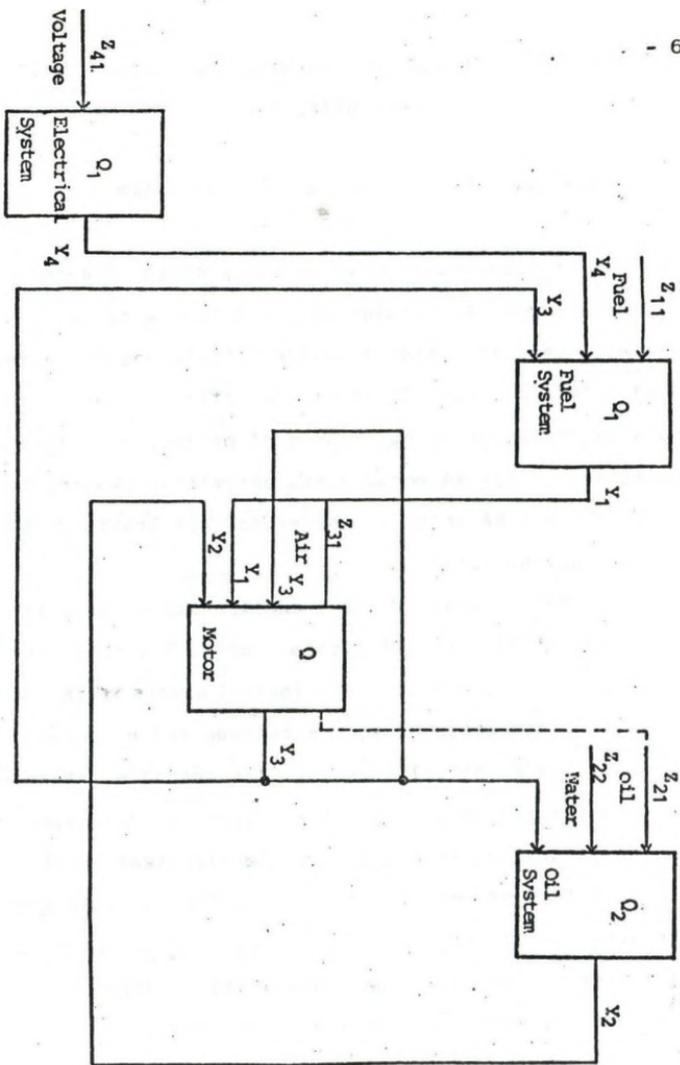
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In this paper we consider the problem of detecting faults in the NPT 401 gas turbine engine fitted with solid state electronic controls, using distillate fuels and operating in a horizontal attitude. The fault detection is formulated here as a binary programming problem of maximizing a likelihood function subject to constraints describing the input-output relations of the units of the engine and prohibiting noncausal illegal output {2}.

The NPT 401 engine under consideration consists of four main subsystems: the oil system, the fuel system, the electrical system and the motor. The logical models of the engine, as well as of its subsystems, are derived defining the output of each unit by a logical function. The method of representing real systems by these logical functions is described in {1}. In the figure,  $y_i$  is a 1/0 variable representing the good/bad of the output of unit  $i$  and  $z_{ij}$  is a 1/0 variable representing the good/bad of external input  $j$  of unit  $i$ . The  $Q_i$ 's are 1/0 variables representing the operativity (good/bad) of the various units, whose reliability is denoted by  $p_i$ .

The problem of fault detection is reduced to the problem of maximizing.

$$J(Q) = \prod_{i=1}^N p_i^{Q_i} (1-p_i)^{1-Q_i}$$



Logical model of the engine

subject to

$$y_i = Q_i F_i$$

$$\left( \prod_{k=1}^{n(\ell)} \bar{y}_{\ell k} \right) A_{\ell} = 0$$

In the above,  $N$  is the number of units,  $n(\ell)$  is the number of units on cycle  $\ell$ ,  $F_i$  is the logical function representing the input-output relation of operative unit  $i$ , and  $A_{\ell}$  is the coefficient denoting the accumulated effect on  $\bar{y}_{\ell 1}$  as it goes around cycle  $\ell$ .

The above mathematical programming problem has been solved by applying various test inputs and observing some of the outputs. The results have been verified in the Propulsion System Laboratory of the Hellenic Airforce Academy.

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## OPTIMAL CONTROL METHODS FOR POWER SYSTEM OPERATION

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The operation of large power systems is typically executed on three hierarchical levels, which are commonly named - regional control centre,

- district control centre,
- substations and power plants.

Power systems have grown continuously in size and complexity over years, while the technological development of control equipment has proceeded much faster during the same period. So every few years new solutions and concepts on process computer hardware and software, as well as on telecontrol systems became available. Today, control equipment used in most large power systems consists at least of two mainly different generations and/or different manufacturers. The introduction of new concepts of power system control at every hierarchical level has to take into account the following main goals:

- system control must not be interrupted,
- cooperation with existing control equipment,
- data reduction to specific needs,
- optimisation of control equipment availability and reliability,
- minimisation of implementation costs.

While planning the modernisation of the control system of the Viennese Electricity Board (WSE) all these facts had to be considered. On the level of the regional control centre, a new command room will be built, which allows parallel operation for the time of transferring control functions from the old to the new centre. A big redundant computer system with 32 bit technology will perform all the above mentioned control tasks.

On the level of the district control centres, several years of experience of using computers for information processing and presentation is available. The first computerised remote control system by WSE was installed in 1977, already with computers in each of the substations too. Since then real time data from the substations is available in the control rooms both on CRT and data loggers. Back up facilities are realised by a reserve system with mimic board and around 100 alarms per substation.

The substations of the network of the WSE are controlled from five district control centres. There are around ten substations controlled from each of these. A bigger substation typically consists of two or three voltage levels, that are 380 kV, 110 kV, 20 kV or 10 kV, with around 100 lines and 2000 alarms.

Besides the well known advantages of having exact data fast at hand, new problems did arise. Computers handle much more data and that much faster than man. With more and more substations connected to the system, 48 in 1985 with up to twelve per control room, the amount of data presented to the operator in case of bigger break-downs reached such proportions, that it could be of no help at all. The effect was even to the contrary, with the operator using the reserve system only, instead of the data presented by the computerised system.

Another problem is the reliability of big centralised information systems. You either have to double it completely or instal a reserve system with severely reduced information. The later means that no more detailed data is available in case of single computer system failures.

So when the opportunity arose to redesign the district control centres, steps were undertaken to overcome above problems. Instead of a single computer distributed system technology will be used. A local area network (LAN) with three computers, one for the communication to the substations, one for the communication to the load dispatch centre and one for detailed data presentation to the operator will be installed.

Instead of leaving out less important information altogether, all available data is still brought to the control room, but can now be sorted in several ways, regarding to the specific needs of the operator. It is also planned to store original data of break-downs that occur and use it for training purposes.

All these features and several others like help functions, alarm descriptions etc. are easily used with a comfortable man-machine- interface based on menue technique.

So after the first step of computerised control, that is including in the system as much information as is possibly needed, the next step will be the reduction to few but highly relevant data. This second step is not only the more important one, but also the much more difficult one. Excepting, transmitting and presenting of information is not any more enough. Concentrating and priority structuring becomes the main point in the use of computerised control methods in energy distribution systems.

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## OPTIMAL CONTROL OF AGE-STRUCTURED POPULATIONS

MARTIN BROKATE (F. R. G.)

In this talk we consider optimal control of an age-structured population. A typical special case is the following optimal control problem:

$$\text{Maximize } \int_0^T \int_0^{\infty} u(t) \rho(a,t) da dt$$

subject to the constraints (on  $0 \leq a, 0 \leq t \leq T$ )

$$0 \leq u(t) \leq u_{\max}$$

$$\left( \frac{\partial}{\partial a} + \frac{\partial}{\partial t} \right) \rho(a,t) + \mu(a,p(t)) \rho(a,t) + u(t) \rho(a,t) = 0$$

$$\rho(a,0) = \rho_0(a) \quad , \quad \rho(0,t) = b(t)$$

$$b(t) = \int_0^{\infty} \beta(a,p(t)) \rho(a,t) da$$

$$p(t) = \int_0^{\infty} \rho(a,t) da$$

Here  $\rho(a,t)$  denotes the population density with respect to age  $a$  at time  $t$ ,  $u(t)$  the harvesting effort (the control),  $b(t)$  the birth rate and  $p(t)$  the total population.

The author has proved a Pontryagin-type maximum principle, which can be applied to the problem above if  $\beta$  and  $\mu$  are differentiable functions of  $p$ .

The proof is based upon evaluation of abstract multiplier rules in Banach space.

If the control appears nonlinearly, the Pontryagin-type maximum condition is obtained for distributed controls  $u = u(a,t)$  ; otherwise, the differential version of the maximum principle holds.

We also comment consequences and special cases of the theorem.

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EXPERIMENTS IN CALCULATION OF GAME EQUILIBRIA  
USING NONSMOOTH OPTIMIZATION

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The paper deals with computational problems in interactive systems aiding analysis and decision making in game situations. New ideas of such interactive system in the case of conflicting goals of players has been proposed by Wierzbicki, 1983. The interactive system is considered as a tool for analysis of conflict processes and for leading special mediation procedure in the game. Analysis of conflict is based on selection of satisficing game equilibria and on definition of constructive and destructive behaviour of players. The selected equilibrium treated as possible status quo point can also be basis for conducting the negotiations. From mathematical point of view, selection of game equilibria boils down to maximization of an appropriate function over the set of Nash equilibria. That is optimization of a nondifferential function over a nonconvex set. Fishing game /Wierzbicki, 1983/ serves as an example in computational experiments reported in the paper. The experiments deal with calculation of satisficing game equilibria for assumed achievement functions and aspiration levels of players. Nonsmooth optimization algorithm, elaborated by Kiwiel, 1984,

has been utilised in the experiments. Penalty function approach has been used to fulfil the Nash conditions. Calculations for different penalty coefficients, assumed accuracy of optimization, different start points and different aspiration levels of players have been performed.

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Antisymmetric Matrices, Staircase Functions and LP

by

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This paper outlines a proof of the Theorem that, for any real  $A = -A^T$ , there exists an  $x \geq 0$  such that  $Ax \geq 0$  and  $x + Ax > 0$ . It shows that, if  $x(\theta)$  is such an  $x$  corresponding to the matrix  $A + \theta(pq^T - qp^T)$ , where  $\theta$  is a scalar parameter,  $p, q \in R^n$ ,  $p, q \geq 0$  and  $p^T q = 0$  then if  $r(\theta) = q^T x(\theta) / p^T x(\theta)$ ,  $r(\theta)$  is a "staircase function" of  $\theta$  and is multi-valued on a finite set of values of  $\theta$ .

The use of these "staircase functions" not only facilitates the proof of the Theorem but enables several (somewhat inefficient at present) algorithms for solving the general LP problem to be constructed.

Finally, it is shown that Farkas's Theorem, and other duality theorems, are but special cases of the above Theorem.

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## STABILIZATION OF THE SECANT METHOD VIA QUASI-NEWTON APPROACH

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We consider the system of nonlinear equations

$$f(x) = 0, \quad (1)$$

where the mapping  $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$  is sufficiently smooth. It is known, that the sequential  $(n+1)$ -point secant method for solving (1) can be written as

$$x_{k+1} = x_k - J_k^{-1} f(x_k), \quad (2)$$

where the matrix  $J_k \in \mathbb{R}^{n \times n}$  approximates  $f'(x_k)$  and is updated by some rank-one quasi-Newton formula.

In [1] a method of the secant type for system (1) with symmetric Jacobian matrix  $f'(x)$  was proposed. It is shown there, that the sequential  $(n+1)$ -point symmetric secant method has the form (2), where  $J_k$  is updated by some rank-two quasi-Newton formula.

The main advantage of the traditional secant method as well as its symmetric variant is that they have a superlinear convergence evaluating the mapping  $f$  only once at each iteration. At the same time their common disadvantage is instability connected with the fact that for  $n > 1$  the secant plane may not converge to the tangent plane. A danger of instability appears when the directions  $\{\Delta x_i\}_{k-n}^{k-1}$  where  $\Delta x_i = x_{i+1} - x_i$ , tend to be linearly dependent. For this reason a superlinear convergence is proved usually under the assumption that there is  $\delta > 0$  such that

$$\left| \det(\Delta x_{k-1}/\|\Delta x_{k-1}\|, \dots, \Delta x_{k-n}/\|\Delta x_{k-n}\|) \right| \geq \delta \quad \forall k \geq 0, \quad (3)$$

but as a rule this inequality is violated in practice.

In the most known stable variants of the secant method some of the vectors  $\{\Delta x_i\}_{k-n}^{k-1}$  are substituted by some of another ones so that the obtained vectors satisfy an inequality analogous to (3). The approximation  $J_k$  is completely determined by the obtained  $n$  vectors. It demands additional (with respect to the sequential

(n+1)point secant method) evaluations of the mapping  $f$ .

Some symmetric and nonsymmetric stable methods of the secant type are proposed in [2], where only part of the vectors  $\{\Delta x_i\}_{k-n}^{k-1}$  is used for computing  $J_k$ , i.e. the vectors  $\Delta x_i$  with the indices  $i=i_1, i_2, \dots, i_l$  where  $i_1=k-1$  and  $l \leq n$  ( $l$  depends on  $k$ ) are used. For stabilization, these indices are chosen to satisfy the inequality

$$\det(\Gamma(\Delta x_{i_1}/\|\Delta x_{i_1}\|, \dots, \Delta x_{i_l}/\|\Delta x_{i_l}\|)) \geq \epsilon^2 \quad \forall k \geq 0,$$

where  $\Gamma$  is the Gram matrix. The matrix  $J_k$  is updated by some rank-one (in nonsymmetric case) and rank-two (in symmetric case) quasi-Newton formulas so that it is determined partly by  $l$  vectors mentioned above and partly by the previous approximation  $J_{k-1}$ . The methods [2] need not additional evaluations of the mapping  $f$ .

The aim of the present paper is to discuss some known stable variants of the secant method and to propose some new variants based on [2].

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Abstract.

Rainer E. Burkard: "Assignment Problems: Recent Solution Methods and Applications".

If person  $P_i$  ( $i=1,2,\dots,n$ ) can handle jobs  $J_{i_1}, \dots, J_{i_k} \subset \{J_1, \dots, J_n\}$  we can ask: Can every job be assigned to a person such that every person handles exactly one job? This question, well-known in combinatorics as "marriage-problem" admits also a weighted version leading to "optimal" assignments of jobs to persons, where optimal refers to a given objective function.

We shall discuss recent computer implementations for solving such problems with a linear objective function. Some general ideas will be outlined which have led recently to a drastical reduction of the computer time spent. Further we will mention some special cases (depending on a special structure of the cost matrix) which can be solved in  $O(n^2)$  time by special purpose algorithms.

Linear assignment problems play an important rôle as sub-problems in many more involved situations. We shall show how an optimal time-slot assignment for a time division multiple access system (TDMA-system) with fixed  $2n$  switch modes can be determined via a linear assignment problem. Such TDMA-systems occur for example in connection with communication via satellites. Another recent application which will be commented on concerns the determination of certain structural properties of

large-scale linear systems.

A mathematically more general formulation of the objective function leads to (linear) assignment problems with objectives, such as bottleneck objectives, lexicographical objectives, time-cost objectives and many others. These problems can be treated in more or less the same way as the classical linear sum assignment problem.

If we consider mutual interdependencies between the elements which are to be assigned to each other, we arrive at the quadratic assignment problem, a model of wide use in different areas of applications:

To be more specific, let us consider a locational decision problem. For erecting  $n$  new buildings (offices, dormitories, shops, etc.) on a campus  $n$  possible sites are available. Let  $a_{ik}$  be the distance between site  $i$  and site  $k$  and let  $b_{jl}$  be a measure how many persons per week walk from a building with function  $j$  to a building with function  $l$ . Since  $a_{ik}b_{jl}$  measures the total walk length if building  $j$  is erected on site  $i$  and building  $l$  is erected on site  $k$ , we want to find an assignment  $\varphi$  of buildings to sites such that

$$\sum_{i=1}^n \sum_{k=1}^n a_{ik} b_{\varphi(i)\varphi(k)}$$

attains a minimal value.

We shall describe in which way QAPs occur in backboard wiring problems, in keyboard design as well as in the analysis of

chemical reactions or at sequencing a team in a relay race. Since QAPs are NP-hard, only heuristic algorithms are available for solving larger sized problems (size  $\geq 15$ ). We shall describe some bound techniques which may be used in optimal or suboptimal enumeration algorithms and focus on a newly developed simulation approach for solving QAPs. Moreover we will point out some special cases which admit a solution in polynomial time.

for any  $\lambda \geq 0$ , then  $y$  is called an infinite nonincreasing direction of  $f(x)$  (at the point  $x$ ); if there exists  $x \in R^n$  such that  $f(x+\lambda y) < f(x) < +\infty$  for any  $\lambda > 0$ , then  $y$  is called an infinite descent direction of  $f(x)$  (at the point  $x$ ).

Definition 3. Let  $y \in R^n$ ,  $y \neq 0$ . If there exists  $x \in C$  such that  $x+\lambda y \in C$  for every  $\lambda \geq 0$ , then  $y$  is called an infinite direction (at the point  $x$ ) in  $C$  or an infinite feasible direction of (P) (at  $x$ ). If an infinite feasible direction  $y$  of (P) is an infinite nonincreasing (or infinite descent) direction then  $y$  is called an infinite nonincreasing (or infinite descent) direction of (P).

Let  $x^{(1)}, x^{(2)}, y \in R^n, y \neq 0$ . If

$$x_i^{(1)} - x_i^{(2)} \begin{cases} \geq 0 & \text{if } y_i > 0 \\ \leq 0 & \text{if } y_i < 0 \end{cases}$$

we write  $x^{(1)} \preceq x^{(2)}$ .

Definition 4. Let  $\phi(x)$  be a real-valued function (include generalized number  $\infty$ ) and  $y \in R^n, y \neq 0$ . If for any  $x^{(1)}, x^{(2)} \in R^n$  and  $x^{(1)} \preceq x^{(2)}$  we have  $\phi(x^{(1)}) \geq \phi(x^{(2)})$ , then  $\phi$  is called a  $y$ -nondecreasing function on  $R^n$ .

Theorem 1. Suppose (P) has feasible solutions, but hasn't any optimal solution and (DP) has a strong feasible solution  $(u^0, x^0)$ . Then for any infinite nonincreasing direction  $y$  of (P) we have

- (I) All  $(u^0, x^0 + \lambda y)$  are strong feasible solutions of (DP) for any  $\lambda \geq 0$ ;
- (II) For any  $\lambda \geq 0$  we have

$$f(x^0 + \lambda y) = f(x^0) < +\infty, u^{0T} g(x^0 + \lambda y) = u^{0T} g(x^0) < +\infty.$$

Corollary 1. Suppose (P) has feasible solutions, but hasn't any optimal solutions, and there exists an infinite descent direction of (P) such that  $y$  is an infinite descent direction of  $f(x)$  for any  $x \in R^n$  and  $f(x) < +\infty$ . Then (DP) hasn't any strong feasible solutions.

Corollary 2. Suppose (P) has feasible solutions and (DP) has a strong feasible solution  $(u^0, x^0)$ . If one of the following two conditions is satisfied;

- (I)  $M(x^0) = \{ t | f(x^0 + ty) = \min f(x^0 + \lambda y) \}$  is bounded;
- (II)  $N(x^0) = \{ t | u^{0T} g(x^0 + ty) = \min_{\lambda \geq 0} u^{0T} g(x^0 + \lambda y) \}$  is bounded,

then (P) has an optimal solution.

$\nabla f(x)$  denotes the gradient of  $f(x)$  at  $x$ .  $G(x)$  denotes the Hessian of  $f(x)$  at  $x$ . Let

$$J(x) = \{i \mid \delta_i(x) \neq 0\}, \quad x \in R^n,$$

where  $\delta_i(x)$  is the characteristic root of  $G(x)$ .

Definition 4. Let  $f(x) \in C^2$ , that is, with continuous second partial derivatives on  $R^n$ , and  $y \in R^n$ ,  $y \neq 0$ . If there exists  $M > 0$  and  $\delta > 0$  such that  $\delta_i(x) \geq \delta$  when  $x \notin My$ ,  $i \in J(x)$ . Then  $f$  is called a singular convex function in the direction of  $y$ .

In the case of linear constraint,  $C = \{x \mid b - Ax \leq 0\}$ , where  $b \in R^m$ ,  $A$  is an  $m \times n$  matrix, we denote the primal and dual programs by  $(P')$  and  $(DP')$  respectively.

Theorem 2. Suppose  $(P')$  has feasible solutions, but hasn't any optimal solution.  $f(x) \in C^2$  and  $f$  is a singular convex function in any infinite descent direction  $y$  of  $(P')$ . Then  $\inf_{x \in C} f(x) = -\infty$ .

From Theorem 2 we can get some well-known existence theorems on extreme value for linear and quadratic convex functions.

Theorem 3. Suppose  $(P')$  has feasible solutions, but hasn't any optimal solution, and  $f(x)$  has finite values on  $C$ . For any infinite descent direction  $y$  of  $(P')$  there exists  $M > 0$  such that whenever  $N \subseteq My$ ,  $D^+ f(x, y)^*$  is a  $y$ -nondecreasing function at points  $x$  where  $f(x)$  has finite value. Then  $(DP')$  hasn't any strong feasible solution.

From Theorem 3 we can get some important existence theorems of the optimal solution of dual program. For example, if  $f(x)$  is a separable function and the primal program has feasible solutions, and it hasn't any optimal solution, then its dual program hasn't any strong feasible solution; if the primal program has feasible solutions and its dual program has a strong feasible solution, then they both have optimal solutions and their optimal values are equal.

\* Note: the right-sided derivative of  $f(x)$  in the direction of  $y$  at  $x$ .

Optimal Feedback Control With Model Uncertainty

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ABSTRACT

The main task of a feedback controller is to provide satisfactory performance of the controlled system in the presence of uncertainty. This requirement was the original motivation for the development of feedback systems, i.e., feedback is required only when system performance cannot be achieved because of uncertainty in system characteristics. Uncertainty may be modeled in many different ways; two that seem easy to use are either as external inputs (e.g., disturbance, measurement noise, etc.) or as perturbations to the nominal model (modeling errors). The performance of a system will be measured in terms of the behavior of the outputs or errors. The assumptions which characterize the uncertainty, performance and nominal model determine the analysis and synthesis techniques to be used.

Although optimal control theory (and Wiener-Hopf-Kalman-Bucy Optimal Control Theory) have been dominant paradigms for the past 20 years, their inadequacies with respect to uncertainties and infinite dimensional systems (except additive noise) has renewed the interest in the classical control "frequency-domain-approach" in recent years. This direction provides useful design tools, including singular value analysis and loop-shaping techniques for systems with linear dynamic models. However, some of the methods which are based on "singular values" still require rather restrictive assumptions on the uncertainties which are essentially modeled as a single norm-bounded

"unstructured perturbations". A structured singular value (SSV) approach has been introduced to cover this deficiency where the uncertainty is in structured block-diagonal form. The SSV approach, together with  $H_{\infty}$  optimization theory, allows one to handle some of the performance and uncertainty aspects of feedback control for linear models in a unified framework. Recent research in robust multivariable control theory has provides some tools for analyzing controllers for uncertain systems and has led to certain techniques for synthesis. We shall see how practical feedback controller design can sometimes be done using this approach. Examples will be used to illustrate the procedures.

The SSV is still much too conservative for many important classes of perturbations, e.g., real-parameter variations, which is a common uncertainty in linear models of a system. Techniques for handling real parameter variations in the analysis and design of feedback controllers will also be addressed. In addition to simultaneous stabilization of all possible such parametric systems, we seek techniques for optimization.

SIMULATION MODEL FOR DETERMINATION OF OPTIMAL RESERVOIR  
CAPACITY IN PRODUCTION SYSTEMS

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Stocks of raw materials, in chemical industry being very often in form of fluids accommodated in reservoirs, have to enable the stable production. The economic goal is to minimize the total costs of reservoir system construction and operation. In the modeling and analysis of various aspects of inventory systems, the simulation technique has shown to be appropriate<sup>1</sup>.

In this paper we shall examine the production system consisting of the arrival of tank cars (or tank trucks) with the fluid, the reservoirs for fluid storage and the production which takes the fluid from the reservoirs as the raw material. The tank cars with the fluid can be of various capacities and types (tank cars or tank trucks). Tank cars arrive to the reservoir area where they wait for unloading. On the other side, the production line has some dynamic of demand for fluid from the reservoirs. The interarrival time of tank cars, the capacity and type of tank cars and the dynamic and intensity of demand for fluid by the production line are of stochastic nature with various probability density functions (theoretical or empirical ones). Described characteristics are corresponding to systems exhibiting large stochastic fluctuations of demands for the raw material and of realization of raw material purchasing.

The determination of optimal capacity of reservoirs can be represented as the problem of minimization of sum of costs of reservoir system construction and costs of waiting of tank cars for unloading the fluid into the reservoirs (waste time). The reservoir volume can be constructed as the set of reservoir modules of given magnitude. To enable the comparison of costs, the reservoir construction cost is reduced to the cost per some time period (e.g. one year) by taking into account the reservoir lifetime. On the other hand, the future costs of tank cars waste time are discounted on the present value by taking into account the time value of money, and are summarized over the same time period (one year). Costs of tank cars waste time depend upon the type and capacity of tank cars.

The basis for seeking the optimum reservoir capacity is the simulation model of arrival, storage and consumption of raw material in production systems<sup>2</sup>. The model is programmed in the GPSS simulation language and implemented on the UNIVAC 1100 computer. The model verification and validation was performed by using tests for program logic correctness, faithfulness of input stochastic variables distributions generation, and comparison of the model reservoir contents, tank cars queues characteristics and system costs with the real system data<sup>2</sup>.

The simulation experiments with the model of the system were performed with the aim of seeking the optimum number of reservoir modules (i.e. for the reservoir system total capacity) with given unit costs, dynamic and structure of tank cars arrivals and dynamic and intensity of production requests for fluid. The elimination of initial bias was taken into account by preliminary simulation of two years of system operation after starting from the empty state, and discarding the accumulated model statistics. For the comparison of alternative configurations of reservoir capacities under the alternative queueing disciplines, we use the correlated sampling variance reduction technique<sup>3</sup>.

The results of the simulation experiments have shown that the optimum reservoir capacity significantly depends upon the queueing discipline used. The first queueing discipline used was one without priorities, while in the second one tank cars types and capacities with higher waste time costs had the greater priorities. The queueing discipline with priorities gives the less total amount of optimum reservoir capacity.

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OPTIMIZATION PROBLEMS FOR TWO-STAGE PROCESS  
OF RESOURCE ALLOCATION

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Consider the process of resource allocation consisting of two successively realized stages. The efficiency  $F(y, t, s)$  of the process depends on the strategy  $y$  of the second stage, an unknown parameter  $t$  and its estimate  $s$  received after the first stage. The strategy  $x$  of the first stage makes information about  $t$  more precise. Values  $x, y, t$  are vectorial. Quantities of the resources intended to accomplish the first and the second stages,  $C$  and  $D$  respectively, depend on corresponding strategies, and their sum does not exceed  $K$ :  $C(x) + D(y) = K$ . It is necessary to choose a manner of the resource allocation between and within the stages.

It may be one of two versions of information for each stage. Before the beginning of the stage the parameter  $t$  is random or uncertain. Before the first stage either a priory frequency distribution function  $p(t)$  or the domain  $T$  of the permissible values of  $t$  is known. Before the second stage either a posterioy frequency distribution function  $p'(t; s, R(x, s))$  where  $s$  and  $R$  are the distribution parameters ( $s$  is the mathematical expectation,  $R$  is the correlation matrix) or the domain  $T'(s, R(x, s))$  of the permissible values of  $t$  where  $s$  determines location of the centre and  $R$  does size of the domain  $T'$  becomes known. In the both cases the function  $R(x, s)$  characterizing the efficiency of the strategy  $x$  is known beforehand, and  $s$  is fixed after the first stage. Information about  $s$  which is available before the first stage is restricted by a priory data concerning  $t$ .

Depending on the character of information about  $t$  before the first and the second stages the following four versions of the problem are possible:

$$\max_{x \in X} \int \max_{y \in Y_x} \left( \int F(y, t, s) p'(t; s, R(x, s)) dt \right) p(s) ds,$$

$$\max_{x \in X} \int \max_{y \in Y_x} \min_{t \in T'(s, R(x, s))} F(y, t, s) p(s) ds,$$

$$\max_{x \in X} \min_{s \in T} \max_{y \in Y_x} \int F(y, t, s) p'(t; s, R(x, s)) dt,$$

$$\max_{x \in X} \min_{s \in T} \max_{y \in Y_x} \min_{t \in T'(s, R(x, s))} F(y, t, s).$$

Here  $X = \{x: C(x) \leq K\}$ ,  $Y_x = \{y: C(x) + D(y) \leq K\}$ , and the variables in all the integrals vary over their natural domains.

On solving these problems having in mind the multidimensionality of the vectors and integral form of the criteria of optimality it is worth-while to use the methods of stochastic optimization.

As an example of practical realization of the described model an economic-mathematical model of search for mineral deposits (oil or gas) can be cited. For this model numerical calculations have been carried out. In this case the parameter  $t$  characterizes location, shape and size of plots under consideration; the first stage is geophysical prospecting, its strategy  $x$  determines the intensity of use of different geophysical methods on different plots; the second stage is exploratory drilling, quantities and location of bore holes depend on its strategy  $y$ ; the value  $F$  is prospected quantity of minerals.

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## Nonlinear Parameter Estimation by Global Optimization

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A method for finding the global minimizer of a nonlinear parameter estimation problem is presented.

Our task was to fit lumped parameter models of the respiratory system to the subject's measured mechanical impedance. We use the following objective function :

$$F(\underline{x}) = 100 \sqrt{\frac{1}{m} \sum_{i=1}^m \frac{|z(f_i) - \tilde{z}(f_i, \underline{x})|^2}{|z(f_i)|^2}}$$

where  $z$  and  $\tilde{z}$  are the measured and modelled complex impedances, respectively,  $\underline{x}$  is the vector of model parameters and the  $f_i$ -s are frequency points. The constraints are represented by simple bounds :

$$a_j \leq x_j \leq b_j ; \quad a_j, b_j \in \mathbb{R}, \quad j = 1, 2, \dots, n.$$

Since this relative least-mean squares problem is in general not unimodal, we applied a global optimization method to find the best fitting model. The use of such

a procedure resulted in a very effective parameter estimation program.

In our algorithm a sampling technique is applied to select promising starting points for the local optimization. A clustering procedure ensures efficiency of repeated local searches. Since our objective function is of the sum-of-squares type, we implemented a combination of Levenberg-Marquardt procedure and random walk to locate local minimizers.

The results of procedure tests and numerical experiences are reported.

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BALANCED REALISATION  
FOR  
DISCRETE-TIME INFINITE DIMENSIONAL SYSTEMS

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For finite dimensional systems the class of balanced realisations is defined as those whose controllability and observability Grammians are both equal of the same positive diagonal matrix. The diagonal entries are in fact the singular values of the Hankel operator of the system and contain essential information about its behavior. Balanced realisations have special structural properties and their truncations usually provide a good reduced order model of the original system. In this contribution we examine the concept of balanced realisations for stable discrete-time infinite-dimensional systems and investigate the possibility of reduced order models based on balanced realisations.

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SYNTHESIS OF NEAR TIME-OPTIMAL CONTROL AND REALIZATION  
OF CONTROLLERS FOR NONLINEAR OSCILLATING PLANTS

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The paper studies the optimal motion of nonlinear oscillating plants described by Duffing and Van der Pol equations. The time-optimal control design of such plants is a hard task as a rule. There are a number of peculiarities concerning these systems. For certain values of the system's parameters there start relaxational oscillations which at given circumstances turn to stochastic oscillations. When the control  $u = 0$ , the studied system has a single periodic solution, all solutions converging into it, no matter what initial conditions may be. To keep the system in its equilibrium state, a suitable  $\mu$  is needed. It is chosen in such a way as to keep the system's motion along trajectories, corresponding to the time-optimal transient processes of the system.

Depending on the dynamic structure of the plant, the configuration of the switching line is more or less complicated. For practical realization are needed control structures which perform a division of the state plane into areas that approximately (with a defined before-hand error) correspond to the areas of optimal division. So the areas are determined through a piecewise - linear approximation of the optimal switching lines.

The paper suggests control structures performing piecewise - linear approximation of the optimal switching lines of plants, described by the equations:

$$\ddot{x} + \mu (x^2 - 1) \dot{x} + x = u ; \text{ (Eq. of B. Van der Pol) } \quad (1)$$

$$\ddot{x} + r (\mu x^2 + 1) x = u , \text{ (Eq. of G. Duffing), } \quad (2)$$

where  $r$  and  $\mu$  are parameters;  $|u| \leq 1$ .

The control structures are set up of elements with linear and relay characteristics. The control design is based on the method of the equivalent signum-functions. The paper

discusses some of the control structures given in applications. These structures are of the so-called parallel type, i.e.

$$u(x) = \text{sign} \left[ \sum_{i=1}^m c_i \text{sign}(c_{i0} + c_{i1}x_1 + c_{i2}x_2) \right] \quad (3)$$

or

$$u(x) = \text{sign} \left[ x_2 + \sum_{i=1}^k f_i \text{sat}(a_i x_1) \right], \quad (4)$$

where  $c_i$ ,  $c_{ij}$  ( $j = 0, 1, 2$ ),  $f_i$ ,  $m$ ,  $a_i$  are parameters;  $x = [x_1, x_2]^T$ ,  $k = (m - 1)/2$ .

Using control structures (3) and (4) we can design controllers for different cases of nonlinear oscillating second order plants from the engineering practice.

The paper illustrates in applications the suggested approach for the design of a closed-loop (on the state coordinates) near-optimal system, involving plants of type (1) and (2).

The presented near-optimal control structures are organized on analog, as well as digital devices-microprocessors and microcomputers. The question of constructing a microprocessor near-optimal control device is also discussed. The hardware capabilities of the device are determined by an 8-bit microprocessor MC6800, the necessary RAM/ROM memory and the input/output drivers.

The functioning of the control device and the necessary software were modelled on a microprocessor development system "Tektronix 3002". The device works in "real time" and "learning".

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THE BENCHMARK CHRONOLOGICAL SIMULATION MODEL, A NEW TOOL FOR A BETTER UNDERSTANDING OF THE ECONOMIC AND PHYSICAL BEHAVIOUR OF ELECTRICAL GENERATING SYSTEMS

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#### INTRODUCTION

Through the cooperative effort of several utility companies, a computer program has been developed to simulate the commitment and dispatch activities that control electric power supply systems.

The simulation tool is a computer program called "BENCHMARK", sponsored by the Electric Power Research Institute of Palo Alto, California and developed by the following universities and electric power utilities: Commonwealth Edison, Duke Power, Northeast Utilities, N.V. SEP (Dutch Electricity Generating Board), Southern Company, Tennessee Valley Authority, Union Electric, Ohio University and University of Tennessee at Chattanooga. It is intended for study of both physical and economic behaviour of power systems.

BENCHMARK can be used in a deterministic mode, in which it is assumed that the availability (or partial availability) of each generating unit is known, or in a Monte Carlo mode, in which outages or partial outages are treated as random events. In both the deterministic and the Monte Carlo mode, it is assumed that the hourly load is known throughout the time interval being simulated. Hydroelectric generation is allocated over a sequence of time intervals each of which may be up to five weeks in duration. Pumped storage and all other generation is dispatched on an hourly basis, one week at a time.

#### THE NEED FOR CHRONOLOGICAL SIMULATION

Attention to the chronological or "time-linked" behaviour of a power supply system becomes imperative when the role of storage units is considered, when ramp-rate limitations affect system response, or other dispatch considerations such as minimum up time or minimum down time are brought into play. Dispatch based on load duration curves suppresses chronology and therefore cannot strictly represent commitment decisions and will produce biased results. Ramp-rate limitations may require peaking units such as combustion turbines or pumped-storage units to be operated whenever base-load units cannot respond rapidly enough to sudden load variations. If ramp-rate limitations were overlooked, the usage of the peaking units or the advantages of dispatcher-controlled load management would then be underestimated. Actual operation of storage units requires tracking of the amount of stored energy available, i.e., the water level in a pumped-storage plant. This is readily accomplished in a chronological simulation, but not in the load duration domain. If this limitation is not observed, the usefulness of the storage plant could be easily overestimated.

#### POTENTIAL APPLICATIONS

- load tracking
- non-dispatchables and cogeneration
- the effect of interruptions of fuel supply or of water supply for hydroelectric plants
- the possible impact of inter-utility power purchases or sales, either short term or long term
- the effect of the fuel cost adjustment on electric power rates
- the effect on the supply system of "load management"
- the cost or benefit of an adjustment of generator maintenance if unexpected opportunities or needs arise
- the benefit of a change in operating characteristics of equipment

## AN ALGORITHM IN CONSTRAINED OPTIMIZATION

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In this paper we shall be concerned with finding solutions to the nonlinear programming problem

(1)  $\min \{ \varphi(x) \mid x \in X \}$ , where  $X = \{x \in D \mid f_j(x) \leq 0, j \in I_0 = \{1, \dots, m\}\}$ , and the functions  $\varphi, f_j: D \subset \mathbb{R}^n \rightarrow \mathbb{R}, j \in I_0$  are continuously differentiable on the open convex set  $D$ .

We shall consider iterative algorithms generating sequences of points  $\{x_k\}$  of the form

(2)  $x_{k+1} = x_k - \alpha_k p_k, x_{k+1} \in X, \varphi(x_k) \geq \varphi(x_{k+1}), k=0, 1, \dots$ , with  $p_k \in \mathbb{R}^n, \alpha_k > 0$  to solve (1).

Under the certain assumptions about the direction vector  $p_k$  we shall separately concern a step-size algorithm for finding a step-size  $\alpha_k$ .

Now we shall define the step  $\alpha_k^*$  obtained by modified Curry-Altman's algorithm (see [1]) (for unconstrained case):

$\alpha_k^* = 0$  if  $\langle \nabla \varphi(x_k), p_k \rangle = 0$ ; otherwise  $\alpha_k^* \in J_k$ , where  $J_k$  is the first interval of positive solutions of the inequality

$$\langle \nabla \varphi(x_k - \alpha p_k), p_k \rangle > \sigma(\langle \nabla \varphi(x_k), p_k \rangle),$$

where  $\sigma: [0, \infty) \rightarrow [0, \infty)$  is a forcing function (see [2]) such that  $\sigma(t) < \delta t$  for every  $t \geq 0$  and some  $0 < \delta < 1$  and  $\alpha_k^* \geq q \bar{\alpha}_k$  for some  $0 < q < 1$  and every  $k$ , where  $\bar{\alpha}_k$  is the smallest positive solution of generalized Curry-Altman's equation

$$\langle \nabla \varphi(x_k - \alpha p_k), p_k \rangle = \sigma(\langle \nabla \varphi(x_k), p_k \rangle).$$

Finally, let  $\alpha_k^*$  be a step defined by

$$\alpha_k^* = \sup \{ \alpha > 0 \mid x_k - \alpha p_k \in X \text{ for all } t \in [0, \alpha) \}.$$

We define  $L^0$  to be the connected component of the level set  $L = \{x \in D \mid \varphi(x) < \varphi(x_0)\}$  containing  $x_0 \in X$ , and for any

$x \in X$ ,  $\epsilon > 0$  we define the index set  $I(x, \epsilon) = \{j \in I_0 \mid -\epsilon < f_j(x) < 0\}$ .

The assumptions about  $X$ ,  $L^0 \cap X$  and  $p_k \in R^n$  at  $x_k \in L^0 \cap X$  are the same as in [2].

**Theorem 1.** (A modification of Curry-Altman's step-size algorithm in constrained case) Suppose that  $\varphi: D \subset R^n \rightarrow R$  is a continuously differentiable function on the open convex set  $D$ . Assume that  $L^0 \cap X$  is compact. Define  $\alpha_k$  by

$$(3) \quad \alpha_k = \min\{\alpha'_k, \alpha^*_k\}.$$

Then: 1)  $x_{k+1} = x_k - \alpha_k p_k \in L^0 \cap X$ ;

$$2) \quad \varphi(x_k) \geq \varphi[\lambda x_k + (1-\lambda)x_{k+1}] \geq \varphi(x_{k+1}) \quad \text{for all } \lambda \in [0, 1];$$

3)  $\varphi(x_k) - \varphi(x_{k+1}) \geq F(\langle \nabla \varphi(x_k), p_k \rangle, v_k, \epsilon_k)$ , where  $F$  is a forcing function of three variables which depends only on  $L^0 \cap X$ .

**Theorem 2.** Consider the iteration (2), where at  $x_k, \epsilon_k$  is chosen by the algorithm for  $\epsilon$  (see [2]),  $p_k \in R^n$  is any direction such that with  $\alpha_k$  obtained from (3) we have that  $x_{k+1} \in L^0 \cap X$ . Let

$$v_k = \| P_{q_k}(x_k) \nabla \varphi(x_k) \|, \quad q_k = \text{kard } I(x_k, \epsilon_k).$$

( $P_{q_k}$  is the orthogonal projection from  $R^n$  onto the orthogonal complement of the column space  $N_{q_k} = (n_{q_1}, \dots, n_{q_k})$ , where  $n_j(x) = [\nabla f_j(x)]^T, j \in I_0$ .) Then, if the sequence  $\{p_k\}$  has the property that  $\| P_{q_k}(x_k) \nabla \varphi(x_k) \| \rightarrow 0$  when  $\langle \nabla \varphi(x_k), p_k \rangle \rightarrow 0, k \rightarrow \infty$  and if the set  $\{x \in L^0 \cap X \mid \| P_q(x) \nabla \varphi(x) \| = 0, q = \text{kard } I(x, 0)\}$  consists of only one point  $x^*$ , it follows that  $x_k \rightarrow x^*, k \rightarrow \infty$ .

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AN EFFICIENT ALGORITHM FOR THE OPTIMAL OPERATION OF A CASCADE  
REACTOR

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The optimal start-up and regulation of a cascade reactor is faced. The satatic optimal configuration of the cascade, with distributed feeding, has been previously studied.

Firstly, the stability problem (in the sense of Lyapunov) of the model of the overall cascade is solved. The model considered and identified is a nonstructured one, based on the biomass, substrate and ethanol concentrations. The rates of reactions are modelled so that it can assume both ethanol and substrate inhibition, in relation to the observed results. It is a complex model, given by (1) or (2), this being due both to the nonlinearity of the state equations and to the high number of variables.

$$\dot{x} = f(x, u) \quad x \in \mathbb{R}^{3N}, \quad u \in \mathbb{R}^{2N}, \quad N = \text{nber of reactors} \quad (1)$$

$$\dot{x}_i = f_i(x_i, x_{i-1}, u_i, \sum_{j=1}^{i-1} u_j) \quad x_i \in \mathbb{R}^3, \quad u_i \in \mathbb{R}^2, \quad i=1 \text{ to } N \quad (2)$$

The overall stability, due to the particular structure of the system, is reducible to the stability of each reactor.

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Then, by reducing the model of each reactor from three to two state-variables, and by using phase-plane representation, the conditions on which the stability of each reactor is assured are deduced. Stable steady-states are two: washout and production one. The initial conditions and the overall specific dilution rate determine which one is obtained.

Then, the Hamiltonian formulation of minimum-time start-up and regulation of the production steady-state is analysed and solved. The time-optimal problem, due to the high number of variables, can not be solved directly. It is approached by solving the two-point boundary-value optimization problem with a quadratic criterium on the trajectory deviation and command effort and with decreasing final time.

The solution of the two-point boundary-value problem is carried out by subdividing the time interval into  $M$  subintervals. The particular stair-case structure of the Hamiltonian allows the implementation of a hierarchical method of sequential decomposition (in space) that, starting from an initial trajectory, finds iteratively the optimal solution.

Results of the method, which show a good convergence, are presented and discussed. Important savings in time can be obtained by optimal control, which is of a high practical interest due to the slow evolution of this kind of process.

Let it be stressed that this method decomposes the overall problem into a series of three-dimensional subproblems and so the possibility of on-line control by microcomputers should be considered.

## EVOLUTIONARY METHOD IN DISCRETE PROGRAMMING

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At present there exists a need to solve tasks for which there are no algorithms or the latter one are noneffective. The large-scale discrete programming, scheduling and some nonanalytical problems belong to the class of such tasks. The paper deals with an evolutionary method which can be applied to solve some of above mentioned problems. In literature the term "evolutionary" refers to more or less stochastic algorithms. The aim of the paper is to present formal description of the method and its applications in discrete programming. As an example, the large-scale task to optimize investments in chemical industry is presented.

The evolutionary method can be applied to such optimization tasks which can be transformed to the form suitable for two-level algorithms. Thus the task: to find  $v^* \in V$  such that

$$f(v^*) = \min_{v \in V} f(v)$$

can be equivalent: to find  $(m^*, w^*) \in M \times W = V$  such that

$$f(m^*, w^*) = \min_{(m, w) \in V} f(m, w) = \min_{m \in M} [\min_{w \in W_m} f(m, w)]$$

where  $W_m \subset W$  is a set determined by means of fixed  $m$ .

Thus the method utilizes a possibility of the division of searched variables into two groups and two-level solving. The values  $m \in M$  are generated by stochastic generator on a high level.

Then the variables  $w$  are computed on a low level as a result of a deterministic procedure, where  $m$  are fixed parameters.

Parameters of probability distributions are modified after each iteration on the basis of information obtained from earlier ones. The main problems which should be solved are:

- a) the proper division of variables into each level
- to solve the low level problem in a possibly short time,
  - to obtain from the low level problem solution the suitable information to correct parameters of a stochastic generator,
- b) a way of utilizing the accumulated information to give the best direction to searching.

During the planning of chemical industry development there was a need to solve the large-scale discrete programming task to optimize investments. This task has been transformed to the following form, for which the evolutionary method can be applied. There are given:

value  $Q$ , vectors  $\bar{b}, \bar{c}_1, \bar{c}_2, \bar{c}_3$ , matrix  $A$ , nonanalytical function  $q(J_1)$ , discrete vector-function  $p(J_1)$ .

The vectors  $\bar{z}, \bar{y}_1, \bar{y}_0$  and set  $J_1$  should maximize:

$$f = \bar{c}_1 \bar{y}_0 - \bar{c}_2 \bar{y}_1 - \bar{c}_3 \bar{z} \quad (1)$$

and satisfy constrains:

$$A\bar{z} + \bar{y}_1 + \bar{y}_0 = \bar{b} \quad (2)$$

$$\bar{z} - \bar{p}(J_1) < 0 \quad (3)$$

$$q(J_1) < Q \quad (4)$$

$$z, y_0, y_1 \geq 0 \quad (5)$$

On the high level subset  $J_1$  which satisfy (4) is determined by a stochastic procedure. Then vectors  $\bar{z}, \bar{y}_1, \bar{y}_0$  are computed by means of linear programming procedure on the low level.

Parameters of stochastic procedure are modified depending on postoptimal analysis of linear programming.

There are computed: a value of performance index assigned to fixed  $J_1$  as  $f(J_1)$ , vector of dual prices  $\bar{\lambda}$  referring to constrains (3), estimated gradient  $\bar{e}$ . This information is utilized to generate new set  $J_1$ .

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A NEW VARIANT OF THE CODE GRECO FOR SOLVING  
NONLINEAR OPTIMAL CONTROL PROBLEMS

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Extended Abstract

The algorithm GRECO - Gradient Réduit pour la Commande Optimale - is a special version of the method GRG (1) for solving Nonlinear Optimal Control problems with time lags and bounded state and control variables of the type:

$$\begin{aligned} & \text{MAX } F | x(0), x(1), \dots, x(T); u(0), u(1), \dots, u(T-1) |, \text{ so that,} \\ & x(t+1) = f_t | x(t), x(t-k_1), \dots, x(t-k_r); u(t), u(t-j_1), \dots, u(t-j_s) |, \\ & a(t) \leq u(t) \leq b(t), \quad t = 0, 1, \dots, T-1, \\ & c(t) \leq x(t) \leq d(t), \quad t = 1, 2, \dots, T, \end{aligned}$$

where for all  $t$ ,

$x(t)$  is the vector of  $m$  state variables,

$u(t)$  is the vector of  $n$  control variables,

$a(t), b(t), c(t)$  and  $d(t)$  are constant vectors,

$k_1, k_2, \dots, k_r$  and  $j_1, j_2, \dots, j_s$  are time lags,

$T$  is the known planning horizon, and

$F, f_t$  are nonlinear differentiable functions.

Numerical solutions for application models of this type in physical, economic, energy and ecologic systems have been obtained using the code GRECO (3). These experiments were performed on several mainframes as IBM/370-168, Burroughs 6700, CDC Cyber, etc..

Considered as large scale nonlinear programs, by the GRG approach, we have modelling and programming facilities derived from the explicit consideration of the nonlinear constraints, the use of sparse matrix techniques, and degeneracy procedures(2).

In this paper we present a variant of the standard algorithm

to allow the incorporation of supplementary inequality and equality constraints, and free planning horizon. The inequalities can be transformed in additional state equations by the introduction of bounded slack variables that will change the dimension of the state vector in the corresponding time periods. For the supplementary equalities we must modify the choice of the basic variables related to these constraints by skipping the standard GRECO principle that introduces with the highest priority the state variable in the basis (2). In the standard GRECO algorithm, when the problem has free initial state variables, they are considered as additional control variables of the first period; something similar we do in the new variant when the model has a free planning horizon:  $T$  is considered as an additional control variable of the first period. For the discretized continuous time problems,  $T$  will be a lagged control variable for the other periods.

Using these simple ideas we could design a new variant of the code GRECO with wider applications and less modelling effort for the users. Some numerical tests have been performed with encouraging results.

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ENVIRONMENTAL SYSTEMS MODELLING WITH RESPECT TO THE FUTURE

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This paper presents some results of the investigations based on the application of environmental modelling. The presented approach can be generally used in the cases of uncertain, time-dependent and multivariate systems. For this reason operational research and probability methods were used to develop a multiperiod future oriented model. The proposed procedure provides the probability ratings of all possible future states of the studied system which elements are composed of nonrecurring discrete events.

Environmental systems are increasingly interdependent and the events occurring within them also interrelated. Consequently the different future versions of the systems cannot be determined relying upon only the events happened earlier. In these types of problems, mostly the experts' estimates represent the future interaction effects in the decision process.

Regarding to the future research point of view, the cross-impact analysis provides the decision maker a systematic manner to search for the causal future events and their interactions. That is, it can handles the interdependences of all pairs of variables during the computations. Besides, the final results are consistent and conformable to standard probability theory axioms. If an environmental system is assumed to be a finite set of the possible future states, then all of the mutually exclusive and collectively exhaustive scenarios can be identified.

The procedure starts with the defining of the environmental system specifying by the list of the separate events. Input data are the time-dependent cumulative probabilities of each event and the cross-impact factors which express the effects of the occurrences of events on each other.

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The main steps of the algorithm are the following:

- Using random numbers a computer simulation is run in each time period sequentially, to attain the modified probability distributions;
- The mutual information scales are calculated measuring the similarity between the event pairs;
- In many instances there are a great number of events in the event set, therefore a clustering technique is used to decompose the original system into closely connected subsystems. This is based on the maximization of the correlation coefficients between each of the possible subsystem and the constituent events;
- Applying linear programming methods a scenario generation of each decomposed subsystem is made for the given binary events;
- The whole systems' scenarios can be computed, after cardinal ranking of the scenario probabilities of each subsystem-as model output.

The paper interprets the most relevant topics of the model applications, such as input data given by the experts, the overall consistency and the cases of non-uniqueness of the linear programme together with the computational background.

A reliability systems' application is presented as an illustration of the method for a tower crane. First the crane was divided into its parts, among which different types of failure interactions exist during the lifetime. Applying the method the resulting failure parameters, the survival curve and the changes of the systems' operation influenced by environmental effects were analysed. The likely reliability scenarios in different points of time were also computed.

ON THE CONNECTIONS BETWEEN  
MATHEMATICAL PROGRAMMING AND  
DISCRETE OPTIMAL CONTROL

- a survey and some new results

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ABSTRACT

This paper gives a review of different theories and topics in the fields of Mathematical Programming and Discrete-time Optimal Control Theory. Well-known and new results (for instance, the generalized Maximum Principle) are presented within a unified framework. Emphasis is placed in the trends of interplay. New research areas are also identified.

Keywords: Optimization, Mathematical Programming, Discrete-time Optimal Control Theory, Duality, Upper Boundary Approach.

### Abstract

Exploration of Non Renewable Resources:

A Dynamic Approach

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Arrow studies a model of exploration of non renewable natural resources, in continuous time, where, the quantity of resources, to be found is a random variable.

In the model there is a T planning period to go.

If society consumes, in one period, an amount c of non renewable resource, it receives a utility u(c). New resources can be found through exploration,  $\alpha$  reflecting the effort applied to get them, spending p per unit of effort.

The number of units found when effort  $\alpha$  is employed is given by a random variable  $w_\alpha$ , having a Poisson distribution with parameter  $\lambda\alpha$ .

Society is then faced with the problem of maximizing the discounted total utility:

$$V^T(y) = \text{Max} \left[ u(c) - pa + \delta E V^{T-1}(y-c+w_\alpha) \right]$$

Subject to:  $0 \leq c \leq y$  ,  $\alpha \geq 0$

where y is the stock of resources available at the beginning of the initial period and  $\delta$  is the discount factor.

The new fact in the above problem, in relation with similar ones in literature, is that the probability distribution is defined by choice variable.

Concavity plays a major role in this kind of model, it can be proved that, under some conditions [related to decreasing risk aversion for  $u'(c)$  and  $u''(c)$ ],  $V(y)$  is a concave function.

The main result is that there exists  $\bar{y}$  such that for  $y > \bar{y}$ ,  $\alpha = 0$  (there is no exploration).

Arrow obtained the same result for a continuous time case.

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CONDITIONAL MOMENTS OF MULTIVARIATE PROBABILITY DISTRIBUTIONS

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Various problems in stochastic programming require the evaluation of conditional moments of a multivariate probability distribution. In chance-constrained programming, for instance, it is required to evaluate the probability mass of rectangles, which can be viewed as a raw moment problem of order 0. Applications to Convex Programming Problems with Recourse [2] require the evaluation of the distribution function as well as conditional means given that the random variable lies within a known polyhedral set.

Formally, we study numerical techniques to solve integrals of the form

$$\int_A dF(x), \quad \int_A x_i dF(x),$$

where  $A$  is a polyhedral set in  $\mathbb{R}^n$  and  $F$  is some continuous probability distribution.

Deák [1] and Szántai [3] have given good sampling methods for the 0th order moment problem for a multivariate normal distribution over rectangular regions  $A$ . Their results are adapted to yield answers to the 1st order moment problem in a straightforward manner.

What is needed is the evaluation of integrals of the form

$$a_1 \int_1^{b_1} a_2 \int_2^{b_2} a_3 \int_3^{b_3} x_k \exp\left(\sum_{j=1}^3 \sum_{i=1}^3 \alpha_{ij} x_i x_j\right) dx_3 dx_2 dx_1.$$

It is shown that this trivariate integral can be computed efficiently by a decomposition involving the evaluation of bivariate normal distributions. Szántai's method uses a decomposition of the sample space  $\mathbb{R}^n$ , while Deák's method is based on a decomposition of the random variable itself. We demonstrate how to combine the two approaches into a powerful hybrid method which utilizes the advantages of both methods simultaneously.

Numerical evidence presented suggests that the hybrid method is by far the best to use if the set  $A$  is known to have large mass, as is the case in most chance-constrained programming applications, and is at least competitive if  $A$  has small mass.

The extension to higher order moment problems is discussed briefly. Other extensions will include the adaptation of the methods to more general polyhedral sets as well as different multivariate distributions.

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PARAMETER TRACKING OF TIME VARYING CONTINUOUS TIME LINEAR SYSTEMS

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In this paper we show that the methods of Ljung (1976) and Gerencsér (1984) can be used for the problem of parameter tracking of time varying continuous-time linear system. We consider linear systems given in innovation representation:

$$\begin{aligned} dx_t &= A(\theta_t)x_t dt + K(\theta_t)dw_t & x_0 &= \xi \\ dy_t &= C(\theta_t)x_t dt + dw_t, \end{aligned}$$

where the parameter  $\theta_t$  is time varying. We assume that  $\theta_t$  belongs to some domain  $D_\theta$  such that for each  $\theta \in D_\theta$  the linear system is stable and inverse stable in some sense uniformly. Furthermore  $\theta_t$  is a smooth deterministic process such that  $|\dot{\theta}_t| < L$ .

The matrices  $A(\theta)$ ,  $K(\theta)$  are sufficiently smooth.  $w_t$  is a standard Wiener process.

Maximum-likelihood tracking is defined as follows. We define an infinitesimal error function by  $\dot{\epsilon}_0^T \frac{\partial}{\partial s} d\epsilon_s(0)$  where the process  $\epsilon_t(0)$  is defined by

$$\begin{aligned} d\hat{x}_t(0) &= (A(\theta) - K(\theta)C(\theta))\hat{x}_t(0)dt + K(\theta)dy_t \\ d\epsilon_t &= dy_t - C(\theta)\hat{x}_t(0)dt, \end{aligned}$$

with  $\hat{x}_0(0) = \xi$ . Subscript  $\theta$  denotes differentiation with respect to  $\theta$  and denotes differentiation with respect to time. The error function incorporating exponential forgetting is defined by

$$dV_t(\theta) = -\lambda V_t(\theta) dt + \lambda \dot{\epsilon}_{0_S}(\theta) d\epsilon_S(\theta)$$

with  $V_0(\theta) = 0$ . The estimator  $\hat{\theta}_t$  will be defined as any solution of  $V_t(\hat{\theta}_t) = 0$ . If there is no solution we put  $\hat{\theta}_t = 0$ . We have the following

Theorem. Under the conditions mentioned above there exist estimators  $\hat{\theta}_t$  such that the random variables  $\hat{\theta}_t - \theta_t$   $t \geq 0$  are stochastically bounded, i.e.

$$P(|\hat{\theta}_t - \theta_t| > C) < \epsilon(C) \quad t \geq 0$$

where  $\epsilon(C) \rightarrow 0$  for  $C \rightarrow \infty$ .

The bounding probability  $\epsilon(C)$  may be arbitrarily small if in  $|\dot{\theta}_t| < K$  the bound  $K$  is sufficiently small by appropriate choice of the forgetting factor.

(Appropriate choice means:  $\lambda(K) \rightarrow 0$ ,  $K/\lambda(K) \rightarrow 0$  for  $K \rightarrow 0$ )

Computational experiences were performed on a Commodore 64 computer. The main objective of the experiments were to test adaptivity.

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In this paper, a Galerkin technique is proposed for computation of an approximate solution of the LQ problem for stochastic linear systems on Hilbert spaces modelled in the white noise framework [1]. Related techniques have been widely considered in the recent literature for infinite dimensional LQ problems, with accessible state [2-7]. In these works the main trouble was to ensure the strong convergence of both the semigroups corresponding to the free dynamics and its adjoint. However, dealing with partially observed cases, some operators have to be assumed Hilbert-Schmidt. This allows to solve this problem more easily, in that it is possible to use the results obtained in [8] on the convergence of the the Galerkin method, for the Riccati equations. The main tool used there was to consider such equations in the space of Hilbert-Schmidt operators.

The problem can be stated as follows: consider the control system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + F\omega(t), & x(t) \in H, \omega(t) \in H_\omega, u(t) \in H_u \\ y(t) = Cx(t) + Du(t) + G\omega(t), & y(t) \in H_o, t \in [0, T] \\ x(0) = x_0 \end{cases}$$

where  $H$ ,  $H_\omega$ ,  $H_u$  and  $H_o$  are real separable Hilbert spaces,  $A$  is the generator of a  $C_0$ -semigroup  $\{T(t)\}$  on  $H$ ,  $B$  and  $F$  are Hilbert-Schmidt operators and  $C$ ,  $D$  and  $G$  are linear bounded operators, such that  $FG^* = 0$  and  $GG^* = -I$ . The noise process  $\{\omega(t)\}$  is an element of the cylindrical probability space  $L_2(0, T; H_\omega)$ , equipped with the Gauss standard measure. The control process  $\{u(t)\}$  is chosen to

be a Volterra transformation of the output process  $\{y(t)\}$  such that the cost functional

$$J(u) = \int_0^T \text{tr } Q E x(t) x^*(t) Q^* dt + \int_0^T \text{tr } E u(t) u^*(t) dt$$

is minimized. It is shown in [1] that this problem has a unique solution. The optimal feedback law

$$u(t) = -B^* K(t) \hat{x}(t)$$

involves the solution of two infinite-dimensional Riccati equations for the feedback gain  $K$  and the estimation error covariance  $P$ ; which in turn is fed into an infinite-dimensional Kalman filter to obtain  $\hat{x}$ . Let now  $V_n$  be a finite-dimensional subspace of  $H$  contained in  $D(A)$ ,  $\Pi_n$  be the orthogonal projection of  $H$  onto  $V$ , and  $T_n(t) = \exp(\Pi_n A \Pi_n t)$ .

We suppose only that

$$\lim_{n \rightarrow \infty} \sup_{t \in [0, T]} \|T(t)x - T_n(t)x\| = 0, \quad \forall x \in H.$$

Necessary and sufficient conditions for this are given by the general Trotter-Kato theorem [9]. Under this hypothesis the solutions of the following ordinary Riccati equations on the space of operators on  $V_n$

$$\begin{cases} \dot{P}_n(t) = (\Pi_n A \Pi_n) P_n(t) + P_n(t) (\Pi_n A \Pi_n)^* + \Pi_n F F^* \Pi_n - P_n(t) \Pi_n C^* C \Pi_n P_n(t) \\ \dot{K}_n(t) = -(\Pi_n A \Pi_n)^* K_n(t) - K_n(t) (\Pi_n A \Pi_n) - \Pi_n Q^* Q \Pi_n + K_n(t) \Pi_n B B^* \Pi_n K_n(t) \\ P(0) = K(T) = 0 \end{cases}$$

converge in the Hilbert-Schmidt norm, uniformly on  $[0, T]$ , respectively to the operators  $P(t)$  and  $K(t)$  [8]. This strong type of convergence is the tool used in proving the convergence of the following sequence of approximate optimal control law

$$u_n(t) = -B^* K_n(t) \hat{x}_n(t)$$

where

$$\begin{aligned}\dot{\hat{x}}_n(t) &= \Pi_n(A - BB^*K_n(t))\Pi_n \hat{x}_n(t) + P_n(t)\Pi_n C^*(y(t) - C\Pi_n \hat{x}_n(t)) \\ \hat{x}_n(0) &= \Pi_n x_0\end{aligned}$$

is an approximated Kalman filter as described in [8]. The following is the main result of the paper:

**THEOREM.** The triple  $(x_n, \hat{x}_n, u_n)$  converges to  $(x, x, u)$ , where  $\{x(t)\}$  is the optimal trajectory of the state, in mean square, uniformly on  $[0, T]$ . Therefore  $J(u_n)$  converges to  $J(u)$ .

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A path-following method for one-parametric nonlinear differentiable optimization

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Path-following methods for Kuhn-Tucker curves will be proposed. There are important applications for

- 1) Dialogue procedures for multiobjective optimization and
- 2) Globalization of locally convergent algorithms using imbedding

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ON THE OPTIMAL NUMBER OF PARALLEL SERVERS  
IN MULTIPROCESSOR SYSTEMS

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Throughout this paper parallel servers mean parallel processors, or a multiple bus system, or also parallel routes through a network to a destination. The paper focuses on the case where processing of tasks requires communication between the processors (in this paper called internal traffic). While parallel processing increases the computation power, the internal traffic reduces the service capacity of a multiprocessor system. Thus, we have two conflicting tendencies: The need of fast and powerful computation tools favours parallel processing while internal traffic restricts the number of processors. In order to obtain the optimum number of parallel processors one has to find the best balance between these two conflicting tendencies.

The mathematical model used in this paper is a many-server queue with service interruptions. In the case of internal traffic (an interrupted service has to be completed at the same server) even for exponentially distributed service times only an approximate approach is available. We arrive at this approach in two steps. First, the approximate approach to a  $M/G/m$  system ( $m \gg 1$ ) is obtained. Then this approach is applied to the investigated many-server queue by

replacing the moments of the service time by the first two moments of the system response time (which in the case of internal traffic equals the completion time).

For this model it will be shown that both the mean system response time as well as the mean queue length of tasks waiting to be processed have a global minimum which can be related to the optimum number of parallel processors. In order to illustrate the results an example will be presented. For other cases results are briefly discussed. Additionally, the approach presented give approximation formulae for a many-server queue with preemptive priorities.

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## OPTIMUM DESIGN OF ONE-STOREY STEEL FRAMES

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The problem considered is the minimum weight design of planar frame steel structures under multiple load conditions subjected to wide range of design requirements. The frame topology as well as the properties of materials are assumed to be fixed. Members of the structures are supposed to be with welded symmetrical  $\text{I}$  cross sections, uniform along the length and secured against local buckling without additional plates, which makes them technological and less labour consuming. The function  $V(\bar{X})$  to be minimized is the structural volume and it is obtained by summing over all the elements the product of its area  $A_i$  and length  $l_i$  of each element, giving:

$$V(\bar{X}) = \min_{\bar{U}} \left\{ V(\bar{X}) = \sum_{i=1}^n A_i(\bar{X}) \cdot l_i \right\},$$

in which  $n$  denotes the number of elements in the frame structure.

A design is defined by a vector of free variables  $\bar{X} = \{X_i\}, i=1, n$ . The free variables are assumed here in to express the dimensions of the cross sections of the frame members. They must be integer and positive in certain bounds defined by the thickness of the available steel sheets.

There are a number of restrictions on the behaviour of the structure which limit the acceptable variation of the free variables and define the set  $\bar{U}$ . These are buckling, displacement and stress constraints as well as side constraints (sizing limitations on member cross sections).

Two different idealized models of the steel frame structures may be considered by the represented optimization technique. On one hand the frames may be treated as elastic, geometrically non-linear system in which the forces and displacements are determined by the displacement method structural analysis, taking into account the effect of the

axial forces.

On the other hand the structure may be considered as being twice nonlinear - geometrically and physically. The forces and displacements in that case are determined by the method of the two idealized cross sections developed in USSR by prof. Gernerling.

In the both cases the problem is multiextreme discrete problem of the nonlinear programming. It is too hard and computer time consuming to solve it directly.

The optimization procedure is simplified by deviding the problem into subproblems and doing suboptimization which leads to considerable reduction of the dimensionality. This is possible because the constraints may be divided into two groups: constraints, concerning the overall reliability of the frames and constraints, concerning the local stability of the cross sections as well as side constraints which take into account the thickness of the available steel sheets. In this way the problem is divided into two subproblems with separate objective functions, free variables and constraints.

The function to be minimized in the first subproblem is the structural volume, the constraints concern the overall stability, the displacements and allowable stresses (deformations) and variables are the area  $A_i$  and moment of inertia  $I_i$  of each cross section. The design procedure is simplified by employing the concept of design variable linking, i.e.  $\{A\} = [L] \cdot \{I\}$ , in which constants  $L_i$  are determined at several or at each iteration solving the second suboptimization problem, from which the optimal dimensions of the cross sections for each element of the frame are obtained. The function to be minimized in that case is the cross sectional area, the constraints concern the local instability of web and flanges, the available steel and additional requirements concerning to which the moment of inertia and the area must be equal or greater than the minimum necessary values determined from the first subproblem.

These latter constraints associate analytically the two subproblems. The aim is to synthesize a cross section with minimum area and stiffness equal or greater than the minimum necessary in order to fulfil the constraints of the first subproblem as well. Free variables are the dimensions of the cross sections. A penalty function based optimization procedure is used.

In accordance with the method a computer program is developed. It is written in FORTRAN - IV language and can design frames with maximum 100 elements and 25 supports. The examples solved show that the optimal projects are 7 to 10 % more economical.

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ON THE OPTIMIZATION OF A SHORT-RUN MODEL  
OF ENERGY-PRODUCTION SYSTEMS

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Edmundo ROFMAN  
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In [1] we have recently presented a general procedure to compute the optimal cost of deterministic control problems. The aim of this paper is to show how that procedure was applied to optimize a short-run model of an energy-production system. The characteristics of the model allow us to introduce several improvements in the algorithm ; doing that we obtain a significant reduction of the time of computation.

The numerical data have been provided by Electricity of France (E.D.F.) ; they describe the French production system and the demand of energy, hour by hour, during a week.

As results we obtain the optimal cost function  $V(x,t)$  ( $x \in \mathbb{R}^{\mu}$  : hydraulic stock of  $\mu$  dams at time  $t$ ) and the optimal production policy  $p(x,t)$  of those dams and the  $v$  thermopower plants (nuclear, fuel, coal and gas plants) of the thermopower system.

\*We recall that our method takes advantage of the characterization of  $V(x,t)$  as the maximum solution of a suitable set of subsolutions of the Hamilton-Jacobi equation associated to the control problem. So, to compute  $V(x,t)$  we will solve the following problem :

Find the maximum element of the set

$$\mathcal{W}^s = \{w_i \in W^{1,m}(Q) / w_i \text{ verifies (1), (2), (3)} ; i = 1, 2, \dots, 2^v\}$$

$$(1) \quad \frac{\partial w_i}{\partial t}(x,t) + \min_{\mathcal{P}_{ad}}(x,t) \left\{ \sum_{\ell=1}^{\mu} \frac{\partial w_i(x,t)}{\partial x_{\ell}} (A_{\ell}^{-} (P_{\ell}^h)^+ + \eta_{\ell} (P_{\ell}^h)^{-}) + \right. \\ \left. + \sum_{\ell=1}^{\mu} C_{\ell}^h(x_{\ell}, t) [(P_{\ell}^h)^+ - \eta_{\ell} (P_{\ell}^h)^{-}] + \sum_{j=1}^v C_j^{\theta} P_j^{\theta} \right\} \geq 0,$$

$$(2) \quad w_i(x,t) \leq w_r(x,t) + k_1^r, \quad \forall r = i,$$

$$(3) \quad w_i(x, T) = 0$$

Where  $Q = \Omega x [0, T]$ ,  $T$  finite horizon ;  $i$  : discrete state variable showing which thermopower plants are working at time  $t$  ;  $\mathcal{D}_{ad}$  : set of admissible production levels (function of  $x$  and  $t$ ) ;  $A_k$  : input of water in the  $k$ -plant ;  $P_k^h$  : production level of the  $k$ -hydraulic plant ;  $\eta_k$  : pumping profit coefficient of the  $k$ -hydraulic plant ;  $C_k^h(x_k, t)$ ,  $C_j^\theta$  : unit production costs ;  $P_j^\theta$  : production level of the  $j$ -thermopower plant ;  $k_i^r$  : cost for passing from state  $i$  to state  $r$ .

Following [1] we discretize the set  $Q$ . In each one of the discretization points  $(x_p, t_q) \in Q^h$  we consider discretized inequations  $(1)^h$ ,  $(2)^h$ ,  $(3)^h$  related to (1), (2), (3). For getting  $(1)^h$  we introduce a new discretization schema for the derivatives  $\frac{\partial w_i}{\partial t}$ ,  $\frac{\partial w_i}{\partial x}$ . For the approximate problems we can show :

- a) Their unique solutions verify the maximum discrete principle used in the proof of the convergence of the method,
- b)  $(1)^h$  can be studied using simple algorithms of convex optimization,
- c) The non linear point fixe problems appearing in the solution of the systems  $(1)^h$ - $(2)^h$  can be considered as dynamic programming problems on a graph.

As a numerical exemple we solve a problem with  $v=7$ ,  $\mu=3$ ,  $[0, T]$  : a week divided in 21 periods. The solution is obtained in 20 seconds on the CII-HB/DPS 68. The program can be runned in small computers (we did it on a PDP 11-23, 128K using 12'30").

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OF ENERGY-PRODUCTION SYSTEMS

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In [1] we have recently presented a general procedure to compute the optimal cost of deterministic control problems. The aim of this paper is to show how that procedure was applied to optimize a short-run model of an energy-production system. The characteristics of the model allow us to introduce several improvements in the algorithm ; doing that we obtain a significant reduction of the time of computation.

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Find the maximum element of the set

$$\mathcal{U}^s = \{w_i \in W^{1,\infty}(Q) / w_i \text{ verifies (1), (2), (3) ; } i = 1, 2, \dots, 2^\nu\}$$

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$$(2) \quad w_i(x,t) \leq w_r(x,t) + k_1^r, \quad \forall r = 1,$$

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Where  $Q = \Omega x [0, T]$ ,  $T$  finite horizon ;  $i$  : discrete state variable showing which thermopower plants are working at time  $t$  ;  $\mathcal{P}_{ad}$  : set of admissible production levels (function of  $x$  and  $t$ ) ;  $A_i$  : input of water in the  $i$ -plant ;  $P_i^h$  : production level of the  $i$ -hydraulic plant ;  $\eta_i$  : pumping profit coefficient of the  $i$ -hydraulic plant ;  $C_i^h(x_i, t)$ ,  $C_j^\theta$  : unit production costs ;  $P_j^\theta$  : production level of the  $j$ -thermopower plant ;  $k_i^r$  : cost for passing from state  $i$  to state  $r$ .

Following [1] we discretize the set  $Q$ . In each one of the discretization points  $(x_p, t_q) \in Q^h$  we consider discretized inequations  $(1)^h$ ,  $(2)^h$ ,  $(3)^h$  related to (1), (2), (3). For getting  $(1)^h$  we introduce a new discretization schema for the derivatives  $\frac{\partial w_i}{\partial t}$ ,  $\frac{\partial w_i}{\partial x}$ . For the approximate problems we can show :

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STABILITY OF SEMILINEAR SYSTEMS IN HILBERT SPACE

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Several problems of mathematical physics, control and circuit theories lead to the following semilinear problem

$$(1) \quad \dot{x}(t) = Ax(t) + B F[x(t)],$$

where  $x(t) \in X$  for fixed  $t \geq 0$ ,  $X$  is a real Hilbert space with scalar product  $\langle \cdot, \cdot \rangle_X$ ;  $A: \mathcal{D}(A) \subset X \rightarrow X$  is a linear operator which is the generator of a linear  $C_0$ -semigroup on  $X$ ;  $U$  is another real Hilbert space with scalar product  $\langle \cdot, \cdot \rangle_U$ ;  $B \in \mathcal{L}(U, X)$ ;  $F: X \rightarrow U$  is a locally Lipschitz function,  $F(0) = 0$ . For every initial condition  $x_0 \in X$  there exists a unique weak (mild) solution of (1.1) prolongable on its right maximal interval of existence  $[0, t_{\max}(x_0))$ .

The following hypotheses will be assumed :

(H1) There exists  $Q \in \mathcal{L}(U, X)$  such that the operator  $QF$  is a gradient type operator,

(H2) There exist  $M = M^* \in \mathcal{L}(X)$ ,  $L \in \mathcal{L}(U, X)$ ,  $K = K^* \in \mathcal{L}(U)$  such that

$$(2) \quad \langle x, -Mx \rangle_X + \langle x, LF(x) \rangle_X + \langle F(x), L^*x \rangle_U + \langle F(x), -KF(x) \rangle_U \geq 0 \quad \forall x \in X,$$

(H3) There exist  $H = H^* \in \mathcal{L}(X)$  and a number  $\varepsilon > 0$  such that

$$(3) \quad \langle Ax, Hx \rangle_X + \langle x, HAx \rangle_X + \langle x, -Mx \rangle_X + \langle x, HBu \rangle_X + \frac{1}{2} \langle u, Q^*Ax \rangle_U + \frac{1}{2} \langle Q^*Ax, u \rangle_U + \langle u, B^*Hx \rangle_U + \langle x, Lu \rangle_X + \langle u, L^*x \rangle_U + \frac{1}{2} \langle u, Q^*Bu \rangle_U + \frac{1}{2} \langle u, B^*Qu \rangle_U + \langle u, -Ku \rangle_U \leq -\varepsilon [ \|x\|_X^2 + \|u\|_U^2 ] \quad \forall (x, u) \in \mathcal{D}(A) \times U,$$

(H4) For every  $\mu \in \mathcal{M} = \{ \mu \in \mathcal{L}(X, U) : (Q\mu) = (Q\mu)^*, \langle x, -Mx \rangle_X + \langle x, L\mu x \rangle_X + \langle x, \mu^* L^* x \rangle_X + \langle x, -\mu^* K \mu x \rangle_U \geq 0 \quad \forall x \in X \}$

the semigroup  $\{ S_\mu(t) \}_{t \geq 0}$ , generated by  $A + B\mu$  is exponentially stable

$$(H5) \quad \forall x \in X, x \neq 0. \exists \mathcal{K} = \mathcal{K}(x) \in \mathcal{M} : \int_0^1 \langle x, Q[F(sx) - \lambda sx] \rangle_{X^*} ds = 0$$

$$(H6) \quad x_n \xrightarrow{w} x_0 \implies F(x_n) \xrightarrow{s} F(x_0) \quad \forall x_0 \in X, \text{ where the}$$

arrows  $\xrightarrow{w}$ ,  $\xrightarrow{s}$  denote weak and strong limits, respectively.

The strengthened hypotheses  $(H4')$ ,  $(H5')$  also will be used.

They arise from  $(H4)$ ,  $(H5)$ , respectively, by taking  $\mathcal{M}_0 = \mathcal{M} \cap$

$\mathcal{C}$  the class of completely continuous linear operators from  $X$  into  $U$  instead of  $\mathcal{M}$ .

The main results are :

Theorem 1.

The equilibrium  $0 \in X$  is globally asymptotically stable provided that  $(H1)$ ,  $(H2)$ ,  $(H3)$ ,  $(H4)$ ,  $(H5)$  hold.

Theorem 2.

The equilibrium  $0 \in X$  is globally uniformly asymptotically stable provided that  $(H1)$ ,  $(H2)$ ,  $(H3)$ ,  $(H4')$ ,  $(H5')$ ,  $(H6)$  hold.

Two examples of the applications of these results will be presented at the conference.

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MINIMIZING A TOTAL SCHEDULE COST FOR TWO-MACHINE FLOW SHOP  
WITH CONTROLLABLE JOB PROCESSING TIMES

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In 1980, Vickson [1],[2] drew attention to the problems of least cost scheduling on a single machine, in which both the sequence of jobs and their processing times were decision variables. The concept of variable but controllable processing times has been taken over from the area of project management where project cost/duration curves have been extensively used in critical path planning with controllable job completion rates. This paper extends the Vickson's initial research in the area of flow shop sequencing.

There are  $n$  independent jobs, numbered  $1, 2, \dots, n$ , to be processed on two machines, A and B. Each job  $j$  is to be processed firstly on machine A for  $a_j - x_j$  time units,  $0 \leq x_j \leq u_j$ , and then on machine B for  $b_j - y_j$  time units,  $0 \leq y_j \leq v_j$ , where  $x_j$  and  $y_j$  are the times by which the processing times  $a_j$  and  $b_j$  are shortened (compressed), and  $u_j$  and  $v_j$  are the maximum compressions on machines A and B, respectively. The cost of processing job, is a linear function  $c_j x_j + d_j y_j$ , with  $c_j > 0$ ,  $d_j > 0$ . Denote by  $C_{\max}(\pi; a_j - x_j, b_j - y_j)$  the maximum completion time for a sequence (a permutation on the set  $\{1, \dots, n\}$ ) and processing times  $a_j - x_j, b_j - y_j, j=1, \dots, n$ . The total schedule cost is equal to the maximum completion time cost plus the total processing cost,

$$K(\pi, x, y) = C_{\max}(\pi; a_j - x_j, b_j - y_j) + \sum_{j=1}^n (c_j x_j + d_j y_j).$$

The problem is to find  $\pi^M$  and  $x^M, y^M$  minimizing  $K(\pi, x, y)$  subject to:  $0 \leq x_j \leq u_j, 0 \leq y_j \leq v_j, j=1, \dots, n$ .

We show that even most simple version of this problem, in which jobs are compressed on only one machine, A, and  $c_j = c$  for all  $j$ , is NP-complete.

A general heuristic method for solving this problem can be stated as follows.

Heuristic G: Step 1) Choose an arbitrary permutation  $\pi^G$ , Step 2) Determine  $x^G, y^G$  minimizing  $K(\pi^G, x, y)$ . An analysis of the worst-case performance ratio shows that  $K^G/K^M \leq 2$ , and this bound is the best possible. In the paper we propose also certain modification of this heuristic.

Heuristic M: Step 1) Find  $\pi^M$  minimizing  $C_{\max}(\pi; a_j - (1-c_j)u_j, b_j - (1-d_j)v_j)$ , Step 2) Determine  $x^M, y^M$  minimizing  $K(\pi^M, x, y)$ . We show that if  $v_j = 0$  and  $c_j = c$  for all  $j$ , then  $K^M/K^M \leq \frac{3-c}{2}$  for  $c \leq \frac{1}{2}$ , and  $K^M/K^M \leq 1 + c(1-c)$  for  $c > \frac{1}{2}$ , and these bounds are best possible. If  $v_j = 0$  for all  $j$ , then

$$K^M/K^M \leq 1 + \frac{\bar{c}(1-\underline{c})}{\underline{c}(1-\bar{c}) + \bar{c}}, \text{ where } \underline{c} = \min_{1 \leq j \leq n} c_j, \bar{c} = \max_{1 \leq j \leq n} c_j.$$

In the general case  $K^M/K^M \leq \min \{ 2, 3 - 2 \min \{ \underline{c}, \underline{d} \} \}$ , where  $\underline{d} = \min_{1 \leq j \leq n} d_j$ . Some other properties and observations are also given.

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A DECOMPOSITION ALGORITHM FOR THE DETERMINATION  
OF OPTIMAL BUS FREQUENCIES

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1. Formulation of the problem.

The following optimization problem will be considered:

$$\min_{n \in \mathcal{N}; \theta_{ij} \in O_{ij}} \sum_{i,j \in I} P_{ij}(n, \theta_{ij}) \quad /1/$$

where:

$$n \in \mathcal{N} = \left\{ n: \sum_{l \in L} n_l \leq N \quad \& \quad n_l \geq n_l^0 \quad ; \quad l=1 \dots L \right\} \quad /2/$$

$\mathcal{N} \subset \mathbb{R}^L$

$$\theta_{ij} \in O_{ij} = \left\{ \theta_{ij} : \xi_{ij}(\theta_{ij}, n) = \emptyset ; \quad i, j \in I \right\} \quad /3/$$

$O_{ij} \subset \mathbb{R}^+$

$$\xi_{ij}: O_{ij} \times \mathcal{N} \rightarrow \mathbb{R}^+$$

$$P_{ij}: O_{ij} \times \mathcal{N} \rightarrow \mathbb{R}^+$$

Let us assume that:

$$\forall n \in \mathcal{N} \exists \theta_{ij} \in O_{ij} : \theta_{ij} > \emptyset$$

This is the mathematical formulation of the passenger-optimized bus allocation problem for a urban transportation network, where:

L - set of routes in the network.

I - set of nodes in the network.

$n = \{n_1, \dots, n_L\}^T$ ;  $n_l$  - number of buses on route l /allocation/.

$\theta_{ij}$  - number of direct passenger trips from node i to node j.

N - number of vehicles in the fleet /resources/.

$n_l^0$  - minimum allocation to routes.

$\xi_{ij}(\cdot)$  - assignment function.

$P_{ij}(\cdot)$  - average travel time for passengers travelling  
from node  $i$  to node  $j$ .

The problem to be discussed can be described in the following general terms: given the matrix of passenger flows on a urban bus network, how optimally to allocate an assumed fleet of buses among these routes? Passenger flows on routes are a function of vehicles allocation due to different travel possibilities between some node pairs /eq.3/. Therefore, calculation of the objective function value requires an expensive computational effort due to allocation-assignment relationships.

For some major transfer points in the network one have successively assign flow matrix to each route, usually for the whole network.

## 2. Decomposition of the problem.

Local optimization problems are introduced. Each of them deals with the local objective  $P_{ij}$  and describes the subsystem for single node pair  $(i, j)$  in the network:

$$\min_{\theta_{ij} \in \mathcal{N}} P_{ij}(\theta_{ij}, b[(n_{ij} - \hat{n})]) \quad /4/$$

$\theta_{ij} \in \mathcal{N}$ ;  $n_{ij} \in \mathcal{N}$ ;  $\hat{n}$ -given as a fixed solution of problem /5/.

Coordination problem:

$$\min_{n \in \mathcal{N}} \sum_{i, j \in I} P_{ij}(\theta_{ij}, b[(\hat{n}_{ij} - n)]) \quad /5/$$

$\hat{n}_{ij}$ - given as a fixed solution of problem /4/.

where:

$b : \mathcal{N} \rightarrow R^+$  penalty function.

## 3. Simulation results.

A sample problem was solved numerically using the penalty function method. An alternative approach was Frank-Wolfe method. Preliminary results can be summarized as follows: i/ A computation time reduction can be achieved, ii/ separate modules in the decomposed problem can be tested in more convenient way.

Elasto-Plastic Analysis of Continuum Structures by Nonlinear Programming

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To solve this problem there are more methods in literature which state the different kinds of plastic stiffness matrices solving nonlinear equations and determine the rates of stresses and displacements with usual iterative methods. The elaboration of a new method is necessary to get the plastic stiffness in every case, for it is not possible to give a general solution.

In this paper a new approach is presented using finite element method; starting by the basic theorems of plasticity the problem is solved by nonlinear programming without using the plastic stiffness matrices. According to the statical theory the minimal value of the potential energy rate function is looked for by satisfying the equilibrium equations and plastic yield conditions.

$$\begin{aligned}
 & x G x^T \rightarrow \min \quad x = (x_{(i-1) \times 6 + l}) \quad \begin{matrix} i = 1, \dots, k \\ l = 1, \dots, 6 \end{matrix} \\
 & A x = b \\
 & f_1(x_{11}, x_{12}, x_{13}) = \frac{1}{6} [(x_{11} - x_{12})^2 + (x_{12} - x_{13})^2 + (x_{13} - x_{11})^2] + x_{14}^2 + x_{15}^2 + x_{16}^2 \leq 0 \\
 & f_2(x_{21}, x_{12}) \leq 0 \\
 & \vdots \\
 & f_k(x_{(k-1) \times 6 + 1}, \dots, x_{6k}) \leq 0
 \end{aligned} \tag{1}$$

where

- G: Hook matrix
- A: transmission matrix (the geometrical data of the structure)
- b: vector of the load rate
- f: yield condition by Mises-Huber-Henky
- k: number of the nodes in the structure
- x: vector of stress rate

According to the mechanical meaning of matrix A it has linearly independent rows. In case of a usual structure the number of the unknown is about 1000 in this quadratic programming problem (1). Using the particularities of problem (1) the feasible direction method was applied.

Let's note the solution of elastic problem by  $x^0$ . It means that  $Ax^0=b$ . In this way the problem can be reduced to linear programming.

$$\begin{aligned} -\nabla G(x^0)r &\rightarrow \max \\ Ar &= 0 \\ \nabla f_1(x_{11}^0, \dots, x_{1m}^0) \cdot r &\leq 0 \\ \vdots \\ \nabla f_m(x_{(m-1)1}^0, \dots, x_{(m-1)m}^0) \cdot r &\leq 0 \end{aligned} \quad (2)$$

A feasible direction  $r$  and a constant  $\lambda$  have to be determined.

$$x^1 = x^0 + \lambda r \quad (3)$$

In (2) the inequality conditions are written on those yield conditions which aren't satisfied in  $x^0$ . The measure of step  $\lambda$  is given by the minimal positive value of  $\lambda$ 's computed from the yield conditions. In this way a yield mechanism is determined. From this point resolving (2) a new direction and constant is got. The iteration is followed till arriving at the yield mechanism which corresponds the minimal value of the energy-rate function.

This method can be used for all kinds of structures, it requires the knowledge of the elastic stiffness matrix only. The method of solution is independent of the actual problem.

Finally, an application of this method is presented by the elasto-plastic analysis of a thick, curved shell structure.

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LOT SIZING IN A PRODUCTION SYSTEM FOR  
PERISHABLE INVENTORIES

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ABSTRACT

The problems pertaining to production scheduling are becoming more complicated issues in modern industries because of uncertainty in the behaviour of men, machines and materials. One of the problems which comprises of material planning has to be identified in relevance to the system parameters to cushion the effect of demand fluctuations over time. The estimation of Economic Order Quantity (EOQ) for raw materials and Economic Production Quantity (EPQ) in a production system facilitates to solve the problems of production scheduling with minimum total annual variable cost. In recent years, the research on perishable inventories are getting due consideration of production researchers for the improvement of materials requirements planning activities.

Realising the practical importance of the control of perishable inventories like gasoline, alcohol, food products and radio active materials in a manufacturing system, an attempt has been made in this paper to develop a mathematical for determining the values of EOQ and EPQ considering the decay nature of the materials. A multistage production inventory system manufacturing multiple products has been assumed for the

model development. Little work has been done considering the decay of the materials during processing. It is obvious to note that the cost incurred due to decay during processing is accountable one if the processing time is longer. A number of decaying materials are being subjected to further processing to attain the finished product form, so the problem of considering the decay during processing is inevitable one.

An integrated production-inventory system has been considered to identify the behaviour of the system considering the perishability of the products during presence in stock and as well as during processing. The model also takes into account the procurement of several production batches of raw material at a time. Exponential decay has been assumed both for in-process and raw material inventory which are based on the on hand inventory. The basic criterion considered here to optimize the batch size is the minimization total annual variable cost. The assumptions made relevant to the system parameters are depicted in appropriate terminology. This paper also throws some light on future research directions.

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## ON THE INVERSE PROBLEMS OF CONTROL SYSTEM DYNAMICS

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The report is concerned with the inverse problems of control theory. These are to identify the initial state and a disturbance or forcing term in the input of a dynamic system on the basis of available measurements of the system output. Such problems may arise, for example, in the solution of one of the two basic problems of dynamics which is as follows: assuming the motion of a mechanical system to be given, one is to determine the forces that generate this motion. On the other hand, the problem under consideration is closely related to those of control and observation under conditions of uncertainty [1,2].

Let the motion of a control system on the interval  $[t_0, \theta]$  be described by the differential equation

$$(1) \quad \dot{x} = f(t, x, u(t)), \quad x(t_0) = x^0, \quad x \in \mathbb{R}^n, \quad u(t) \in \mathbb{R}^z,$$

where the initial state  $x^0$  and the function  $u(\cdot)$ , that represents the disturbance (the control), are assumed to be unknown in advance.

Suppose the output of the system is given by the following equation

$$(2) \quad y(t) = h(t, x(t), u(t)), \quad t \in [t_0, \theta],$$

where the function  $h: [t_0, \theta] \times \mathbb{R}^n \times \mathbb{R}^r \rightarrow \mathbb{R}^m$  is continuous.

All the information on  $w = (x^0, u(\cdot))$  accessible a priori, is restricted to the inclusion  $w \in W$ , where  $W$  is a preassigned set in the corresponding functional space.

The problem is to identify the term  $z = Fw$  under the restriction  $Aw = y$ ,  $w \in W$ , where the operator  $A$  transforms the pair  $w = (x^0, u(\cdot))$  into  $y = y(\cdot)$  according to (1), (2). (Here for example  $z = w$ ,  $z = u(\cdot)$  or  $z = P_L u(\cdot)$  where  $P$  is a projection operator on the subspace  $L$ ).

Due to the noninvertibility of  $A$  the solution of the problem in general is nonunique. Following [2] let us consider the informational domain, consistent with  $y$  i.e.,  $Z(y) = \{z: z = Fw, Aw = y, w \in W\}$ . If  $y$  is given with an error that does not exceed a  $\delta > 0$  ( $\|y - y_\delta\| \leq \delta$ ), then  $Z(y_\delta)$  may acquire an arbitrarily large deviation from  $Z(y)$  no matter what small is the  $\delta$ . Therefore, in order to solve the problem it is necessary to combine the methods of the theory of illposed problems [3,4] and theory of observation under uncertainty conditions [1,2].

It is possible to introduce some extensions  $Z_{\epsilon, \alpha}(y)$  of  $Z(y)$  (the regularisations of  $Z(y)$ ) that depend on positive parameters  $\epsilon, \alpha$ , and also to establish some results on the convergence of  $Z_{\epsilon, \alpha}(y_\delta)$  to  $Z(y)$  under  $\epsilon \rightarrow 0, \alpha \rightarrow 0, \delta \rightarrow 0$ , assuming the equations (1), (2) to be linear in  $u(t)$ . For systems (1), (2) that are linear in  $x, u(t)$  a description of  $Z_{\epsilon, \alpha}(y)$  may be reduced to the solution of some standard variational problems. Further on the convergence of a proximal point algorithm to the solution of the basic problem is examined.

A dynamic variant of the above problems with varying  $\theta$  is then considered and the dependence of the solutions on  $\theta$  is finally discussed.

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A SUBGRADIENT TECHNIQUE FOR SOLVING A  
PARTITIONED TRAFFIC ASSIGNMENT PROBLEM

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Moderately large traffic assignment problems can nowadays be solved due to the discovery by Beckmann (1956), that solving a convex optimization problem yields a solution fulfilling Wardrop's first principle. This principle states that all used routes in a traffic network have equal cost and that all unused routes have a larger cost.

Many road networks today are so large that it is impossible to model and solve traffic assignment problems involving all the links and nodes in the network. Traffic planners are compelled to make a number of simplifications. Usually, only a subnetwork of special interest is studied. This way of working can be viewed as a scheme of aggregation/disaggregation. Two classes of methods have been suggested for creating the new subnetwork, "aggregation by link extraction" and "creation of artificial links". Only the first method has been applied to large scale road networks.

Hearn (1978) applies known decomposition techniques to the problem of aggregating by link extraction. A drawback with Hearn's method, "transfer decomposition", is that a near-optimal solution may be far from reproducing optimal link flows within the subnetwork of interest.

In this paper we present a general subgradient search procedure which is believed to give better approximations to the optimal solution, measured in link flows. Some of the algorithmic details are based on a result by Nguyen (1979), showing that a specific traffic assignment problem with elastic demand can be solved to reproduce optimal link flows. The requirement is that the optimal travel time between every origin - destination pair is known. Under certain assumptions, the true travel times in the subnetwork are computed during the subgradient steps.

The suggested procedure is related to both the "transfer decomposition" technique proposed by Hearn and "geographical decomposition" (Dantzig et al. 1976).

When problems are solved through an aggregation/disaggregation scheme, it is often useful to know how much loss in accuracy there is in the objective function value when the aggregated problem is solved instead of the original problem. Such bounds can be derived during the process of solving the partitioned traffic assignment problem, but since the objective function is of an "artificial" type, it would be more valuable to compute a measure of "goodness" based on the solution in terms of links'flows instead.

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THE EFFECT OF POINT-LIKE NOISE SOURCES ON  
CHEMICAL DISTRIBUTED PARAMETER SYSTEMS

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The modeling of stochastic distributed parameter systems has a great importance in chemical engineering from the viewpoint of fault detection, measurement device placing and control. Although several attempts have been done in the recent literature to make suitable models for such systems but the solution of real industrial problems is mainly based on the heuristic linear multivariable approximation of the complicated nonlinear stochastic distributed parameter chemical systems.

In our paper the following integral equation has been chosen for the stochastic model of a one dimensional chemical system in transient state

$$\int_0^t [T(x, \tau, \omega) - T_b(\tau)] d\tau = \int_0^t \int_0^L K(x, \xi) f(\xi, \tau, \omega) d\xi d\tau -$$

/1/

where  $-\int_0^L K(x, \xi) [T(\xi, t, \omega) - T^0(\xi)] d\xi$

x space coordinate ( $0 \leq x \leq L$ )

t time ( $0 \leq t$ )

$\omega$  elementary event

$T(.,.,.)$  system state variable / temperature, concentration, etc./

$f(.,.,.)$  system source variable / heat transfer, chemical reaction, etc./

$K(.,.)$  kernel function / describing the mixing conditions in the system /

$T_b(\cdot)$  input function /in time at  $x=0$ /

$T_0(\cdot)$  initial state function /in space at  $t=0$ /

The source of the fluctuations in  $T(\cdot, \cdot, \cdot)$  is assumed to be in the system source variable concentrated in one  $x = \xi$  point:

$$f(\xi, \tau, \omega) = [cT^* + f^0(\tau, \omega)] \delta_{\xi} \quad /2/$$

where .

$c, T^*$  constants

$f^0(\cdot, \cdot)$  steady state process with continuous trajectories  
w.p.l.

$\delta_{\xi}$  Dirac-delta distribution

$E[f^0(\tau, \cdot)] = 0$  for all  $\tau$  .

Under these assumptions it can be shown that the process  $T(\cdot, \cdot, \cdot)$  is also steady-state one and its correlation function fulfills an explicit operator equation which is closely related to the operator equation of the solution.

It has already been developed an efficient numerical method for solving such operator equations thus it is possible to obtain the correlation function for practical cases. The proposed method can be easily extended to more complicated noise sources too.

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Effects of power-converters, compensation  
equipment and filter circuits on the volt-  
age trajectories in electrical distribution  
systems.

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By means of modern power converters an optimal and dynamic control of the power flow in electrical distribution systems with d-c-engines can be achieved. Reactive power flow and network losses are minimized by means of electronical or mechanical operated compensation equipment. On the other hand more and more low voltage electronical equipment is used for control, protection, calculation, research and others. The power converters and the power compensation equipment generate upper harmonics and under-/overvoltages which may disturb each other or damage electronical equipment.

For the compensation of the upper harmonics in the a-c-currents filter circuits must be installed. The planning of these filter circuits in general is based on the conventional ideal rule for the harmonics, which means that only characteristic harmonics occur. For example in case of a six-pulse bridge converter, only filter circuits for the 5th and 7th harmonics are considered. But our calculation and measurements show that in the dynamic operation mode not only these characteristic but also non-characteristic upper harmonics of the order 2 and 4 occur, which have amplitudes up to 7 % of the fundamental harmonic. This effect is very important, if we take into account that an installed filter circuit for the 5th harmonic together with the inductance of the feeding network will have another resonance frequency below the order 5, in general near to the order 4!

For the fundamental harmonics the filter circuits operate capacitive. This must be considered in the case of dimensioning the reactive power compensation equipment. If this compensation should be realized by means of controlled capacitors, under- and overvoltages occur in case of switching, which may be dangerous for electronical equipment. The values of these under-/overvoltages depends on the ratio of the switched capacity to the capacity, which is already connected to the system. Considering this rule prohibited voltage deviations can be avoided.

On principle the interaction of power converters, power compensation equipment, filter circuits and low voltage electronical equipment should be taken into account already in case of planning and if possible studied by means of transient analysis.

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1 - THE LONG-TERM SCHEDULING OF THE ELECTRIC ENERGY POWER SYSTEM

ORELIA is a tool made available to those involved in the scheduling of the main grid at EDF.

In more precise terms, this tool concerns planners seeking the target towards which the system should head in 20 years or more as modifications are made on the system.

Determining a target system is obviously a difficult problem : careful consideration must be especially given to the impact of the planned investment on the transmission system when siting the new generation units. The solution being sought - the localization of the new generation means and reinforcements of the system - should minimize the sum of investment annuities and the estimated planning cost for the year under consideration.

The planning cost itself takes into consideration, on the one hand, the operating cost of the system for EDF, and on the other hand, the cost of the unsupplied energy to the customers. In order to estimate the planning cost, the various unavailabilities (failures, maintenance) liable to affect the generation and transmission equipment must be taken into account. This is essential, since a large part of investments on the system corresponds to the need to be able to cope with randoms involving the availability of equipment. Moreover, the planner must also study several development scenarios in regard to consumption, techniques, etc.

2 - THE ORELIA MODEL

ORELIA was thus built in order to "optimize" simultaneously the introduction of new means of generation and the development of the transmission system, while taking into account the random aspect of the problem. The use of a new method - the "subgradient stochastic method" - has made this possible. However, this method furnishes results in terms of continuous variables. The method was thus complemented, in ORELIA, by an heuristic method supplying results in integers for lines or units. The execution of both methods is quite similar from an algorithmic point of view.

3 - MATHEMATICAL MODELLING OF THE PROBLEM

Before briefly describing both methods, it would be useful to formulate the problem in more precise terms. The problem may be stated with the following formula :

$$\text{Min}_{y \in D} [a \cdot y + E f(y,s)]$$

- y is the vector of the unknown investments (generation and transmission) subject to stay in set D.
- D is the intersection of a block and hyperplanes. The block reflects the bound constraints which each of the potential investments (investments already undertaken, site or corridor constraints, etc.) must respect. The hyperplanes express the possible knowledge of the investment volumes per type of generation facility.
- a designates the vector of unitary costs of investment in terms of annuities,
- s is a random vector reflecting the various possible situations of availability of generation and transmission equipment,

f is the planning cost function associated with a vector of investments and an availability situation. Its computation is that of an "economic dispatch" modelled as a minimal cost transportation problem, making it a convex function.

(To carry out this modelization, which neglects KIRCHHOFF's second law of the d.c. approximation, the capacity of the system lines is weakened).

E designates the mathematical expectation of  $f(y,s)$  on the probability space associated with  $s$ .

#### 4 - THE STOCHASTIC SUBGRADIENT METHOD

The principle of this method was the subject of two papers (PSCC 1981 and IFAC 1982). Since then, the method has been put into practice with a certain degree of success and it shall be briefly described here. Moreover, it is the basis for the integer method. It consists essentially in the construction of a sequence :

$$y_{n+1} = P[D, y_n - r_n g(y_n, s_n)] \quad \text{where}$$

$s_n$  is a sequence of availability situations selected at random,

$g(y,s)$  the gradient (or the subgradient) of the function  $a \cdot y + f(y,s)$  computed from the dual solution of the transportation problem,

$r_n$  the element of a sequence of type  $a / (b + n)$ ,

$P(D,y)$  the projection of  $y$  in set  $D$ .

This method converges in theory and, to a considerable extent, in practice, at the cost of tens of thousands of situations selected at random (for 100 connections and 30 installations of units app. 30 mn IBM 3081).

#### 5 - THE INTEGER METHOD

In certain ways, this method is similar to the "cut method". Its execution is similar to that of the stochastic gradient method.

The main difference is that a sequence  $D_m$  of values for set  $D$  is made to be descending until there remains only one point. Every  $m$  (presently 1000th) iteration, some of the bounds between which the potential investments must remain (and which define the block having  $D$  in it) are brought closer together. The choice of which bounds will be modified is heuristic and depends, in particular, on the values taken - at the time of the choice - by vector  $y_m$  of the investments and by a vector means of the gradients on the last  $m$  iterations. In addition to this, the choice is carried out so as to restrict set  $D$  in a way which is neither too fast, nor too slow. In practice, ten or twenty packages of  $m$  iterations are needed for an example like the preceding one, with the necessary CPU time being approximately the same.

This method, which features a rather simple design, supplies nevertheless good integer solutions, related to the "continuous variable" solutions of the same problems.

SENSITIVITY ANALYSIS OF AN OPTIMAL CONTROL PROBLEM  
ARISING FROM AIR QUALITY CONTROL IN URBAN AREA

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The object considered in the paper has a form of a computer model for short-term prediction of air pollution in a city, predestined for supervising  $\text{SO}_2$  emission of power plants in Warsaw Metropolitan Area [1]. The process of pollutant dispersion in the atmosphere is described by two-dimensional advection-diffusion equation (2), averaged over the mixing layer height.

Basing on such a model, the real time emission control problem for the network of heating plants serving the area was formulated:

$$J(\underline{u}) = \int_0^T \int_{\Omega} r(c-c_d)_+^2 d\Omega dt + \frac{\beta}{2} \int_0^T \sum_{j=1}^n \beta_j u_j^2(t) dt \rightarrow \min, \quad (1)$$

with state equation:

$$c_t + \underline{w} \text{grad } c - K \Delta c + \alpha c = Q + \sum_{j=1}^n \chi_j(x,y) u_j(t) \quad \text{in } \Omega \times [0, T] \quad (2)$$

$$\partial c / \partial n = 0 \quad \text{if } \underline{w} \cdot \underline{n} \geq 0, \quad c = 0 \quad \text{if } \underline{w} \cdot \underline{n} \leq 0 \quad \text{on } \partial \Omega \times [0, T]$$

$$c(0) = c^0 \quad \text{in } \Omega,$$

and constraints

$$\forall_j, t \in [0, T] \quad \underline{u}_j \leq u_j(t) \leq \bar{u}_j \quad (3)$$

$$A \underline{u}(t) \geq \underline{b}(t), \quad \underline{u} = (u_1, \dots, u_n)^T \quad (4)$$

The problem may be interpreted as the minimization of the total loss (1) containing environmental damage incurred by air pollution as well production cost. The controlled emission sources  $u_j$  are subject to technological constraints (3) and minimal supply requirements (4).

The computation of wind field  $\underline{w}$  takes into account the structural data (geometry of the city, topography ect.) and meteorological forecast, while horizontal diffusion coefficient  $K$  depends on atmospheric stability. Hence both of these parameters as well as deposition factor  $\alpha$  are subject to significant error. Using the method proposed in [2], we investigate the lo-

cal sensitivity of an optimal control for the problem (1)-(4) with respect to the perturbations of the coefficients of state equation (2); e.g. given diffusion coefficient

$$K_\varepsilon = K_0 + \varepsilon K_1, \quad \varepsilon > 0 \quad (5)$$

where  $K_0, K_1$  are given elements, then it can be shown that the optimal control for problem (1)-(4), for  $\varepsilon > 0, \varepsilon$  small enough, takes the form:

$$\underline{u}_\varepsilon = \underline{u}_0 + \varepsilon \underline{q} + \underline{o}(\varepsilon). \quad (6)$$

Here the element  $q$  is the so-called functional sensitivity coefficient for the optimal control problem (1)-(4).

We derive the form of the functional sensitivity coefficient for the problem (1)-(4). It is shown that the sensitivity coefficient is given by a unique solution of an auxiliary, convex, control constrained optimal control problem. Using this results we propose a numerical method for the determination of the sensitivity coefficients. Numerical results for the test optimal control problems are presented.

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## ON MATHEMATICAL MODELS OF IMMUNOLOGICAL TOLERANCE

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Experimental evidence suggested that tolerance to human serum albumin induced in chickens shortly after hatching could be caused predominantly by B cell tolerance. A simple mathematical model was developed assuming that tolerance was due to elimination or irreversible inactivation of B cells and subsequent recovery from tolerance caused by the differentiation of new immunocompetent cells. The model assumed two compartments of B cells, immature and mature ones. However, the experimentally observed recovery from tolerance was much slower than the values calculated by means of the mathematical model, even when two collaborating B cell populations reactive to human serum albumin considered.

Therefore T helper ( $T_h$ ) cell tolerance was included into the mathematical model. As in the case of B cell tolerance,  $T_h$  cell tolerance is assumed to be due to irreversible inactivation by antigen and the recovery of  $T_h$  activity is ascribed to differentiation of new T lymphocytes. In the original version, only the mature T cell compartment was considered. Then also the case of two compartments of T cells (immature and mature) was investigated. In both cases, the calculated values of recovery from tolerance were compared with experimental data. We concluded that  $T_h$  cell tolerance explanation can lead to the assumption of two families of  $T_h$  cells.

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Our model assumes as the major mechanism of induction of tolerance direct inactivation of cells by antigen. However, other mechanisms were observed to cause or participate in effecting tolerance. Therefore, alternative models of tolerance are being considered which involve other mechanisms of tolerance induction, e. g. suppressor cells.

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AN INTERVAL ARITHMETIC METHOD TO FIND THE GLOBAL MAXIMA  
OF MULTIVARIABLE FUNCTIONS

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The problem considered in this paper is to locate the point(s) such that the values of a multiextremal function  $f(x_1, x_2, \dots, x_k)$  has the greatest value in a given region. Various methods, both deterministic and probabilistic, are developed to solve this global optimization. However, very few methods guarantee the true optimum. We use the method of interval analysis where the lower and the upper values of  $f$  are calculated as an interval for any interval value of  $x$ . Suppose that  $f$  belongs to  $C^2$  and the original region is a hyperrectangle. This region is divided successively, and in each subregion the interval value of  $f$  is estimated. We can discard subregions where the true maximum cannot exist, and finally the global maximum point(s) can be obtained. The difficulty is that the memory size and the computational time grow rapidly as the number of variables increases. To overcome this difficulty we make use of the interval version of Newton method to find relative extrema in the region and on the boundary. In the latter case one or more variables are fixed as constants, so that dimension-varying Newton method is necessary. The algorithm is summarized as follows. The capital letters denote intervals.

Step 1 In each subregion calculate  $F' = (\frac{\partial F}{\partial x_1}, \dots, \frac{\partial F}{\partial x_k})$  and  $D = \det$  (Jacobian matrix of  $F$ ). If the subregion includes the boundary, set flag = 1.

Step 2 If  $0 \notin F'$  the subregion can be discarded since no relative extrema exist in it. However, in case that flag = 1, its intersection with the boundary must be reserved as a subregion. Otherwise test whether  $0 \in D$  or not. If it is true then divide the subregion and go to Step 1, else apply Newton method and conserve the result. Note that the degree of Newton method is less than  $k$  when the subregion is a part of the boundary.

Step 3 We can discard subregions that have the upper values of  $F$  less than the greatest lower value of  $F$  so far computed.

Step 4 If the size of each remaining subregion is greater than the prescribed criterion, then divide the subregion and go to Step 1. Otherwise they give the final solution.

By the present method, we have found the global maxima of several test functions up to five variables.

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NONCOOPERATIVE GAMES DEFINED BY RELATIONS

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A noncooperative game defined by relations  $\Gamma = \{(A_i, \phi_i, <_i) \mid i \in I\}$  is a nonempty set  $I$  of players and for each  $i \in I$ : a set  $A_i$  of decisions of the player  $i$ , a function  $\phi_i: A \rightarrow 2^{A_i}$  which restricts decisions of the player  $i$  ( $A = \prod \{A_i \mid i \in I\}$ ,  $2^L$  - the family of all subsets of a set  $B$ ) and a relation function  $<_i: A \rightarrow 2^{A_i \times A_i}$ , such that  $<_i^a$  is a relation on the set  $\phi_i(a)$ . Let  $\bar{\phi}_i(a)$  denote the set of maximal elements for  $<_i^a$  in  $\phi_i(a)$ . An element  $a \in \prod \{\bar{\phi}_i(a) \mid i \in I\}$  is called an equilibrium in  $\Gamma$ .

Let  $E$  be a real Hausdorff topological vector space. A set  $K \subset E$  is approximatively finite convex if for each neighbourhood  $V$  of  $0 \in E$  there exists a finite subset  $\{x_i \mid i \in I\} \subset K$  and a finite family of convex sets  $\{C_i \mid i \in I\}$  such that  $C_i \subset V$  for each  $i \in I$  and  $K \subset \cup \{x_i + C_i \mid i \in I\}$ .

**THEOREM.** Let  $\{E_i \mid i \in I\}$  be a family of real Hausdorff topological vector spaces. If for each  $i \in I$ :  $A_i$  is a nonempty convex subset of  $E_i$ ;  $\phi_i: A \rightarrow 2^{K_i}$  is a continuous function, where  $K_i$  is a compact subset of  $A_i$  and  $2^{K_i}$  is the family of nonempty, closed subsets of  $K_i$ ;  $\phi_i(A)$  is approximatively finite convex; for each  $a \in A$   $<_i^a$  is an irreflexive, transitive relation defined on  $\phi_i(a)$ , such that the set  $\{(a, b, c) \in A \times A_i \times A_i \mid b, c \in \phi_i(a) \text{ and } b \not<_i^a c\}$  is closed in  $A \times A_i \times A_i$ ;  $\bar{\phi}_i(a)$  is convex for each  $a \in A$ , then there exists an equilibrium in  $\Gamma$ .

This theorem generalizes Theorem 2 in (1), Theorem 7 in (2), and Theorem 17.1 in (3).

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Dynamic optimization method for plant expansion planning  
with probabilistic operation simulation

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The aim of expansion optimization is to adjust a power system to the alterations of the demand for load and energy occurring during the course of years, and at the same time to minimize costs for all periods of time. Reasons for plant expansion are the deficiency in load or energy or high operating costs. For every year you have several alternatives for system expansion: adding new plants (nuclear, coal, gas, etc.), stoppage of old plants or making improvements on old plants, for example upon efficiency or diminution of polluting emission. Thus you get a solution tree which in each period you take the best branches from of the then different power systems as a basis for the optimization in the next period.

The periodical overall cost of the power system is the sum of annual capital cost and the fixed and variable operating costs of all power plants. Fixed and capital costs only depend on the features of the plants, the variable costs depend on the operation time of the plants resp. the generated energy.

Power plants with low fuel cost should have high operation time and vice versa. In order to achieve this you have to load the power plants following the order of increasing fuel cost. But the operation time of each plant is restricted by the load duration curve at the load position of the plant. Because of failure probability the plants don't generate the amount of energy that is the partial integral under the load duration curve in their order position. High cost plants must make up for the loss of energy of the low cost plants so that even the plants with high fuel cost have to generate a small amount of energy. Therefore you develop step by step the so called equivalent load function from the load duration curve and the loads and failure

probabilities of the power plants.

This was the basic idea of the earlier works of Marquis, Jamoule and Dillon. But power generation of most plants and energy types is restricted by reason of maintenance or even non-technical reasons, so that free operation simulation doesn't work. Therefore you first have to load all plants up to their lower bounds of generated energy and then up to the lower bounds of the generated energy of the energy types. Finally you load all plants according to increasing fuel cost and order their simulated operation times by decreasing values, so developing the operation function. Of course you have to consider the upper bounds of generated energy for each plant and energy type.

With pump storage stations in the system some other plants have to generate the required pump energy. You can derive specific variable costs for pump storage stations from the fuel cost of the pump energy generator and the efficiency factor of the pump storage station.

You have to take into consideration, that the integral of the operation function does never exceed the integral of the equivalent load function except in case of generated pump energy because otherwise the power system will generate unwanted and useless energy.

The optimum loading order which was used to construct the equivalent load function differs from the real order of loading the power plants on account of the restrictions in energy generation for plants and energy types. That's why the energy generated by the whole power system often differs a little from the periodical demand for energy. A slight approximation of the equivalent load function corrects this defect. A prototype of a computer program, written in FORTRAN, required only two approximation steps to get the correct results.

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Practical Methods in Computational Geometry and Their  
Applications to Geometrical and Geographical Optimization Problems

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Progress in the algorithmic theory of computational geometry for the past decade is one of the most remarkable we have ever seen in many fields of computer science. However, it seems that the progress has been so rapid that we have not been able to carefully evaluate the theoretical results from the practical standpoint, nor have we had enough time to implement them as computer programs and to apply them to optimization problems in other disciplines such as operations research.

This is a survey of part of the ideas, their implementations and the results of computational experiments on model problems as well as real-scale problems which have been got, developed and carried out by the author's research group for fundamental problems in computational geometry and geographical optimization problems.

1. Linear-time approximation algorithm for the minimum-length matching for points in the plane and its application to the improvement of the pen-movement of a mechanical plotter:--- A graph to be drawn by a mechanical plotter is first transformed to a Euler graph by adding a few fictitious edges as possible (their number is easy to determine), and then is drawn unicursally. Fast (i.e., linear-time) exact algorithms are known for the latter stage, whereas the known exact algorithm for the former (which contains the matching problem for the odd-degree vertices of the graph), although it is of polynomial order, say, of cubic order, is too much time-consuming for large graphs with as many as tens of thousands of vertices and edges.

The approximation algorithm for the former stage, which makes use of the technique of "bucketing", i.e., partitioning the planar region concerned into equal square "buckets", and of traversing those buckets in a highly "snaky" way, runs in time linear in the size of the graph to be drawn, and, having been applied to determining a proper order of drawing edges of large road networks, Voronoi diagrams and VLSI patterns, has been proved to drastically reduce the plotting time.

2. Practical algorithms for the point-location and the range-searching problems:--- The point-location problem is: Given a partition of the plane into polygonal regions, to determine to which region the query point belongs each time a query point is given. The range-searching problem is: Given a set of points in the plane, to enumerate all those points which are included in the query polygonal region each time a query region is given. There are several variants as well as extensions of those problems.

There have been proposed a number of quite sophisticated algorithms highly efficient in the sense of the worst-case time/space-complexity, but they run as fast (or as slow) even in the average case as in the worst case, the coefficient of proportionality hidden in the capital O of their complexity function is usually fairly large, and, what is worse from the practical standpoint, they are not so easy to implement due to the level of sophistication in algorithm and in data structure.

The idea of using buckets to localize the entire problem to some smaller problems of a constant size (at least on the average)---constant independent of the size of the entire problem however large it may be ---, and then applying a little elaborate strategies to solving the thus

localized problems, has been pursued to get easily implementable algorithms for the point-location and the range-searching problems. They were proved to run very fast on the average (in linear time with a small coefficient of proportionality for most problems) at the sacrifice of slight deterioration in the worst-case performance. (The worst-case is rarely met in practice.)

Specifically, the point-location algorithm makes use of the bucketing technique in combination with "lazy or retarded application" of the "slabbing" technique within a bucket, where the minimum amount of preprocessing is done to produce the data structure for the later use in slabbing.

The range-searching algorithm partitions the plane into buckets and produces an artificial graph with the given points as the vertices at the preprocessing stage. When a query polygon is given, the intersections of the polygon with the artificial graph are searched, where the buckets make the intersection search efficient. Then, the required points can be enumerated by the help of the Jordan curve theorem.

Computational results show these algorithms are more than satisfactory for practical problems.

3. Improved incremental algorithm for the Voronoi diagram for points and that for line segments:--- Nobody will doubt that an algorithm of the divide-and-conquer type is the best algorithm for constructing the Voronoi diagram for many points given in the plane because it is proved to be the optimum in the worst-case complexity and is not very difficult to implement. However, for  $n$  points, it takes  $O(n \log n)$  time even on the average, and, as experience has taught us, the most familiar divide-and-conquer algorithm which divides the given set of points into subsets according to their  $x$ -coordinates often suffers from a certain kind of numerical instability for large  $n$ , say, more than several thousands.

In such a context, it is interesting to see that the more primitive incremental algorithm --- which, instead of giving all the points at a time, adds them one by one and modifies the diagram step by step accordingly --- can be improved to run in linear time on the average for randomly uniformly distributed points and, furthermore, that it is highly robust against the nonuniformity of the distribution of points. This improvement has been achieved by means of an elaborate order in which to add the points, i.e., by adding the points one by one in such a way that, at any intermediate stage, the points which have been added may be distributed in the plane as uniformly as possible. In order to design such an ordering algorithm, the partitioning of the plane into buckets numbered in a tricky order is used. Computational experiments have shown that the improved incremental algorithm constructs the Voronoi diagrams for as many as tens of thousands of points in CPU time about a quarter millisecond per point on a standard large computer (of speed about 17 MIPS).

Existing algorithms of the divide-and-conquer type for the Voronoi diagram for  $n$  straight-line segments run theoretically in  $O(n \log n)$  time, but they seem to be more difficult to implement than those for points are. Moreover, numerical instability will be more serious because it is necessary to exactly determine the intersections of straight lines and parabolas which are often tangent to one another. In contrast, if line segments are regarded as consisting of their end points and the interior "open" segments and if the Voronoi diagram is constructed first for the end points and then the interior open segments are added one by one, a line-segment version of the incremental algorithm is obtained. This algorithm has qualitatively the same properties as that for points, i.e., it runs in linear time on the average and is robust against the nonuniformity of the given configuration of line segments. Computational experiments have

shown that it constructs the Voronoi diagram for line segments in CPU time a little more than one millisecond per segment. Numerical instability is easier to circumvent in the incremental algorithm than in the divide-and-conquer algorithm.

4. Various kinds of geographical optimization problems:--- Many problems of optimizing the locations of facilities in the plane are mathematically formulated as the problem of determining the coordinates of  $n$  points,  $x_1, \dots, x_n$ , in such a way that a certain function  $F(x_1, \dots, x_n)$  may take the minimum or maximum value. Typical examples are as follows.

<PI> To minimize  $F(x_1, \dots, x_n) = \int f(\min_i \|x - x_i\|) d\mu(x)$ , where  $x_1, \dots, x_n$  are the locations of  $n$  facilities,  $\mu$  is the measure representing the population density and  $f$  is a monotone increasing function representing the loss for an inhabitant to gain access to the nearest facility. This is the problem of determining the facility locations so as to minimize the total loss of the inhabitants assuming each inhabitant receives service from the nearest facility. In this case,  $F$  can be expressed in terms of the Voronoi diagram for points  $x_1, \dots, x_n$  as  $F(x_1, \dots, x_n) = \int \sum_i V_i f(\|x - x_i\|) d\mu(x)$ , where  $V_i$  is the Voronoi region (i.e., the "territory") of point  $x_i$ .

<PII> To maximize  $F(x_1, \dots, x_n) = \int \sum_i A_i \cap V_i d\mu(x) = \int \mu(A_i \cap V_i)$ , where  $\mu$  and  $V_i$ 's are as above, and  $\{A_i\}$  is a given partition of the plane into  $n$  polygonal regions  $A_1, \dots, A_n$ . This is the problem of "approximating" the given partition by a Voronoi diagram.

Sometimes, a moving facility will offer service at places  $x_1, \dots, x_n$ , say, on each day. Then the distance  $\|x_i - x_{i+1}\|$  may be restricted within a specified value, to give rise to the following constrained version of <PI>.

<PIII> To minimize  $F(x_1, \dots, x_n) = \int \sum_i V_i f(\|x - x_i\|) d\mu(x)$  subject to the constraints  $\|x_i - x_{i+1}\| \leq D$  ( $i=1, \dots, n-1$ ) (and  $\|x_n - x_1\| \leq D$  for a periodic problem).

If a facility can offer service while moving and if the path of its movement is assumed to be polygonal, then the Voronoi diagram for the edges of the polygonal path will be needed. Sometimes, the constraint on the total length of the path may further be imposed.

<PIV> To minimize  $F(x_1, \dots, x_n) = \int \sum_j V_j f(\|x - P_j(x)\|) d\mu(x)$  subject to the constraint  $\sum_j \|x_{j+1} - x_j\| \leq D$ , where  $x_1, \dots, x_n$  are the vertices of the polygonal path  $P$ ,  $V_j$  is the Voronoi region for the  $j$ -th edge  $E_j$  of  $P$  connecting vertex  $x_j$  to  $x_{j+1}$ , and  $P_j(x)$  is the point on  $E_j$  that is the nearest to  $x$ . (In a periodic problem,  $x_{n+1}$  is regarded as  $x_1$ .)

All the problems <PI> through <PIV> have the two-fold difficulties. One is the "combinatorial" difficulty due to the nonconvexity of the function  $F$ , and the other is the difficulty in actually computing the value of  $F$  and its derivatives. The former is so serious that we may be contented

with a local optimum. However, by virtue of the fast Voronoi-diagram algorithms mentioned in the above, the latter is no longer a difficulty. Computer programs, which are basically rather primitive descent methods calling the fast Voronoi-diagram algorithm repeatedly for computing the objective function and its derivatives, have succeeded in finding local optimum solutions for the above-mentioned problems with more than one hundred points and segments in a practicable time.

A number of theoretically interesting phenomena are observed experimentally. For example, for  $n$  large, the "density" of optimally located facilities is in a simple relation to the population density for each specific  $f$  in <PI>; if  $D$  is very large, the solution to <PIV> seems to approach a kind of "space-filling curve" as  $n$  increases; etc.

Dynamical versions of the problems can also be considered. For example, if  $n$  facilities offer service at  $\mathbf{x}_{1,j}, \dots, \mathbf{x}_{n,j}$  at noon on the  $j$ -th day of the week and if the loss for a demand occurring at place  $\mathbf{x}$  at time  $t$  to gain access to the  $i$ -th facility open on the  $j$ -th day ( $t_j$  = the noon of the  $j$ -th day) is a function of  $\|\mathbf{x} - \mathbf{x}_{i,j}\|^2 + \alpha^2 |t - t_j|^2$  ( $\alpha$  being the conversion factor of a temporal interval into a spatial distance), then the following dynamical version of <PI> will arise.

<PV> To minimize  $F(\mathbf{x}_{1,1}, \dots, \mathbf{x}_{n,1}; \mathbf{x}_{1,2}, \dots, \mathbf{x}_{n,2}; \dots; \mathbf{x}_{1,7}, \dots, \mathbf{x}_{n,7}) = \sum_{i,j} \int_{V_{ij}} f(\|\mathbf{x} - \mathbf{x}_{i,j}\|^2 + \alpha^2 |t - t_j|^2) d\mu(\mathbf{x}, t)$ , where  $\mu(\mathbf{x}, t)$  is the measure representing the distribution of demands in the space-time, and  $V_{ij}$  is the space-time Voronoi diagram with respect to the metric  $\|\mathbf{x} - \mathbf{y}\|^2 + \alpha^2 |t - s|^2$ .

If the space is one-dimensional, the space-time is two-dimensional, so that it is now computationally easy to deal with the problem <PV>. However, for the two-dimensional space, a faster algorithm is to be developed in order to handle large problems of type <PV>.

References [omitted]

MODELLING THE UNIT COMMITMENT  
PROBLEM BY A DECOMPOSITION METHOD

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Abstract:

Managing the continually increasing systems for production and distribution of electric energy is becoming more and more difficult. One of the significant requirements is to save primary energy and costs by means of optimum power plant utilization. In this field which so vitally affects power generation and distribution, and with due regard to all the given conditions of energy management, technology and policy, computer centers and powerful optimization programs may be used with advantage to assist the load dispatcher in planning the operation of generation plants and supply systems.

The result is a saving in operation costs and more efficient power plant operation as compared with previous operation planning and management schemes. The outlined procedures will assist the load dispatcher to find an optimal operation of generation plants and supply systems.

Naturally, such methods can also be used to simulate the effects of power system failure or maintenance, forecast reserve capacities and plan the layout of new power stations.

Depending on the tasks to be handled, it is necessary to select a suitable planning time-scale. Long-term optimization (new plant planning) extends over a period of several years. Medium-term optimization involves the determination of basic data for the management of large water reservoirs, for power supply contracts and for primary energy planning. Depending on reservoir size or length of contract, the period concerned may range from several weeks to one year. The aim of short-term optimization is

to plan power plant utilization for the next day, the time-scale often being extended to one week. A further subdivision of periods for instantaneous optimization (minute-hour), automatic generation control (second-minute) and primary control (second) is necessary for operations management, but is less relevant to operations planning.

The applied algorithms make it possible to handle thermal systems, as well as hydrothermal systems. Further it is possible to take into account a system of combined water reservoirs as well as the use of power supply contracts.

The aim of this planning algorithms is to built up a unit commitment, combined with a load dispatch schedule for the selected power plants, so that the total operating costs for the given time period are kept to a minimum, taking into account all relevant conditions - static, dynamic and integral contingent conditions. For example, the start-up costs of a generating unit modelled as a function of the numbers of hours the unit has been down, the minimum up-time and the minimum downtime are influencing the production costing during the optimization procedure.

The decomposition technique which is used here splits up the whole problem to a sequence of detailed tasks. Every task is now be handled with an optimizing algorithm of its own, integrated into an overall optimum at the end of each iteration cycle.

Thus the unit commitment problem in the first stage involves a suboptimal selection of power plants for the considered time-period. The second stage deals with the optimal load-dispatch, concerning the power plants selected above. These two stages are combined within a Gauss-Seidel iteration strategie to find an overall optimum.

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## ONE-MACHINE SCHEDULING PROBLEMS WITH RESOURCE CONSTRAINTS

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Til now one-machine scheduling problems have been considered under assumption that operations processing times are constant (e.g. [1]). In the presented paper these problems are generalized to the case when operations processing times depend on an allotted amounts of resource (e.g. energy, fuel, oxygen, catalyst raw material, money); at the same time both the amounts of resource allotted to each operation and the global amount of resource being constrained.

One machine scheduling problem with allocation of continuously-divisible constrained non-renewable resource can be formulated as follows. There are  $n$  jobs  $J_1, \dots, J_n$  that are to be processed on one machine. Each job  $J_i$  consists of one operation  $O_i$ . The machine can handle only one job at a time. The operation  $O_i$  corresponds to the processing time  $p_i$ . We shall assume that  $p_i \hat{=} p_i(u_i) \hat{=} b_i - a_i u_i$ , where  $u_i$  is the resource amount allotted to  $J_i$  and  $a_i > 0$ ,  $b_i > 0$ . We assume, moreover, the following set of feasible allocations of resource:

$$U \hat{=} \left\{ \bar{u} \in R^n : \bar{u} = [u_1, \dots, u_1, \dots, u_n]' \wedge \sum_{i=1}^n u_i \leq \hat{U} \wedge \right. \\ \left. 0 = \alpha_i \leq u_i \leq \beta_i \leq \frac{b_i}{a_i}, i=1, 2, \dots, n \right\},$$

where  $\hat{U}$  is the global amount of continuously-divisible resource and  $\beta_i$ ,  $i=1, 2, \dots, n$ , are known. For each job  $J_i$  the release date  $r_i$ , i.e. the moment at which the job  $J_i$  is available for processing, and the due date  $d_i$ , i.e. the moment

by which the job  $J_1$  should be completed, may be also given. There may exist precedence constraints between jobs.

One-machine scheduling problem with resource constraints consists in finding such a permutation of jobs on machine and such an allocation of resource  $\bar{u} \in U$  that one of the following criteria be minimized:

1) maximum cost  $C_{\max}(\pi, \bar{u}) \triangleq \max_i \{c_i(C_i(\pi, \bar{u}))\}$ , where  $c_i(t)$  is a non-decreasing function in the time variable  $t$  and  $C_i(\pi, \bar{u})$  is a completion time of job  $J_i$  in a permutation  $\pi$  under an allocation of resource  $\bar{u} \in U$ ;

2) maximum completion time  $C_{\max}(\pi, \bar{u}) \triangleq \max_i \{C_i(\pi, \bar{u})\}$ ;

3) maximum lateness  $L_{\max}(\pi, \bar{u}) \triangleq \max_i \{C_i(\pi, \bar{u}) - d_i\}$ ;

4) weighted sum of completion times  $\sum_{i=1}^n w_i C_i(\pi, \bar{u})$ ,

where  $w_i$  is a weight attached to  $J_i$ ;

5) weighted sum of tardinesses  $\sum_{i=1}^n w_i T_i(\pi, \bar{u})$ , where  $T_i(\pi, \bar{u}) \triangleq \max\{0, C_i(\pi, \bar{u}) - d_i\}$ .

Some properties of the optimal control in the one-machine scheduling problems are shown. In particular, it is shown which one-machine scheduling problems with resource allocation are NP-complete and which ones have polynomial algorithms.

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ON THE TIME-OPTIMAL RESOURCE ALLOCATION  
IN A SEQUENCE OF PROJECTS

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The contribution presents an approach, based on control theory concepts, to the problem of time-optimal allocation of a single, continuously divisible, constrained resource in a sequence of projects of independent activities when the maximum level of total usage of the resource is time-variable. The concept of the control of a project of activities was introduced by Burkov [1] as a special approach to resource allocation in PERT networks. Its distinctive feature is that the activities are continuous dynamical systems which mathematical models are given in the form of functions relating at any time the performance speeds of the activities to the amounts of resource, i.e. the activities are described by the equations

$$\dot{x}_i(t) = f_i(u_i), t \geq 0,$$

where  $x_i(t)$  and  $u_i(t)$  are the state of an  $i$ -th activity and the amount of resource allotted to it at the time  $t$ , respectively. The terminal states of activities, which must be reached in order to complete the activities, are also given.

Until now, the problem of time-optimal control of the projects under resource constraints has been satisfactorily solved in the literature only for the case when at every time of the project duration the maximum level of the total usage of resource is constant. In practice, however, this level is frequently subject to variations during the control of project.

Also the performance of the independent activities of the project can not be frequently started at the same time; some of the activities may appear during the performance of the project. In such a situation a sequence of projects is a good model of the system of activities.

In this paper we shall consider the time-optimal control of a sequence of projects of independent activities, when the maximum level of total usage of renewable, continuously divisible, resource is time-variable and piece-wise constant. The solution concept, based on convex analysis and the notion of the sets of reachable states is presented. It allows reduction of the dynamic problem of optimization to the static one. An algorithm to find an optimal resource allocation is developed. Some properties of the time-optimal control are given, with special attention paid to concave models of activities. The results obtained can be also applied to the time-optimal control of one project of independent activities under time-variable amount of resource.

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## MULTIOBJECTIVE STRUCTURAL OPTIMIZATION

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### Abstract

The optimization of building and machine structures usually involves a number of requirements that should be met at the same time to obtain the fully useful design. In the case of single criterion optimization, one of the requirements is selected as the criterion, while the remaining ones are met by including them into the constraint set. But with such an approach, it is necessary to determine a priori the bounds which these requirements should fulfill. Multiobjective optimization enables us to take into account numerous criteria that are often mutually conflicting. It is then possible to find a compromise and preferable solution which - although none of the criteria involved attains its extremum - guarantees meeting all the requirements in the best way possible. The paper deals with the problem formulation of multiobjective structural optimization and methods for generating the set of compromise solutions and selecting a preferable solution. The criteria appearing most frequently in the theory of structures are as follows:

- i/ minimum volume or weight of the structures,
- ii/ the minimum potential energy or the maximum structural stiffness,
- iii/ minimum displacements at selected points or regions of the structure,

- iv/ maximum critical force,
- v/ maximum frequency of free vibrations,
- vi/ maximum moment of inertia,
- vii/ maximum safety or reliability.

It should be emphasized that to satisfy any of the criteria mentioned above, it is necessary to find the extremum of an objective function taking into account the related constraints. In the theory of structures, the most important constraints are related with:

- permissible stresses or the safety factor of the structures under all possible loading states,
- permissible displacements in a given structure,
- minimum and maximum sizes that are permissible in view of services and constructional reasons.

The solution obtained after meeting a few of the criteria mentioned above has usually the form of the compromise set. The optimal structure should be selected from this set on the basis of a global criterion which can take various forms in different optimization problems. Its choice is particularly vital because it will be decisive for results of the entire optimization problem. With this paper an attempt to fill the gap between the theory of multiobjective optimization and its application in optimum structural design will be presented.

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Some numerical experiments with the application of sequential  
quadratic programming to state-constrained  
optimal control problems

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Abstract

In an earlier paper (ref.[1]) a numerical method for the solution of state-constrained optimal control problems was presented. The main purpose of the present paper is to present some numerical experience with the method.

The method is essentially an infinite dimensional analogue to sequential quadratic programming. As such, the method can be considered as an application of the method of Newton to the necessary conditions for optimality.

In the setting of state-constrained optimal control problems, the analogue to the inversion of the Hessian matrix of the Lagrangian is the solution of a linear two point boundary value problem.

Each iteration of the method involves the solution of two linear two point boundary value problems (LTPBVP).

Numerically, a collocation method is used for the solution of these LTPBVPs.

The resulting set of linear equations is solved by Gauss elimination.

The method is derived considering only constraints of the equality type. Inequality constraints are transformed into equality constraints by means of non-negative slack-variables. Direct application of our method to the transformed problem would in general lead to singular (sub)problems, thereby destroying the expected rate of convergence. Therefore a special modification is necessary to avoid these problems.

Essentially this modification introduces the complementarity condition in our algorithm, thereby avoiding singularity for problems where strict complementarity holds.

At the final stage of the method an active set strategy is used to assure convergence to multipliers with the correct sign.

Reference:

[1] de Jong J.L., Machielsen K.C.P.

On the application of sequential quadratic programming to state-constrained optimal control problems.

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On degeneracies in semi-infinite optimization

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A semi-infinite optimization problem (S.I.P.) can be formulated in the following way:

$$(S.I.P.) \left\{ \begin{array}{l} \text{Minimize } f \text{ on } M, \\ M = \{x \in \mathbb{R}^n \mid h_i(x) = 0, i = 1, \dots, p; g(x, y) \geq 0 \text{ for all } y \in Y\} \end{array} \right.$$

where  $f, h_i, g$  are continuous functions, whereas  $Y \subset \mathbb{R}^m$  is a compact subset, independent of  $x$ .

A typical problem of this kind is that of Chebyshev approximation.

For  $\bar{x} \in M$ , the points  $\bar{y}_j$  for which  $g(\bar{x}, \bar{y}_j) = 0$  are (global) minima for the function  $g(\bar{x}, \cdot) \mid Y$ . In fact, the problem, minimize  $g(\bar{x}, \cdot)$  on  $Y$ , is a special case of a general parametric programming problem.

Under some additional assumptions, especially that every  $\bar{y}_j$  is a nondegenerate minimum for  $g(\bar{x}, \cdot) \mid Y$ , (i.e. we have strict complementarity and the Hessian of the corresponding Lagrangian is positive definite), the feasible set  $M$  is locally described by means of a finite number of equality constraints and a finite number of inequality constraints.

Optimality criteria and numerical methods for (S.I.P.) are based on this local reduction, by means of the implicit function theorem, to a usual (finite) mathematical programming problem in  $\mathbb{R}^n$ .

In general, however, one cannot avoid that at some points  $\bar{x} \in M$  the corresponding points  $\bar{y}_j$  are degenerate minima.

As a consequence, the local structure of the set  $M$  can be much more complicated.

A structural analysis of the feasible set  $M$  as well as the behaviour of  $f$  on  $M$  is done in the case that the index set  $Y$  is a compact interval.

Via a decomposition, based on singularity theory, the feasible set will be described locally by means of a finite set of polynomial functions.

Via a partition of the feasible set into manifolds, we introduce a critical point concept. A special subset of the critical point set consists of Kuhn-Tucker points. Using this critical point concept, we can show that the easiest local optimality criteria (using local reduction by means of the implicit function theorem) are generically satisfied.

Unfortunately, it is not possible to generalize these ideas to higher dimensional index sets.

However, by relaxing the strict complementarity assumption and strengthening the standard second order sufficient conditions for (local) strict optimality, we used some results from sensitivity analysis in nonlinear programming to obtain results for (S.I.P.), almost similar to the nondegenerate case.

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Models and Methods for Estimating an Origin-Destination  
Trip Matrix from Network Data

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In this paper we present mathematical programming models for the problem of estimating a trip matrix from network data. The models presented are based on the results of Nguyen (1977) who has shown that trip matrices that reproduces observed linkflows in a congested network can be obtained by solving an elastic demand traffic assignment problem with a specific linear demand function.

It can be shown that this elastic traffic assignment problem has a unique optimal solution with respect to the linkflows but that the resulting trip matrix not necessarily is unique.

The mathematical programming models presented will deal with the problem of deriving one of the optimal trip matrices from Nguyens elastic demand traffic assignment problem.

Models with different choice criteria will be discussed and solution methods based on decomposition techniques will be presented for the various models.

The presented models will all be examples of implicit optimization models, i.e. optimization models in which the constraints involve another optimization model.

Single as well as multiple criteria models will also be discussed.

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STOCHASTIC PROCESSES WITH UNCERTAINTY ON  
THEIR TRANSITION MOMENTS

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Most of the problems encountered in stochastic processes are considered in the framework of the Ito stochastic differential equation; and in our opinion there are two main reasons to that. i) The Ito equation has nice properties that other representations have not, and ii) the mathematical assumptions which support the Ito equation are generally satisfied in most practical situations. Clearly these assumptions are related to the transition moments and state that

$$E(\Delta x_t/x, t) = a(x, t)\Delta t; \quad E((\Delta x_t)^2/x, t) = b(x, t)\Delta t; \quad E((\Delta x_t)^k) = o((\Delta t)^2)$$

In some cases, it happens that the moments  $a(x, t)$  and  $b(x, t)$  are not exactly known, but rather are defined with some uncertainty, in such a manner that they can be represented in the form  $a(x, u, t)$  and  $b(x, u, t)$  where  $u$  denotes the time dependent parameter associated with the definiteness of the system. This uncertainty may be of random nature, in which case  $u$  itself is a stochastic process; or else it may be of non random nature, and for instance, one may think of  $u$  as a fuzzy variable.

Uncertainty of random nature. In this case the use of stochastic differential equations is very questionable on a mathematical standpoint. Instead of using a markovianization technique which increases the scale of the system, we rather employ the state moments of the process. By using the transition moments, one can obtain the state moments in terms of  $a(x, u, t)$  and of  $b(x, u, t)$  together with the probability density  $p(x, u, t)$ , and furthermore, in some situations of importance, one can derive the differential equations for the state moments themselves.

We also describe the stochastic process by the infinite set of differential equations of the state moments, and all the problems are solved by means of the latter: for instance filtering, optimum control, estimation of  $u$ , and so on.

To summarize, we so have an approach to stochastic processes without using stochastic differential equations.

Uncertainty of non random nature. In this case one has

$$E(\Delta x_t/x, t) \in a(x, t)\Delta t \quad E((\Delta x_t)^2/x, t) \in b(x, t)\Delta t$$

while the other transition moments behave like  $(\Delta t)^2$ .

By using some very simple realistic mathematical assumptions, it is then possible to convert these equation above into the form

$$E(\Delta x_t/x, t) = \alpha(x, u, t) t ; \quad E((\Delta x_t)^2/x, t) = \beta(x, u, t) t$$

and then we use a distributed variational approach on the Fokker-Planck-Kolmogorov equation to solve various questions.

LINEAR APPROXIMATION OF SURFACES WITH A CRITERION  
"MINIMUM OF AVERAGE QUADRATIC TIME" TREATING  
OPTIMAL CONTROL PROBLEMS

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A b s t r a c t

When solving the optimal control problem with an integral criterion of the quality of motion, it usually becomes necessary to determine and memorize multidimensional surfaces in the state space. Considering the peculiarities of the surfaces and the requirements for convenient technical realization, especially suitable appears the piecewise-linear approximation. In this case the error of the near-optimal system with respect to the optimal one (in first iteration) is proportional to the time of "erroneous" motion of the system (the motion with control, different from the optimal control law). A natural norm of nearness of the approximation appears therefore the time of erroneous motion of the system, or some functional involving it

Consider a system, defined by the following nonlinear equation:

$$\dot{x} = f(x, u, t) \tag{1}$$

Suppose, in the stage of optimal control design a surface has been determined, defined usually by a supporting net of S points.

We seek to determine the vector of parameters C of the approximating surface with the equation

$$C^T x = \alpha \tag{2}$$

where  $\alpha$  is a free parameter, so that the functional

$$J = \sum_{i=1}^S \Delta t_i \tag{3}$$

In (3)  $\Delta t_i$  is the time of motion of the system from the i-th point on the surface to the approximating surface.

It is proved in the paper that the vector  $C = C_{opt}$  satisfies the following system of nonlinear quadratic equations:

$$\alpha e^k C + C^T D^k C = -\alpha^2 g^k \quad (k = 1, \dots, n), \quad (4)$$

where  $e^k$  is  $n$ -vector,  $D^k$  -  $(n \times n)$  - matrix,  $g^k = \text{const}$ .

The paper presents the formulae to calculate those matrices. Some essential properties of the matrices  $D^k$  are proved, which make easier the design of methods to solving eqs.(4). A programme on FORTRAN IV is set up, which computes and defines the matrix parameters of (4) and solves it for  $n = 2$ .

In an example involving a second order oscillating plant a piecewise linear approximation of the optimal switching line, was carried out with the suggested nearness norm  $L_{2t}$ . For comparison, another approximation was carried out with a norm-minimum of average quadratic deviation  $L_2$ . With both criterions, the approximation was performed with different number of approximating straight lines. Using the computed coefficients of the straight lines were designed the corresponding near-optimal control structures.

The computer simulation of the so-designed control systems and the comparative analysis led to the following conclusions:

- 1/ The suggested nearness criterion defines a dynamic norm in the state space, which depends on the motion of the system;
- 2/ The greater the distortion of the phase portrait of the system near the approximated surface, the greater the difference between the parameters of the designed control structures with the two different norms.

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Operation Planning decisions in hydro-thermal power systems  
under uncertainty

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(Abstract)

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One of the main problems when planning the operation of power stations in a hydro-thermal system consists in evaluating the effects of external stochastic parameters: flow to run-of-river stations and temperature dependent electricity consumption. The large variance and the non-linear functional relationship of these parameters regarding the supply implicates tail events with low probability but extraordinarily high costs. Therefore it is necessary to consider not only the most probable or mean value of the parameters but to include the entire "stochastic range".

Among these decision problems are the fuel stock management and the storage operation planning. In the first case it is important to weigh the risk of spot market purchases against stock surplus whereas the fuel demand in the individual time interval depends on the operation of the thermal power plants. In the case of the storage operation planning the problem consists in balancing extreme storage utilization with reserve planning.

These problems are mathematically represented as an adaptive sequential decision process with weekly time intervals over the period of one year. In every time interval a risk matrix  $R_{ij}^t = R^t(s_i^t, d_j^t)$  depending on events  $s_i$  and decisions  $d_j$  is formed. Given the probability distribution for  $s_i$ , the expectation value is chosen as decision criterion. If only an interval can be given for  $s_i$ , the decision is made according to the "minimize maximum risk" (MinMax) criterion.

Methodically these problems are solved by means of Stochastic Dynamic Programming. In every given time interval a suboptimal decision  $d_j^t$  ("action") is determined at each discrete state (remaining storage or stock level) in order that together with the remaining still open decisions at this state ("afteraction") an optimal solution is found.

The EDP programs support the dispatching centres in making their short- and medium-term decisions.

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A NEW GLOBAL OPTIMIZATION TECHNIQUE FOR SOLVING  
PARTIAL DIFFERENTIAL EQUATIONS.

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1. SCOPES.

We propose to use an original global optimization method for solving functional equations and particular partial differential equations. We know that a lot of biological or physical phenomena can be represented by such equations.

Let us consider a general functional equation which has the form :  $Au(x,t) = f(x,t)$  with initial and boundary conditions (A being a linear or non linear differential operator).

Several numerical methods allow to approach its solutions. For example the operator A is approached using the finite differences method. On the other hand, with the help of the finite elements method (3), we keep A unchanged but we are looking for the solution in an approached set, that is to say that we choose an approximation for  $u(x,t)$ .

The proposed method can be related to the second eventuality when the operator A remains unchanged,

2. DESCRIPTION OF THE METHOD.

Using a transformation related to the Archimed's spiral (2) (Space filling curve (1)) and considering the following relations :  $x = a\theta \cos \theta$ ,  $t = a\theta \sin \theta$ ,  $\theta = (1/a) \cdot \sqrt{x^2 + t^2}$  we can reduce the variable  $x, t$  of the function  $u(x,t)$  to a single one  $\theta$  involving a function  $G(\theta)$  deduced from  $u(x,t)$ .

This transformation can be easily generalized to  $n$  variable ( $n > 2$ ).

This function  $G(\theta)$  can be approximated for instance, by a polynomial function :  $G(\theta) = \alpha_1 \theta + \dots + \alpha_n \theta^n$ .

If  $A_1$  and  $A_2$  are respectively the boundary and initial conditions, we get the following penalized problem, (4) :

$$(P_{\text{con.}}) : \underset{\alpha_i}{\text{Min}} \left\{ \|AG(\theta) - F(\theta)\|_{L^2(\Omega)}^2 + \frac{1}{\varepsilon_1} \|A_1\|_{L^2(\Omega)}^2 + \frac{1}{\varepsilon_2} \|A_2\|_{L^2(\Omega)}^2 \right\} .$$

with  $\Omega = \{(x,t)/(x,t) \in [0,M] \times [0,t]\}$ .

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A discretisation leads to the problem :

$$(P_{dis.}) : \min_{\alpha_i} \left\{ \sum_j \left( A \frac{\alpha_j}{a} (x_j^2 + t_j^2)^{1/2} + \dots + \frac{\alpha_n}{a} (x_n^2 + t_n^2)^{n/2} \right) + \frac{1}{\epsilon_1} \sum_k A_{1,k}^2 + \frac{1}{\epsilon_2} \sum_m A_{2,m}^2 \right\}.$$

If the operator A is linear, the functional problem is transformed into a linear algebraic system  $\frac{\partial J}{\partial \alpha_i} = 0, \tau = 1, \dots, n$ .

This method was tested on the heat equation :  $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$   
 with the boundary conditions :  $u(0,t) = u(1,t) = 0$   
 and the initial conditions :  $u(x,0) = 2 \sin x, 0 \leq x \leq 1$ .

### 3. LAGRANGE METHOD, (5).

Consider the functional :  $J = \Sigma(Au-f) + \lambda_1$  (the boundary conditions:  $A_1$ ) +  $\lambda_2$  (the initial conditions :  $A_2$ ).

We want to minimize :  $\min_{\alpha_1, \dots, \alpha_n, \lambda_1, \lambda_2} J(\alpha_1, \dots, \alpha_n, \lambda_1, \lambda_2)$ .

This involves a linear system having (n+2) equations with (n+2) unknowns :

$$\frac{\partial J}{\partial \alpha_i} = 0, \quad i=1, \dots, n \quad A_1 = 0 \quad A_2 = 0$$

The previous methods was numerically tested on microcomputers. (Apple, Commodore).

### 4. CONCLUSION.

This technique has all the advantages of the finite elements methods. But on top of that, this method is simple, fast and can be used on microcomputers because we don't need a large memory space.

This method can also be used for approaching functions with several variables.

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Modelling and optimization of buffer stocks in a  
production line

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We consider a multi-stage production system where not only the demand is subject to uncertainty but the production process may also be disturbed. On some stages of the production process machine failures, faulty products, breakdowns may occur. To describe the disturbed production different models of random processes are considered. In many cases simple continuous random processes are good approximations but the batch production systems often have to be described by random step functions. Some new type of these processes have been formulated for the problem.

In practice buffer stocks ensure the continuous production and demand satisfaction for the system considered. It is a great difficulty to provide for continuous production with reasonable low level of in - process inventories. Stochastic programming models are formulated for the optimal allocation of the buffer stocks in raw material, in in-process stocks on the different production levels and in

finished goods. The optimality criterion is the maximal service level of the system, where the service level is measured by the probability of non-interruption in supply of processing and in demand satisfaction. In an other type of models a prescribed high service level of the production - supply system should be ensured with minimal investment in buffer stocks.

The solution of the above stochastic programming problems is rather sophisticated in some cases, so for the practical application simple approximate solution methods are derived, too. These results have made it possible for us to solve these problems effectively and have given an efficient tool for decision makers in planning and production managers in control of buffer stocks.

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New Type of Triangulations for  
Simplicial Methods of Solving Equations

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1. Efficiency of the simplicial algorithms for computation of the fixed points of continuous mappings, for solving nonlinear equations, computation of economic equilibria and cores of  $n$  person games depends heavily on the choice of the type of triangulation used.

2. New interpretation of existing methods of triangulation is discussed permitting to obtain these triangulations in the unified scheme.

3. New types of effective triangulations are proposed based on the new scheme.

4. Pivot rules are described corresponding to the new triangulations.

5. The algorithms of variable dimensions using new types of triangulations are proposed.

6. The results of computational experiments show the efficiency of the new triangulations.

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## AN ALGORITHM FOR FAST DIFFERENTIATIONS AND ITS APPLICATIONS

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It was implied until lately that  $T(F')$  - the complexity of gradient evaluation for function with  $n$  variables is  $O(nT(F))$ . Recently other algorithms for gradient evaluation were presented in [1] and independently in [2]. The running time of these algorithms does not depend on the number of variables and satisfies the relationship  $T(F') = O(T(F))$ . Such decreasing of running time for gradient evaluation makes it possible to increase considerably the efficiency of nonlinear minimization methods.

Let  $\mathcal{F}$  be a basic set of differentiable functions. For each  $f \in \mathcal{F}$  numbers  $T(f)$  and  $T(f')$  denote the complexity of  $f$  and of the gradient  $f$  evaluation. We suppose  $\forall f \in \mathcal{F} \quad T(f') \leq CT(f)$ , where  $C$  is a common constant for the whole basic set. Let  $(x_1, \dots, x_n)$  be the arguments of the function  $F$  defined in  $R^n$ . The function is called factorable if an algorithm of its evaluation may be presented in such a form:

$x_{n+1} = f_1(x_1)$  Here  $f_i \in \mathcal{F}$ ,  $x_i$  is the set of its arguments. The  
 $x_{n+2} = f_2(x_2)$  complexity of  $F$  evaluation is  $T(F) = \sum T(f_i)$ . We  
 . . . suggest such an algorithm for  $F'$  evaluation for  
 $F = x_{n+k} = f_k(x_k)$  a factorable function  $F$  that  $T(F, F') \leq DT(F)$ , where

$D$  is a constant depending only on the basic set and not depending on the complexity and the number of variables of  $F$ . The algorithm presented in [1] is intended for a gradient evaluation of the rational function. In this case the basic set is  $(+, -, \cdot, /)$ . If  $T(+)=T(-)=0$  and  $T(\cdot)=T(/)=1$  then  $D=3$  and if  $T(+)=T(-)=T(\cdot)=T(/)=1$  then  $D=4$  for our algorithm. These

estimations are just like the estimations presented in [1] for their algorithm.

Let  $F''$  denote the matrices of the second derivatives. Generalizing the method presented in [2] we can suggest algorithms for  $F''$  evaluation with the estimation  $T(F'')=O(nT(F))$  and for  $F''y$  evaluation with the estimation  $T(F''y)=O(T(F))$ , where  $y \in R^n$ . The latter algorithm can be used for constructing Newton direction  $(F'')^{-1}F'$ . This algorithm does not need memory to store matrices and the estimation of its running time is  $O(nT(F))$ .

Applying the new algorithm makes it possible to analyse the spreading of computational errors because evaluating the gradient we calculate additionally  $\partial F/\partial x_i$  when  $i > n$ . It enables us to see how the error of calculating  $f_i$  influences the error of calculating  $F$ .

Some functions actually used are not factorable. Calculating these functions we use conditional expressions and do-loops. We can generalize our result for the function defined by an arbitrary code, but with a considerable growth of memory. However in some cases we can present an algorithm for gradient evaluation with  $O(T(F))$  estimation of running time without such growth of memory.

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CORPORATE PLANNING AND THE VALUE OF INFORMATION

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ABSTRACT

The strategies and policies of information systems development are created or should be created at the level of the whole organization, i.e., at the corporate level. Therefore, it is important to assess the information requirements of the organization at the corporate level, too. Otherwise partial, conflicting and inconsistent system structures may appear. Specially, the assessment of the corporatewide information requirements is pertinent today when the data processing techniques allow us to develop total, even worldwide information systems (data nets, hierarchical consolidation models).

One way to evaluate the information requirements is just to ask, to interview the decision makers. This approach may be relevant in very early phases of the assessment process but when searching and selecting new alternatives this direct approach is not sufficient enough. Ackoff puts it as follows: "One cannot specify what information is required for decision making until an explanatory model of the decision process and the system involved has been constructed and tested. Information systems are subsystems of control systems. They cannot be designed adequately without taking control into account. ... A decision model identifies what information is required and, hence, what information is relevant."

Particularly, the relevancy or the value of information can be evaluated by the 'explanatory' corporate model. By a corporation model is generally meant a model where the relations between the finance, marketing and production functions are described simultaneously.

When we are investigating the value of information at the corporate level it is difficult if not impossible to determine only one quality attribute of information which could characterize the relevancy (value) of information. Furthermore, it is impossible to determine only one final evaluation criterion. Therefore, in this study a multiattribute and multicriteria approach is used to assess the value of information at the corporate level. For instance delay, inconsistency, underestimation, and overestimation may be used as the quality attributes of information and total profits, cash balance, market share, or the value of the production plant may be used as evaluation criteria.

The model illustrating the whole corporation should be multidimensional and complicated enough to describe the structure, dynamic action and goals and objectives of the real corporation. Specially, the description of the corporate planning, decision making and control processes is important.

In our experiments we have used a methodology based on optimal control approach to optimize the selected criteria. After the integration of some features of system dynamics to the optimal control approach we have developed a useful tool to model economic and business problems.

The results of our experiments with a corporate model, which represents a firm operating in the metal industry, indicate for instance that the recommendable characters of information systems highly depend on the evaluation criteria. For some lower level functions in an organization information and its particular attribute may prove to be important. Instead, the criteria which are used to evaluate the performance of the whole corporation, for instance profit or financial stability, are not sensitive to the same information attribute.

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On some inverse problems in the system of partial differential equations for the carrier distributions in semiconductors

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The paper deals with some parameter determination problems (inverse problems, deterministic identification, optimal control) by optimality criteria in different boundary value problems for the parametric system of stationary partial differential equations describing the carrier distributions in bounded domains  $G \subset \mathbb{R}^n$ ,  $n=1,2,3$ , of semiconductors:

- (1)  $\operatorname{div} J_0 = q(N - y_1 + y_2)$  ,  $J_0 = -D^0 \operatorname{grad} y_0$  ,
- (2)  $\operatorname{div} J_1 + R(y_1, y_2) = 0$  ,  $J_1 = -(D^1 \operatorname{grad} y_1 - y_1 \mu_1 \operatorname{grad} y_0)$ ,
- (3)  $\operatorname{div} J_2 + R(y_1, y_2) = 0$  ,  $J_2 = -(D^2 \operatorname{grad} y_2 + y_2 \mu_2 \operatorname{grad} y_0)$ .

Here  $y_0$  is the electrostatic potential,  $y_1, y_2$  are the mobile electron and hole current densities,  $D^0$  is the dielectric coefficient,  $N$  is the net density of ionized impurities,  $D^1, D^2$  are the coefficients of diffusivity for  $y_1$  and  $y_2$ ,  $J_1, J_2$  are the electron and hole current densities,  $\mu_1, \mu_2$  are the electron and hole mobilities,  $R$  the Shockley-Read-Hall recombination. The boundary value problems are considered in corresponding subspaces of the product

$W_2^1(G) \times W_p^1(G) \times W_p^1(G)$  with Sobolev spaces  $W_q^1(G)$ ,  $q=r$  and  $q=p$ ,

$\frac{2n}{n+1} < p < +\infty$ . As parameters we use the coefficients

$D^i = D^i(s)$ ,  $s \in G$ ,  $D^i = D^i(|\operatorname{grad} y_1|)$ ,  $i=0,1,2$ , and the dotands  $N$ .

We give sufficient conditions for the existence of optimal parameters. Furthermore we estimate from above the diameter

of the set of optimal parameters and consider the problems of sensitivity and tolerance for this set in form of concrete estimates which means Hölder continuity in the case of unique optimal parameter.

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DUALITY IN PROBABILISTIC CONSTRAINED LINEAR PROGRAMMING

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We are concerned with the following problem:

(1) Find  $\hat{x}$ , if it exists, such that

$$\sup_{u \in U} uA\hat{x} = \min_{x \in X} \sup_{u \in U} uAx$$

$$\hat{x} \in X = \{x \in R^n \mid \mathcal{P}(Ax \geq \beta) \geq p, x \geq 0\}$$

$$U = \{u \in R^m \mid \mathcal{P}(uA \leq r) \geq q, u \geq 0\}$$

where  $A$  is a given  $m \times n$  dimensional deterministic matrix;  $\mathcal{P}$  denotes probability;  $\beta$  is a  $m$ -dimensional random vector and  $r$  is a  $n$ -dimensional random vector with given joint probability distribution functions  $F_\beta$  and  $F_r$  respectively;  $p$  and  $q$  are prescribed reliability levels,  $0 < p < 1$ ,  $0 < q < 1$ ; the vectors  $x$  and  $u$  are comprised of variables.

The interpretation of this problem is similar to the game theoretical interpretation of linear programs. We investigate conditions under which  $uAx$  has a saddlepoint with respect to minimizing over  $X$  and maximizing over  $U$ .

Define the sets  $Y$ ,  $X(y)$ ,  $C$ ,  $U(c)$  to be

$$Y = \{y \in R^m \mid y \in \text{supp } F_\beta, F_\beta(y) \geq p\},$$

$$X(y) = \{x \in R^n \mid Ax \geq y, x \geq 0\} \text{ for given } y \in R^m,$$

$$C = \{c \in R^n \mid -c \in \text{supp } F_r, F_r(-c) \geq q\},$$

$$U(c) = \{u \in R^m \mid uA \leq c, u \geq 0\} \text{ for given } c \in R^n$$

where  $\text{supp } F_r$  and  $\text{supp } F_\beta$  are the support sets of the probability distributions  $F_r$  and  $F_\beta$ , respectively.

Then  $X = \{x \mid \exists y \in Y : x \in X(y)\}$

and  $U = \{u \mid \exists c \in C : u \in U(c)\}.$

The proof of the following theorem is based on stochastic programming results of Prékopa and the author, on linear programming theory, on the Kuhn-Tucker saddlepoint necessary optimality theorem, and on Rockafellar's convex analysis results.

Theorem. Suppose  $\text{supp } F_{-r}$  is bounded,  $F_{-r}$  is strictly logarithmic concave and continuously differentiable on  $\text{int } \text{supp } F_{-r}$ . Suppose  $\text{int } \{c \in C \mid U(c) \neq \emptyset\} \neq \emptyset$ . Suppose  $F_p$  is quasiconcave and  $\text{int } \{y \in Y \mid X(y) \neq \emptyset\} \neq \emptyset$ . If (1) has an optimal solution  $\hat{x}$  then there exists  $\hat{u} \in U$  such that  $\hat{u}$  maximizes  $uA\hat{x}$  over the set  $U$ . Furthermore, for  $\hat{y} = A\hat{x}$  and  $\hat{c} = \hat{u}A$  it holds

$$\begin{aligned}
 \sup_{c \in C} \sup_{u \in U(c)} \inf_{y \in Y} uy &= \hat{u}\hat{y} = \\
 &= \sup_{u \in U} \inf_{x \in X} uAx = \\
 &= \inf_{x \in X} \sup_{u \in U} uAx = \\
 &= \inf_{y \in Y} \inf_{x \in X(y)} \sup_{c \in C} cx = \\
 &= \hat{c}\hat{x} .
 \end{aligned}$$

On the Manifold of Control Processes  
in Lagrange Problems

András Kósa

The Mosre theory turns out to be a highly efficient tool in the global study of extremal problems. In order to set up an appropriate theory for general Lagrange problems of calculus of variations, first of all, a convenient differential manifold structure must be given on the set of the admissible processes.

Suppose we are given  $m, n \in \mathbb{N}$ , an open domain  $D \subset \mathbb{R} \times \mathbb{R}^n$  and functions

$$f: D \rightarrow \mathbb{R}^n, \quad g: D \rightarrow \mathbb{R}^{n \times m}.$$

Assume that the partial derivatives  $\partial_2 f$ ,  $\partial_2 g$  exist and the functions  $f, g, \partial_2 f, \partial_2 g$  satisfy some Carathéodory type conditions. Let  $(t_0, \xi_0), (t_1, \xi_1) \in D$ ,  $t_0 < t_1$ , and

$$A := \{(x, u) \in H_n^1[t_0, t_1] \times L_m^2[t_0, t_1] \mid (t, x(t)) \in D \quad (t \in [t_0, t_1])\}.$$

Put  $j := \text{id}_{[t_0, t_1]}$  and for all pairs  $(x, u) \in A$  consider the system

$$(1) \quad \dot{x} = f \circ (j, x) + g \circ (j, x) u,$$

$$(2) \quad x(t_0) = \xi_0, \quad x(t_1) = \xi_1.$$

The set of admissible processes is defined in the following way:

$$M := \{(x, u) \in A \mid (1) \text{ and } (2) \text{ are fulfilled}\}.$$

An embedded manifold structure is given on  $M$  provided that the variational equations of (1) taken along fixed processes have certain properties of reachability in system-theoretical sense. As an application of this constructuin to Lagrange problems, in an earlier paper the homotopy type of the manifold of the admissible processe has been calculated.

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On the Numerical Solution of Certain

Time-Minimal Control Problems

of Second Order

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If one applies Galerkin's method to the approximate solution of time-minimal distributed control problems for wave or plate equations, then one obtains a sequence of finite-dimensional linear control problems with state equations of the form

$$\dot{Y}(t) + A_n y(t) = b(t), \quad t > 0, \quad (1)$$

where  $A_n$  is a constant symmetric and positive definite  $n \times n$  - matrix and the control function  $b = b(t)$  is allowed to vary in  $L^2((0, \infty), \mathbb{R}^n)$ . Initial conditions are given of the form

$$y(0) = y_0, \quad \dot{y}(0) = \dot{y}_0 \quad (2)$$

with fixed vectors  $y_0, \dot{y}_0 \in \mathbb{R}^n$  and the problem consists of finding the smallest time  $T = T(M)$  and a control function  $b_M \in L^2((0, \infty), \mathbb{R}^n)$  with  $\|b_M\|_{2, T(M)} \leq M$  for some given positive constant  $M$  such that the corresponding solution  $y = y(t)$  of (1) and (2) for  $b = b_M$  satisfies

$$y(T) = \dot{y}(T) = \theta_n \quad (3)$$

for  $T = T(M)$  where, for any  $T > 0$  and  $b \in L^2((0, \infty), \mathbb{R}^n)$ , the norm  $\|b\|_{2, T}$  is defined by

$$\|b\|_{2, T} = \left( \int_0^T b(t)^T b(t) dt \right)^{1/2}.$$

It can be shown that for any choice of  $y_0, \dot{y}_0 \in \mathbb{R}^n$  and  $M > 0$  there is exactly one time-minimal null-control  $b_M \in L^2((0, \infty), \mathbb{R}^n)$  which can be characterized as follows: Let  $b_T \in L^2((0, \infty), \mathbb{R}^n)$ , for every  $T > 0$ , denote the minimum norm control (which is

uniquely defined) such that the corresponding solution  $y = y(t)$  of (1) and (2) for  $b = b_T$  satisfies (3). Then

$$T = T(M) \Leftrightarrow \|b_T\|_{2,T} = M \quad (4)$$

in which case  $b_T$  is the unique time-minimal null-control.

Since  $b_T$  can be explicitly expressed in terms of the eigenvalues and eigenvectors of  $\Lambda_n$  and also  $\|b_T\|_{2,T}$  can be easily computed, the determination of the time-minimal null-control  $b_M$  can be conveniently achieved by solving the right-hand equation of (4). This can be done by using the secant method which only requires the calculation of function values.

Numerical results are presented.

## OPTIMAL LOCATION UNDER EQUILIBRIUM ALLOCATION

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We consider the problem of locating a number of facilities in order to provide service at least cost to a given set of users with pre-specified demands. Both in the real world and in the models set up for analyzing such locational decision problems, two conceptually interdependent phases should be distinguished: the location phase in which a specific site for each facility is chosen, and the allocation phase in which each user is assigned to a specific subset of the facilities established.

The far majority of existing models assume both phases to be under the decision maker's (DM) control. This means that the DM has full freedom to choose among the locations available and that the DM can determine which facilities are to serve which users. Such an assumption is quite plausible in cases where the service is delivered to the users. Another category of service, however, is the so-called travel-for service, that is, service is consumed at the facilities where it is supplied. In such cases, the DM's control over the allocation phase becomes rather questionable. Even if users are assigned by the DM to facilities in some overall optimal manner, there may well exist a different assignment pattern which better reflects the users' individual preferences. Typically, such a situation may occur if a large number of users patronizes the same facility and congestion results. In such cases, the DM might rather leave it to the users to determine the allocation pattern on an individual basis and take the users' behaviour explicitly into account when the location of the facilities is decided upon.

For the standard prototype problems of discrete location theory, congestion at the facilities has not yet been investigated from an equilibrium point of view. Our aim is to study such phenomena under the assumption that the users so to speak "solve" the allocation part of the problem by assigning themselves to facilities such that an equilibrium state results.

As part of the investigation we will demonstrate that an allocation of users to facilities so obtained may differ from an overall optimal solution. We will furthermore devise algorithms, partly borrowed from the field of traffic assignment, for solving extensions of well-known prototype location problems in the sense that equilibrium (allocation) solutions are searched for.

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## NASH GAME WITH REGULAR PERTURBATION

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Up to now, the theory of differential Nash games has been considered in category of a linear model and quadratic cost functionals. Very little attention has been paid to games with a nonlinear model. In this paper we consider the Nash differential game whose linear model is perturbed by nonlinear terms, viz.:

$$\dot{x} = Ax + \epsilon f(x, t) + B_1 u_1 + B_2 u_2, \quad x(0) = x_0 \quad (1)$$

$$J_i = \frac{1}{2} x'(T) Q_{if} x(T) + \frac{1}{2} \int_0^T (x' Q_i x + u_i' R_{ii} u_i + u_j' R_{ij} u_j) dt \quad (2)$$

where  $i, j=1, 2, i \neq j$ , and, vectors and matrices are of appropriate dimensions.

The necessary optimal conditions for the Nash feedback strategy give a nonlinear two-point boundary value problem

$$\dot{x} = \frac{\partial H_i}{\partial p_i}, \quad x(0) = x_0 \quad (3)$$

$$\dot{p}_i = -\frac{\partial H_i}{\partial x} - \left( \frac{\partial u_j}{\partial x} \right)' \frac{\partial H_j}{\partial u_j}, \quad p_i(x, T) = Q_{if} x(T) \quad (4)$$

where  $H_i$  denotes the Hamiltonian for the  $i$ -th player,  $i, j=1, 2, i \neq j$ .

Generally (3)-(4) is difficult to solve and we suppose that  $p_i$  can be extended into a power series with respect to  $\epsilon$

$$p_i = \sum_{k=0}^{\infty} \epsilon^k p_i^{(k)}(x, t) \quad (5)$$

Substituting (5) into (4) and equating powers of  $\epsilon$  one obtains

coupled Riccati matrix differential equations (for  $k=0$ ) and a series of quasi-linear partial differential equations. Considering the perturbation term  $f(x,t)$  as a polynomial in  $x$ , it is proved that  $p_i^{(k)}(x,t)$  is also a polynomial in  $x$ . The coefficients of the polynomials can be determined in a pretty simple way by solution of a proper set of linear ordinary equations.

Several theorems have been proved about asymptotic properties of the approximation of the Nash equilibrium strategies.

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A Worst Case Analysis for the Optimal Gradient Method

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A frequent experience in continuous models of applied mathematics is that the actual behaviour of an algorithm does not verify pessimistic forecasts suggested by theoretical considerations. In the following we discuss one possible reason of this phenomenon.

Theoretical convergence analysis of an algorithm considers in most cases only exact arithmetic. It is generally accepted that inexact arithmetic has only disadvantages. There are situations however, when roundoff errors, composing a random noise, prevent the realization of the worst case predicted by theory, i.e. the worst case is numerically instable.

We can demonstrate this on unconstrained optimization. As a simple illustration, we choose first the optimal gradient method. Suppose that  $f(\underline{x})$  is convex, twice continuously differentiable and has a minimal value  $M$  on an open convex set  $A \subseteq R_m$ . Let  $D \subset A$  be convex compact. Suppose that  $D$  contains  $\bar{x}$ , the minimum set of  $f(\underline{x})$ . Let  $\{\underline{x}_n\}_{n=0}^{\infty}$

be the trajectory of the optimal gradient method, where  $\underline{x}_0 \in D$ . These assumptions are sufficient to ensure that  $f(\underline{x}_n) \rightarrow M, n \rightarrow \infty$ . If the Hessian of  $f(\underline{x})$  is positive definite on  $D$ , then the convergence is at least linear. Otherwise only sublinear convergence is proved in the literature, more precisely, convergence of  $O(1/n)$ . In the paper we prove that this latter estimation cannot be improved in general /it is sharp for certain functions/. At the same time a simple probabilistic error model guarantees that the trajectory cannot follow a route which might produce sublinear convergence. The reason is that the set of such routes is of measure 0 within the set of all possible routes. The idea of the proof is

$$\left( \frac{f(\underline{x}_n) - M}{f(\underline{x}_0) - M} \right)^{1/n} \leq \frac{1}{n} \sum_{i=1}^n \frac{f(\underline{x}_i) - M}{f(\underline{x}_{i-1}) - M}$$

Here the quotients on the right hand side are random variables and their sum can easily be estimated by means of martingale theory.

We point out that similar, though not identical arguments also apply to more advanced methods, even for Newton's method and some quasi-Newton methods.

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ANALYSIS OF DNA DISTRIBUTIONS FROM FLOW CYTOMETRY  
BY MEANS OF AN OPTIMIZATION PROCEDURE

by F. LAMPARIELLO\* and S. LUCIDI\* (ITALY)

ABSTRACT

The problem of estimating the fractions of a population corresponding to the various phases of the cell cycle from the DNA content distribution obtained by flow cytometry is an important problem in the diagnosis and treatment of tumor diseases.

Since the introduction in 1969 of flow cytometry in the study of cell kinetics, several methods have been proposed for the case of cytogenetically homogeneous populations (for references see [1]). These methods are based on a mathematical model of the DNA distribution and of the corresponding fluorescence distribution, which determines a theoretical histogram to be compared with the measurements.

The application of flow cytometry to the study of human malignancies reveals very often an abnormal DNA content consisting in the presence of abnormal stemlines (aneuploidy) with respect to the normal diploid cell population. Recently a graphical method for estimating the fraction of cells in S-phase from clinical tumor samples containing aneuploid cell populations has been proposed [2].

However a qualitative or semi-quantitative analysis of DNA histograms may lead to somehow unreliable results particularly when the different components in the mixture exhibit a high overlapping degree or when the aneuploid component fraction is relatively small, as may be in the so-called precancerous conditions. On the other hand it appears of great interest to give a quantitative evaluation of aneuploidy as a tool of predicting to some extent the clinical behaviour of the neoplastic disease or for confirming a possible suspicion of malignant lesion.

In this paper we present an automatic procedure for recovering the unknown model parameters from samples containing mix-

tures of two or more cell populations.

At first we propose some modeling assumptions for DNA and fluorescence distributions. Then we present a parameter estimation procedure based on the minimization of an error index resulting from the comparison between the theoretical and the experimental histograms. In order to take constraints on the parameters into account, we introduce suitable changes of variables so that the original minimization problem is transformed into an unconstrained one. The solution is attained by means of a Newton-type method which offers, as known, fast convergence properties. As regards the line search technique, we adopt the non monotone steplength selection rule suggested in [3], which allows a considerable saving in computations.

The performance of the procedure has been tested against simulated data and the results obtained show its reliability. The procedure has been also applied to some clinical samples with satisfactory results as far as the final fit is concerned.

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An Automated System of Planning Computations  
of a Long-Range Planning on the Basis of  
DISPLAN

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An automated system of planning computations in conditions of a long-range planning we mean a relatively rounded system serving to objectification of a long-range planning in the process of its formation on the basis of economic and mathematic computations using the computation technique.

A consolidation of the information system of economic and mathematic methods with the methodology of the planning activity in the form of DISPLAN and choice procedures. DISPLAN - a dialogue system of planning - will be the basic technical component for building up of a dialogue between the user and the system.

DISPLAN in a long-range planning will represent a complex of economic, mathematic, statistical and technical means bounded with the data base which will be formed on line and in which the user will handle the extra prepared language or instructions.

The task of the DISPLAN will be to secure the management by the parameters of the formed balance system, the choice procedures, the goal programming and resource approach in a long range planning on the basis of forecasting and economic projections.

The function of this system in a broader sense of the term will be a demonstration of mutually tuned quantificated conceptions of the development of national economy and balancing of complex programs of the economic development on the basis of formulated goals and effectiveness criteria.

In planning of the long-range outlook there exist multidimensional aspects of economic rationality often of a different and conflict nature.

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Boundary control problems for systems described  
by hyperbolic partial differential equations.

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Abstract

This talk will focus on various aspects of the boundary stabilization problem for systems described by partial differential equations of hyperbolic type in a bounded open domain in  $R^n$ : these will include the related topic of algebraic Riccati equations, as well as a discussion of numerical schemes for computing Riccati operators. Parabolic problems will also be mentioned but mostly for the sake of comparing results, and techniques which are available in the two cases. More specifically, we shall divide our talk in three parts.

Part 1: Exponential versus stabilization with feedback control action exercised on the boundary of the spatial domain.

Part 2: Related stabilization problems via the Algebraic Riccati Equation.

Part 3: Numerical techniques for solving Algebraic Riccati Equations. Positive results and open questions.

In part 1), we shall first dwell on various positive and negative stabilization results in the case of hyperbolic equations and then we shall treat in more detail the exponential decay of hyperbolic feedback solutions with feedback control action on the velocity exercised in the Dirichlet boundary conditions of a convex domain.

Examples of hyperbolic feedbacks will be given which produce strongly stable but not exponentially stable solutions: these will involve suitable feedback operators of finite rank. In the case of the wave equation defined on the convex domains it will be shown that the feedback operator

$Fy = -\frac{\partial}{\partial \bar{\partial}} A^{-1}y_t$  (where  $A^{-1}$  is the inverse of Laplasian) inserted in Dirichlet

boundary conditions produces exponential decay of all corresponding solutions.

The afore mentioned positive results of exponential stabilization may be taken to constitute the starting point of the analysis in part 2) leading to the Riccati operator of the regulation problem. The Riccati operator yields another way of obtaining exponential decay by boundary feedback. The advantage here is that the Riccati operator has a potential of being computed numerically from the appropriate approximation of algebraic Riccati equation. This leads naturally to part 3). There the main theoretical difficulties in the numerical computations of Riccati operator are related to its low regularity; to lack of faithfulness of spectral properties of the approximation to those of the original system. To overcome the low regularity of the Riccati operator, a procedure which combines approximation with regularization may be applied. However, the issue

of selecting the appropriate approximation schemes is a delicate one. An appropriate approximation scheme must have the properties that preserves the spectral properties of the original feedback system. This is not a fact that can be taken for granted, since some approximation schemes commonly used like Finite Elements Methods do not possess these properties even in a very simple case of first order hyperbolic equation.

Positive results based on Finite Difference scheme applied to the computation of Riccati Operator via approximation of Algebraic Riccati equation will be presented.

UNCERTAIN SYSTEMS: ROBUSTNESS OF ULTIMATE BOUNDEDNESS CONTROL WITH RESPECT  
TO NEGLECTED DYNAMICS

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Abstract: Feedback control of a class of imperfectly known dynamical systems is considered. On the basis of known functional properties and bounds relating to the uncertain elements in the generic system, and initially neglecting actuator and sensor dynamics, a feedback structure is determined (using established Lyapunov-based techniques [1],[2]) which guarantees uniform ultimate boundedness of all motions of the system. The effects on performance of the introduction of uncertain actuator dynamics and uncertain sensor dynamics are subsequently investigated.

In particular, the class of systems considered typically consists of:

(a) a dynamical process modelled by

$$\begin{aligned} \dot{x}(t) &= A x(t) + \bar{B}(t)z(t) + F(t, x(t)), \quad x(t) \in \mathbb{R}^n \\ \bar{B}(t) &= B + \Delta B(t) \end{aligned} \quad (1)$$

which is to be regulated via an appropriately determined feedback operator  $\Phi$ , operating on the output  $y(t)$  of (b) a dynamical state sensor given by

$$\begin{aligned} \mu_s \dot{y}(t) &= \bar{D}(t) [y(t) - x(t)], \quad y(t) \in \mathbb{R}^n \\ \bar{D}(t) &= D + \Delta D(t), \quad \sigma(D) \subset \mathbb{C}^- \end{aligned} \quad (2)$$

and generating  $u(t) = \Phi(y(t)) \in \mathbb{R}^m$  at the input of (c) a dynamical actuator given by

$$\begin{aligned} \mu_a \dot{z}(t) &= \bar{C}(t) [z(t) - u(t)], \quad z(t) \in \mathbb{R}^m \\ \bar{C}(t) &= C + \Delta C(t), \quad \sigma(C) \subset \mathbb{C}^- \end{aligned} \quad (3)$$

Here,  $A$ ,  $B$ ,  $C$ , and  $D$  are known matrices; in (2) and (3),  $\sigma(\cdot)$  denotes spectrum (so that  $C$  and  $D$  are asymptotically stable) and  $\mu_s \geq 0$ ,  $\mu_a \geq 0$  are parameters, the values of which reflect (inversely) the "fastness" of the sensor and actuator dynamics. The overall system is subject to uncertainty: the functions  $\Delta B: \mathbb{R} \rightarrow \mathbb{R}^{n \times m}$ ,  $\Delta C: \mathbb{R} \rightarrow \mathbb{R}^{m \times m}$ ,  $\Delta D: \mathbb{R} \rightarrow \mathbb{R}^{n \times n}$ , and  $F: \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}^n$  are unknown members of known uncertainty classes  $B$ ,  $C$ ,  $D$ , and  $F$ , respectively, which are implicitly defined via the functional properties and bounds alluded to above. The classes  $B$  and  $F$  are such that, in the absence of sensor and actuator dynamics (i.e.  $\mu_s = 0 = \mu_a$ ), the feedback operator can be determined so as to yield satisfactory performance (in the sense that, for arbitrary uncertainty realizations  $\Delta B \in B$  and  $F \in F$ , global uniform ultimate boundedness with respect to an acceptable neighbourhood  $N$  of the zero state is guaranteed). The main question addressed is essentially that of robustness with respect to neglected dynamics, viz. how does the presence of sensor and actuator dynamics affect the performance of the feedback controlled uncertain system?

Under the assumption that the uncertainty classes  $C$  and  $D$  consist of bounded  $C^1$  matrix-valued functions with known bounds (sufficiently small), two cases are investigated: (i)  $\mu_a > 0$ ,  $\mu_s = 0$  (actuator dynamics only); and (ii)  $\mu_s > 0$ ,  $\mu_a = 0$  (sensor dynamics only). In the former case, the existence of a threshold value  $\mu_a^* > 0$  is established such that, for each value  $\mu_a \in (0, \mu_a^*)$ , the process (1) remains globally uniformly ultimately bounded (under arbitrary uncertainty realizations  $\Delta B \in B$ ,  $\Delta C \in C$ ,  $F \in F$ ) but now with respect to a larger set  $N_a \supset N$ . In other words, the property of global uniform ultimate boundedness of the feedback-controlled reduced order system (i.e. process (1) with  $z(t) = u(t) = \Phi(y(t)) = \Phi(x(t))$ ) is structurally stable in the sense that it is qualitatively retained when (sufficiently fast) actuator dynamics are introduced. Analogous results for case (ii) are subsequently deduced. Finally, an example illustrating these robustness properties is presented.

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ESTIMATIONS OF COMPLEXITY OF THE  
TABULATION FOR DISCRETE EXTREMAL PROBLEMS

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The notions of the radius of stability and the sphere of stability for discrete extremal (trajectory) problems were introduced and studied in [1-2]. The radius of stability of the matrix  $A$  is the supremum of radii of open spheres with the centre in  $A$ , and any matrix from this spheres have no optimal trajectory differ from optimal in  $A$ . This spheres is called the spheres of stability for the matrix  $A$ . If all trajectories in  $A$  are equal to each other, then the radius of stability in this matrix is equal to zero by definition.

Thus, if we solve the problem on matrix  $A$ , then we define an optimal trajectories in all matrices from the spheres of stability of the matrix  $A$ . All definitions and theorems on the theory of stability in trajectory problems are contained in [1-2]

$R_{mn}$  is the matrix's space with the Chebishev's metric, ( $m \times n$  - dimensions of the matrix). A trajectory problem is given on matrices from  $R_{mn}$ .  $R_0$  is subspace of  $R_{mn}$ , the lengths of all trajectories on any matrix from  $R_0$  are equal to each other. The dimension of  $R_0$  is equal to  $k$ . The maximum number of elements in trajectory is  $n_0$ .  $S_r(A)$  is open sphere with the radius  $r$ , and  $A$  is the centre of  $S_r(A)$ .  $\bar{S}_r(A)$  is the closure of the  $S_r(A)$ .

Definition. The set  $W \subseteq R_{mn}$  is the covering of the set  $V \subseteq R_{mn}$ , if for any matrix  $B \in V$ ,  $B \in S_r(A)$  and  $S_r(A)$  is the sphere of stability of matrix  $A$ .

In what minimum number of matrices we need ( for example, from  $S_1(0)$  ) to solve the problem for knowing the optimal trajectory in any matrix from this sphere, that is the question on minimum cardinality of covering of given sphere by spheres of stability.

Theorem 1. There is no countable covering of  $\bar{S}_1(0)$  with spheres of stability or there closures.

The following theorem gives an answer to the question on the estimation of cardinality of the minimum covering  $\bar{S}_1(0)$  with spheres of stability except for the set  $U \subseteq S_1(0)$  with the Lebeg's measure  $\epsilon$ . This covering is called  $\epsilon$ -covering.

Theorem 2. For any  $\epsilon > 0$  there exists  $\epsilon$ -covering  $W$  of the set  $S_1(0)$  such, that

$$|W| \leq 2^{mn} (3t^2/\pi^2 + O(t \ln t))^{mn},$$

where  $t = (\max(2n_0, ] \epsilon^{k-mn} [ ))^{mn}$ .

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DISCRETE STABILITY OF STOCHASTIC PROGRAMMING  
 PROBLEMS WITH RECOURSE

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It is well-known that the dynamic (canonic) formulation of stochastic programming problem with recourse is defined recursively in finite-dimensional space, whereas the static formulation defines the problem in a function space. It is pointed out by Olsen [1], that the static formulation is more computationally tractable than the dynamic one and it can be solved in some cases by solving a sequence of finite-dimensional "discretizations". In this report we present sufficient conditions for discretization of the stochastic programming problem in  $L_p$ -space to be stable.

Consider the problem

$$\min \{f_1(x) + \int_S f_2(s, x, y(s)) \sigma(ds)\} = f^*$$

$$x \in X$$

$$g_{1j}(x) \leq 0, \quad j \in J_1, \tag{1}$$

$$y(s) \in Y \text{ for a.a. } s \in S,$$

$$g_{2j}(s, x, y(s)) \leq 0 \text{ for a.a. } s \in S \text{ and all } j \in J_2$$

(a.a. denotes almost all).

Here  $f_1: R^r \rightarrow R^1$ ,  $f_2: R^k \times R^r \times R^m \rightarrow R^1$ ,

$g_{1j}: R^r \rightarrow R^1$ ,  $j \in J_1$ ,  $g_{2j}: R^k \times R^r \times R^m \rightarrow R^1$ ,  $j \in J_2$ ,

$x \in R^r$ ,  $X \subset R^r$ ,  $y(s) \in L_p(S, B(S), \sigma; R^m)$ ,  $Y \subset R^m$ ,

$\sigma$  is the probability measure induced by a random vector  $s$ ,

$S \subset R^k$ ,  $\sigma(S) = 1$ ,  $J_1$  and  $J_2$  are finite sets of indices.

Consider instead of problem (1) the following discrete problem:

$$\min_{x \in X} \{ f_1(x) + \sum_{i=1}^n f_2(s_{in}, x, y_{in}) \sigma_{in} \} = f_n^* \quad (1n)$$

$$g_{1j}(x) \leq 0, \quad j \in J_1,$$

$$y_{in} \in Y, \quad i=1,2,\dots,n,$$

$$g_{2j}(s_{in}, x, y_{in}) \leq 0, \quad i=1,2,\dots,n, \quad j \in J_2.$$

Here  $n \in N = \{1,2,3,\dots\}$ ,  $s_{in} \in S$ .

Suppose that

1) the functions  $f_2(s, x, y)$ ,  $g_{2j}(s, x, y)$ ,  $j \in J_2$ , are continuous in  $(s, x, y)$ , convex and smooth in  $y$  for all  $(s, x)$ ;

2) the functions  $f_1(x)$ ,  $g_{1j}(x)$ ,  $j \in J_1$ , are convex and continuous;

3) the support  $S$  is bounded;

4) there exists a point  $[\bar{x}, \bar{y}(s)]$  such that  $g_{2j}(s, \bar{x}, \bar{y}(s)) < 0$  for a.a.  $s \in S$  and all  $j \in J_2$ .

Theorem. Suppose that a sequence of discrete measures  $\sigma_n$  ( $n \in N$ ) converges weakly to the probability measure  $\sigma$  and that the conditions 1)-4) are fulfilled. Then

$$f_n^* \rightarrow f^* \quad \text{as} \quad n \rightarrow \infty$$

and all limit points of solutions of problems (1n) ( $n \in N$ ) solve the problem (1).

Remark. If measure  $\sigma$  has a density, then it must be Riemann integrable.

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## Exact boundary controllability of an integro differential equation

Let  $\Omega = (0,1)^2$  be a given square in  $R^2$  with boundary  $\partial\Omega$ . We consider a viscoelastic incompressible and isotropic Kirchhoff plate. Let  $\Delta$  be the Laplacian on  $\Omega$ ,  $\rho_0$  the reference density, and  $G(t)$  be the relaxation modulus representing the memory of the material. Then the displacement  $u(t,x)$  of the material point  $x$  at the actual time  $t$  is governed by the integrodifferential equation

$$\rho_0 u_{tt}(t,x) + \Delta^2 u(t,x) + \int_0^t G(t-s) \Delta^2 u(s,x) ds = 0 \quad (1)$$

on  $\Omega \times (0,T)$ , with initial conditions on  $\Omega$

$$u(0,x) = u_0(x), \quad u_t(0,x) = v_0(x) \quad (2)$$

Here  $u_t$  means partial derivative wrt  $t$ ,  $G(t) = \frac{d}{dt} G(t)$ . The plate is considered to be supported along the  $\partial\Omega$  boundary and we are supposed to control the bending moments there, i.e.,

$$u(t,x) = 0, \quad \Delta u(t,x) = f(t) \quad \text{on } \partial\Omega. \quad (3)$$

Define  $H := L_2(\Omega)$ ,  $Q := H^2(\Omega) \cap H_0^1(\Omega)$ . Then the control problem can be stated as follows:

(CP) Specify subspaces  $Q_1 \subset H$ ,  $H_1 \subset Q$  such that given any pair of initial and final conditions  $(u_0, v_0), (u_1, v_1)$  in the subspace  $Q_1 \times H_1$  of the shifted energy space  $H \times Q$  we are to find a suitable control  $f \in L_2(0,T; L_2(\partial\Omega))$  steering the solution  $u(\cdot)$  of (1), (2), (3) from  $(u_0, v_0)$  to  $(u_1, v_1)$  in finite time  $T > 0$ .

Fattorini's method of lifting the boundary input into the state equation, see [2], based on Necas' extended Dirichlet input map, i.e.  $B: L_2(\partial\Omega) \rightarrow H$ ,  $Bf$  'solves'  $\Delta u = 0$  on  $\Omega$ ,  $u = f$  on  $\partial\Omega$ , see [3], culminates in an abstract integrodifferential equation with distributed load input: decomposing the solution  $u(t) = \exp(-a(\cdot)/2)t \cdot (v(t) + w(t))$ , where  $w(\cdot)$  solves the 'elastic' part of (1) with zero initial conditions and boundary conditions (3),  $f$  replaced by  $\exp(-a(\cdot)/2) \cdot t f(t)$ , then defining  $b(t) := -\int_0^t (\rho_0^{-1} s) \exp(-a(\cdot)/2) ds$ , and  $A := A + (a(\cdot)/2)I$ ,  $Au := \Delta^2 u$ ,  $c := 3/4 \cdot a(\cdot)^2 + \dot{a}(\cdot)$ ,  $v(\cdot)$  has to solve

$$v_{tt}(t) + A_a v(t) + \int_0^t b(t-s) A v(s) ds = a(\cdot) v_t(t) + cw(t) + \int_0^t b(t-s) w(s) ds. \quad (4)$$

Under natural differentiability assumptions (4) can be solved explicitly by Tsuruta's method [4]. For 'small' kernels  $b(\cdot)$  ( $a(\cdot)$ ) we recover the same controllability results, obtained for the unperturbed plate equation by a classical result of Ingham, applying a standard perturbation argument.

If  $a(0) = a'(0) = 0$ , and if  $a(\cdot)$  is analytic with exponential decay the perturbation due to the long memory of the material is in fact compact, since in this case the distributed load input in equation (4) is then continuously differentiable with values in  $H$  such that under suitable assumptions on the initial data the 'state'  $(v, v_t)$  keeps in the subspace  $D(A) \times Q$  of the energy space  $Q \times H$ , the embedding being compact. Since the output operator associated with the unperturbed plate equation is surjective acting from  $L_2(0, T, L_2(\partial\Omega))$  to  $Q_1 \times H_1$  it suffices to show that the adjoint of the solution operator for equation (1), (2), (3), is injective. This is done using the Laplace transform of the solution of the adjoint equation associated with (1), (3), which can be given explicitly. Then by a continuation argument similar to [1], developed for heat equations with memory, exact boundary controllability is shown.

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STABILIZABILITY BY THE FEEDBACKS  $-B^*$  AND  $-B^*P$ 

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ABSTRACT

Let  $(A, B)$  denote the linear dynamic system described by the abstract differential equation:

$$\dot{x} = Ax + Bu,$$

where  $A$  is the generator of a strongly continuous semigroup of bounded linear operators  $T(t)$ ,  $t \geq 0$ , (say), on a Hilbert space  $H$  (the state space), and  $B$  is a bounded linear operator from another Hilbert space  $U$  (the control space) to  $H$ . Suppose that the system is not stable, i.e., for some  $x (\neq 0)$  in  $H$ :  $T(t)x \not\rightarrow 0$  (in a prescribed sense),  $t \rightarrow \infty$ , and suppose that there is a bounded linear "state feedback" operator  $F: H \rightarrow U$ ,  $u = Fx$ , so that the "closed loop" system

$$\dot{x} = (A + BF)x$$

is stable, i.e., the semigroup  $S(t)$ ,  $t \geq 0$ , (say), generated by  $A + BF$  is such that, for each  $x$  in  $H$ :  $S(t)x \rightarrow 0$  (in a prescribed sense), as  $t \rightarrow \infty$ . Then the system  $(A, B)$  is said to be stabilizable by  $F$ . The stabilizability problem is an important problem in Control Theory.

In this paper we study stabilizability by means of the feedbacks  $-B^*$  and  $-B^*P$  --where  $P$  is a bounded linear, self-adjoint and non-negative operator on  $H$ .

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Now if the generator  $A$  is dissipative then so is  $A - BB^*$ .

Consequently,  $S(t)$ ,  $t \geq 0$ , is a semigroup of contraction operators over  $H$ . In this case weak and strong stabilizabilities can be studied, using the canonical decomposition of Hilbert space contractions of Bèla Sz-Nagy and C. Foias.

Here we study the case in which the generator  $A$  is not dissipative and we find necessary and sufficient conditions for the feedback  $-B^*$  weakly and strongly stabilizes the system  $(A, B)$ .

Next we turn to the feedback  $-B^*P$ . In general, as in the finite dimensional case, one can take  $F = B^*P$ , where  $P \geq 0$  satisfies the Steady State Riccati Equation (SSRE). Necessary and sufficient conditions for  $S(t)$ ,  $t \geq 0$ , to be exponentially stable are known. In this paper we concentrate on the case in which  $P$  is a solution of the SSRE, while  $S(t)$ ,  $t \geq 0$ , is not exponentially stable. It will be shown that the semigroup  $S(t)$ ,  $t \geq 0$ , is a quasi-affine transform of a contraction semigroup, as soon as the self-adjoint operator solution  $P$  is positive. Moreover, weak and strong stabilizabilities can only be achieved, in general, on a dense subspace. The important case in which the operator  $B$  is compact will also be studied. Finally, we discuss the case in which  $P \geq 0$  is not a solution of the SSRE. Necessary and sufficient conditions for  $A - BB^*P$  to generate a stable semigroup  $S(t)$ ,  $t \geq 0$ , will be given.

We must note the fact that  $A - BB^*P$  --where  $P > 0$  satisfies the SSRE-- generates a quasi-affine transform of a contraction semigroup has not been observed before. This is also the case for sufficient conditions for "approximate" stability --i.e., stability on a dense subspace-- of a quasi-affine transform of a contraction semigroup.

OPTIMAL IDENTIFICATION PROBLEMS  
OF REGIONALIZED VARIABLES SYSTEMS

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Constantly increasing demand for adequate description of natural phenomena is giving rise to the development of mathematical methods of system modelling and optimization.

In geology, hydrology and environmental sciences one searches for appropriate models of real systems which make possible the precise analysis and prediction of features of a system under consideration. Regionalized variables models may serve as the base for decision making in many fields of applied sciences, e.g. mining decisions, agriculture planning, civil engineering projects, economic forecasts. The high cost of a great deal of data to be collected and analysed and the importance of their accuracy resulting from large scale of a system render the application of optimization techniques indispensable.

In this paper mathematical models for optimization problems for the regionalized variables system are presented. It is assumed that the phenomenon of our concern is described by an unknown function  $z(\mathbf{x})$  which is considered to be one realization of a random function  $Z(\mathbf{x})$ . The function  $z(\mathbf{x})$  represents a system parameter of our interest, e.g. the thickness of a deposit or its quality parameters in geology, the water level in hydrology, soil para-

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meters in agriculture, the degree of pollution in a certain area.

The paper deals with the optimization problem of additional data points choice strategy in order to increase the accuracy of the given parameter estimation. A discrete optimization problem is considered. It is presumed that  $N$  possible locations for observation points are given and among them there are  $n$ ,  $n < N$ , already existing data points. On the base of obtained experimental data an approximate variogram has been determined. The optimization problem consists in selecting certain additional data points from among  $N-n$  points left. The choice should extremize a given objective function representing the quality and/or the cost of system identification.

The mathematical formulation of the problem presented, considerably modified in comparison with previous works, is introduced. A new adaptive multistage strategy for the optimal choice of additional data points is proposed. The strategy provides the advantage the successively obtained data are fully used and at each stage the decision is made on the base of the updated model.

Moreover, a multicriteria discrete optimization model for the choice of additional data points is presented and a complete method for solving such a problem is given. The proposed method consists in introducing an appropriate order relation with regard to values of objective functions given for the set of admissible solutions.

Some remarks related to the application of the proposed methods and possible extensions are enclosed.

On Markovian Representation of Stationary Gaussian Processes

Anders Lindquist and Giorgio Picci

Abstract

The following inverse problem is of central importance in stochastic systems theory. Given a stationary Gaussian vector process  $\{y(t); t \in \mathbb{R}\}$  of smallest possible dimension so that

$$(1) \quad y(t) = Cx(t)$$

for some matrix  $C$ , and determine a stochastic differential equation for  $x$ . This is the stochastic realization problem, first formulated by Kalman in 1965, and the representation is called a minimal stochastic realization.

As it stands, this problem may not be meaningful unless the given process has a rational spectral density, and hence a finite-dimensional representation is possible. Moreover, the concept of minimality needs a natural dimension-free formulation which also covers the infinite-dimensional situation. Finally, we want a coordinate-free theory which allows us to factor out, in the first analysis, the properties of the realizations which depend only on the choice of coordinates and may unduly complicate the picture.

To this end, we have developed a geometric theory of stochastic realization, in which the idea of state is defined through a fundamental property of conditional independence (splitting), a natural generalization of the property of state in the deterministic theory. This point of view provides a general framework for stochastic modeling in which problems of stochastic systems theory can be set.

Important areas for potential application of this theory include identification, stochastic model reduction, and stochastic control, and there is preliminary evidence that the basic ideas presented here will prove to be fruitful.

Moreover, there are already problems in estimation theory which have been successfully tackled by such an approach. Some cases in point are smoothing, interpolation, and, in general, problems with a noncausal information flow. Possible extensions of the theory presented here to the nonlinear (nonGaussian) case will provide solution to even wider areas of important application. For example, realization theory of finite-state processes would provide powerful technics to solve important problems in communication theory.

MATHEMATICAL SIMULATION MODEL OF POWER TRANSFORMER  
FOR ELECTRICAL POWER SYSTEM PROTECTIVE SCHEMES

This paper presents a mathematical model and a digital simulation model of three-phase transformer. The model covers the following transformer features:

- topology of electric and magnetic circuits,
- characteristics of the transformer plates,
- transient eddy-current losses in the transformer plates,
- transformer winding connections and star-point connection.

The novel approach consists in the application of equivalent gyrator circuit and the presentation of transient eddy-current losses, which is different to that conventional. The following assumptions have been made for building the model:

- the transformer components are lumped, linear and stationary except for the magnetizing characteristic which is non-linear,
- no interaction of leakage fluxes is taken into account,
- no inter-coil or inter-winding capacitance is taken into account,
- the distribution of winding leakage fluxes obeys the Rogowski's formulas,
- the magnetizing characteristic employed in this model is assumed to be explicit /in a function or table form/ but this does not eliminate the possibility of magnetic hysteresis loop feature to be taken into account, and
- the mathematical model is a set of hybrid equations representing an n-terminal pair network.

The digital model of transformer was built by the authors on the basis of simulation languages. The numerical model

building technique used up to now for transformers employs the state space approach where a mathematical model of the system under investigation is built in the form of a set of differential equations and proposes matrix analysis algorithms for solving these equations. Such a model-building technique is very difficult, time consuming and not capable of determining the currents and voltages at freely selected points of the modelled system in a simple and quick way.

While building the digital simulation model, the authors first built an analog block diagram /structural analog model/ of the system under investigation and then employed this diagram to write a program which took the system structure into account by describing the individual subassembly functions and their interrelations. Both analytic model-building and structural model-building techniques were used for creating the analog block diagram. The former took the present mathematical model of the system under investigation as the starting point while the latter used the system structure for building individual component models of the real system and involving them into a structural scheme.

The proposed model provided means of determining both electrical quantities, such as phase and line currents and voltages, and magnetical quantities. The authors employed this model to simulate and investigate the performance of 16 MVA transformer under transient conditions /inrush current/.

The paper presents examples of test results provided in the form of time diagrams/<sup>The results/</sup>which were obtained correspond closely with those obtained experimentally at laboratories and power system facilities as discussed in the available bibliography. These results were used as the basis for evaluating the protection schemes chosen for the transformer.

HIGHER ORDER RIGHT-DIFFERENTIABILITY OF SOLUTIONS TO CONVEX PROGRAMMING PROBLEMS WITHOUT STRICT COMPLEMENTARITY

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Let  $H=(-\delta, \delta)$  be a fixed set of real parameters.

Consider a family  $\{P_h\}$  of the following convex programming problems depending on  $h \in H$ :

$$(P_h) \quad \begin{cases} \text{find } u(h) \in R^n \text{ such that} \\ f(u(h), h) = \min_{u \in \phi_h} f(u, h) \end{cases}$$

where

$$\phi_h = \{u \in R^n \mid \phi^i(u, h) \leq 0, i=1, \dots, m\}$$

Assume:

- (i)  $f(\cdot, \cdot)$  is  $(p+1)$ -times continuously differentiable function on  $R^n \times H$
- (ii)  $f(\cdot, h)$  is strongly convex, uniformly with respect to  $h$
- (iii)  $\phi^i(\cdot, \cdot)$  are  $(p+1)$ -times continuously differentiable functions on  $R^n \times H$
- (iv) for each  $h \in H$   $\phi^i(\cdot, h)$  are convex functions of  $u$
- (v)  $\phi_h \neq \emptyset$  for each  $h \in H$
- (vi) for each  $h \in H$  the gradients of constraint functions binding at  $u(h)$  are linearly independent.

It follows from the result due to K. Jittorntrum [1] that under the above assumptions the solutions  $u(h)$  of  $(P_h)$  and the associated Lagrange multiplier  $\lambda(h)$  are right-differentiable functions of  $h$ .

In this paper it is shown that these functions are actually  $p$ -times right differentiable, and the respective right-derivatives are given as the solution and the associated Lagrange multiplier for an auxiliary quadratic programming problem.

The method of constructing of these problems is presented and the form of the second derivative ( $p=2$ ) is derived.

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TIME-OPTIMAL CONTROL OF ACCELERATION CONSTRAINED MECHANICAL  
SYSTEMS

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Abstract

It is well known that in the present status of technology, many mechanical systems work as positioning devices. The path between motion end points is not specified and depends on the control algorithm and hardware. The limitations of the strains and vibrations of the mechanical systems in the case of higher operation speeds and heavier manipulation loads, have to be taken into consideration. So, the subject of this report is the problem for time-suboptimal point-to-point control in the presence of control and acceleration constraints.

A direct method for control synthesis of the corresponding two-point boundary-value problems, combined with a parametric optimization procedure is proposed. Control laws are obtained on the base of the relay principle--the motion of the mechanical system is such that in every sampling interval, either the control functions are extremal or the dynamic constraints have their limit values. In the latter case, the control functions are determined using the equations of motion and the equations of acceleration constraints. The boundary-value problem reduces the solution of a system of  $N$ -shooting equations with  $N$ -unknowns (switching values). The optimization procedure is performed over the set of all fea-

sible solutions.

Using backward integration in time, we can solve the problem when deceleration constraints are imposed only in some fixed final intervals--which is of special practical interest.

To be more illustrative, the dynamical model of a manipulator with cylindrical coordinates is taken into consideration. As dynamical constraints for this numerical example, the limitations on radial and centrifugal accelerations of the gripper have been set up.

The algorithm of the proposed direct method for time-suboptimal control synthesis, in the presence of control and acceleration constraints is workable even though the identification of the parameters in the dynamical model is not satisfactory and can be realized on the real system as in a self-learning adaptive procedure.

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COMMITTEES FOR BINARY RELATIONS BY DECISION MAKING

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1. A system of binary relations is the model of different situations of decision making such like search of most preferable elements from the feasible set, like problems of pattern recognition, like search of information in data base, like interpretation of  $N$ -dimensionale scenes etc. We consider here the problem of finding the element

$$x \in X, \forall a \in A : x \wp a,$$

where  $\wp$  is some binary relation,  $X$  is the feasible set.

In case of nonsolvability of this system, it is possible to use some corrections of this model or some generalizations of a concept of solution. The generalizations such like committee solutions were used in spheres of pattern recognition and optimization [1]. We consider here the existence of committee solutions for abstract binary relations and the applications.

2. Let  $L$  be the linear topological space over the set  $R$  of real numbers, and  $\wp$ -some binary relation over  $L$ .

Assume that

$$(A1) \forall \lambda > 0 : a \wp b \iff (\lambda a) \wp b \ \& \ (\neg (-\lambda a) \wp b)$$

where " $\neg$ " is the sign of denial;

$$(A2) a \wp b \implies \exists \mathcal{V}_a \ni a, \forall c \in \mathcal{V}_a : c \wp b,$$

where  $\mathcal{V}_a$  is the neighbourhoods of  $a$ ;

$$(A3) a \neq 0 \implies a \wp a$$

where  $0$  is the zeroth vector;

$$(A4) \forall \alpha, \forall \beta: \alpha \beta \vee \alpha \tilde{\beta} \vee (-\alpha \beta)$$

where  $\alpha \tilde{\beta} \Leftrightarrow (\neg \alpha \beta) \& (\neg (-\alpha) \beta)$ ;

$$(A5) \alpha \tilde{\beta} \& \beta \gamma \Rightarrow (\alpha + \gamma) \beta.$$

Let's consider following system on  $\alpha$  :

$$\alpha \in A, \forall \beta \in B: \alpha \beta \tag{1}$$

where  $A, B$  are given subsets of  $\mathbb{L}$ .

If (1) is inconsistent then we consider the committee  $K$  [1]:

$$K \subset A, \forall \beta \in B, \forall \alpha \in K: \alpha \beta, |K| < +\infty, \tag{2}$$

where " $\forall \alpha \in K$ " means "for the majority of elements  $\alpha \in K$ ",

$|K|$  is the number of elements of  $K$ .

Then let's assume

$$(A6) \forall \beta \in B, \exists \alpha_\beta \in A: \alpha_\beta \tilde{\beta} \& (\forall c \in B, c \neq \beta: \neg \alpha_\beta \tilde{c});$$

$$(A7) 0 \notin B, |B| < +\infty.$$

Theorem.  $(A1) \div (A7) \Rightarrow \exists K: (2)$ .

3. We consider following applications: the contradictory problems of selection of decision in spheres of economics, technic, medicine, by search of information in data bases, by interpretation of multivalued scenes, in pattern recognition. Specially we consider some cyclic dynamic of state in problem of selection of cure in medicine.

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AUGMENTED FUNCTION METHOD  
FOR THERMAL SENSITIVITY ANALYSIS

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Design sensitivity analysis (DSA), which determines the sensitivities (or gradients) of the objective function and constraints for changes of the design variables, is the most critical step in an optimization process. In this paper a method for DSA is presented and called the augmented function method (AFM).

The AFM is related to the adjoint variable method of structural optimization. However, it is more straightforward in obtaining the adjoint equations and signifies the role of the adjoint variables as the Lagrange multipliers and as the sensitivity coefficients in some load optimization problems.

The present paper is concerned with the static load optimization problems of thermally loaded solid bodies. A general formulation is given in the analysis in which a continuous variable approach is adopted. Nonlinear types of distributed heat sources and/or boundary conditions are accommodated in the study. Design (or control) variables are the applied temperature on the solid boundary, and also the space dependent functions in the heat source and boundary heat flux expressions. By suitably choosing these design variables it is desired to minimize a given objective function, while satisfying a number of behavior and side constraints.

The AFM is employed for the thermal sensitivity analysis and the boundary element method (BEM) is utilized for the spatial discretizations of the primary and adjoint problems. The BEM proves to be most effective especially

in the example problem chosen in which only boundary controls and observations are analyzed.

It is concluded that the proposed DSA technique of using the AFM for derivation of functional gradients and the space discretizations by the BEM is a very efficient computational method for thermal load optimization problems. The outlined procedure should easily be extended to material and shape optimization problems of engineering design.

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AN ADAPTIVE COORDINATION ALGORITHM  
FOR INTERACTION BALANCE METHOD WITH FEEDBACK

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Interaction Balance Method with Feedback is applied to control large scale system. Each subsystem is controlled by a local optimizer which calculates control values heving regard to coordination variables  $p$ . The coordination problem is to appoint  $\hat{p}$  minimizing the difference between real system inputs and ones calculated by the local optimizers. This difference is called discoordination.

In the paper the coordination algorithm in the presence of disturbances has been considered. This problem was previously analyzed by Ruszczyński ("An Algorithm for Real System Coordination") in Large Scale System Theory and Application, Proc. of the IFAC Symp. Udine 1976). He proposed a very simple updating algorithm consisting of the following linear transformation:

$$p^{i+1} = p^i + F \cdot D(p^i) \quad (A)$$

where  $D(p^i)$  - discoordination depended on the coordination variables  $p$ .

The matrix  $F$  in the above formula had been computed basing on the model of the system. In the above mentioned paper the con-

vergence of this algorithm in the case of constant disturbances under the assumption of sufficiently small difference between the system equations and the model ones had been proved.

In my paper the case of variable, drifting disturbances has been investigated. In this case optimal values of the coordination variables are unknown functions of time, and the difference between the system equations and the model ones could increase to excess. Consequently the convergence property of the algorithm (A) could be lost. In the paper the adaptation of the operator  $F$  basing on the collected measurements of discoordination has been proposed. The special variable metric method presented by the author at the 11-th IFIP Conference on System Modeling and Optimization in Copenhagen has been applied as the adaptation algorithm. This method was prepared for the problem of tracking minimal point of nonstationary objective function.

In the paper the convergence of algorithm (A) with the adaptation of the matrix  $F$  in the case of linear system has been proved. Several cases of assumptions concerned with variability of the disturbances have been analyzed. The application of this adaptive algorithm to control more complicated system has been also discussed. High effectiveness of the presented method has been finally illustrated by some numerical experiments.

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COMPUTER ASSISTED MODELLING OF  
LINEAR, INTEGER, SEPARABLE AND FUZZY  
PROGRAMMING PROBLEMS

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ABSTRACT

For mathematical programming (MP) to have greater impact upon the decision making process, MP software systems must offer suitable support in terms of conceiving and describing programming models. In this presentation: (i) the basic modelling ground rules for linear programming are first introduced. (ii) Techniques of modelling which allow logical restrictions to be reformulated as integer programs are described. (iii) It is shown that many classes of nonlinearities which are not variable separable may be reformulated in piece-wise linear form. (iv) Fuzzy linear programming formulation of multi-attribute and goal programming problems is discussed briefly.

It is shown that analysis of bound plays an important role in the following contexts: model reduction, formulation of logical restrictions as 0-1 integer programs, reformulation of nonlinear programs as variable separable programs and formulation of fuzzy linear programs. It is observed that as well as incorporating an interface between the modeller and the optimiser there is a need to provide the modeller with software facilities which support the modelling techniques presented in this talk.

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Title: Markov Random Fields and Problems in Computational Vision

Abstract

In this lecture I discuss various problems in Early Vision using models of Images as Markov Random Fields corrupted by noise. In theories of vision, it is believed that most problems of early vision are of a computational nature. Examples of such problems are edge detection, surface reconstruction and extraction of depth information from stereo images. We show that all these problems can be formulated in a unified way in the framework of Bayesian Estimation Theory and this also provides the 'correct' method for incorporating a priori information. We discuss optimal estimates and show how the concept of an innovations field plays an important role in these problems.

The actual computation of these estimates can be done using Monte Carlo Methods such as the Metropolis Algorithm or Simulated Annealing. Some results on the asymptotic analysis of simulated annealing are presented. Moreover, the solutions of these estimation problems can be implemented in a distributed architecture and we discuss these issues and present experimental results.

The models considered in this lecture correspond (in the simplest situations) to Ising Models with a random external field. Understanding the order-disorder phenomena for these systems is a topic of current interest in mathematical physics. The concept of temperature has a natural meaning in the context of estimation of random fields, and the problem of estimation of parameters such as temperature from noisy observations is also discussed.

The research discussed in this lecture represents joint work with J. Marroquin and S. Gelfand of the Massachusetts Institute of Technology.

From: Prof. Sanjoy K. Mitter

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ON THE BAYESIAN METHODS OF OPTIMIZATION  
IN THE PRESENCE OF NOISE

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As the Bayesian we shall define such methods of optimization which minimize the expected deviation from the optimal value of function  $f(x)$ ,  $x \in A \subset R^m$ . The well known Bayesian methods, see Mockus (1972) and (1981), converge to the minimum of any continuous function with or without the noise. But those methods are rather complicated and can be used easily if the number of observations (function evaluations) does not exceed 100.

The following simplified version of the Bayesian method was developed by Mockus (1983)

$$x_{n+1} \in \arg \max_{x \in A} \min_{1 \leq i \leq n} \sigma_i / (\mu_i - y_{0n} + \varepsilon_n) \quad (1)$$

Here  $\mu_i = y_i$  is the expectation of  $f(x)$  with regard to the pair  $(x_i, y_i)$ ,  $\sigma_i$  is the standard deviation of  $f(x)$  with regard to  $(x_i, y_i)$ ,  $x_i$  is the point of the  $i$ -th observation,  $y_i$  is the observed value of function  $f(x)$  plus the noise  $y$   
 $y_{0n} = \min_{1 \leq i \leq n} y_i$ ,  $\varepsilon_n$  is a positive number and  $n$  is the number of observations. The expression (1) happened to be so simple because the Kolmogorov's consistency conditions were replaced by the conditions of continuity of the Bayesian risk function and of the convergence to the minimum of any continuous function without noise. In the presence of noise the method (1) will not satisfy the first convergence condition by Mockus (1981), namely that the "conditional" expectation  $\mu_i$  should converge to the true value of function  $f(x)$  when  $n \rightarrow \infty$ . The convergence conditions can be provided if the observed values  $y$  are replaced by the average values around

the best point in the sphere of radius  $\varrho_n = (\beta)^{1/m} L_A^{-\alpha n}$   
 Here  $L_A$  is Lebesgue measure of  $A$ . In the presence of noise  
 the minimal observed value is not the unbiased estimate of  
 the true minimal value, so some correction is needed. In  
 the case of Gaussian independent noise to each of  $n$  points  
 (without averaging) the following correction, see Kramer  
 (1946), should be added:  $\delta_n = \delta(\sqrt{2 \lg n_s} - 1/n_v \sqrt{2 \lg n_k})$   
 Here  $n_k$  is the number of clusters of points with averaging,  
 $n_v$  is the number of points in one cluster, for simplicity  
 supposedly the same in all clusters. In this case the simp-  
 lified version of the Bayesian method will satisfy the con-  
 vergency conditions also in the presence of noise and so  
 can be used for the multimodal stochastic programming prob-  
 lems as well.

The corresponding calculations concerning the test and  
 real problems were carried out by V. Tieshis and L. Zukauskaite.

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A Stochastic Multiobjective Dynamic Programming Method  
with Application to Energy Modelling

by

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Abstract

A stochastic multiobjective dynamic programming model is described for the optimal utilization of mineral resources for satisfying the energy demand of the national economy. The model is based on a special block-structure, where each block is characterized by its input and output vectors and state variables. The whole system is considered as a superposition of the subsystems defined by the individual blocks. Input-output and state transition relations are defined for each block, and the overall objectives of the whole system are defined by investment cost, production cost, manpower demand and environmental effect.

A special stochastic multiobjective dynamic programming algorithm is developed for the numerical solution of the model, which is the common generalization of deterministic single or multiobjective dynamic programming algorithms and Bayesian decision methods.

A case study illustrates the model and the solution methodology.

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## FUNCTIONAL-INFORMATION MODEL OF HUMAN OPERATIVE MEMORY

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An isofunctional programme model of the human operative memory is created, as the last is accepted like quasiequivalent of the mnemonic activity of the human being during the teaching. The model of the operative memory (OM) is learned according to the idiosyncrasies of a certain human being and then is used like means of teaching the same person. Microelectronic realization of the model is possible like a build in system in the micro-computer receiving plus this artificial intelligence.

The work consists of two parts :

FIRST: The model of OM (build isofunctionally) is described (a multi-graph with a global criterion for effectiveness and local criteria for volume, speed, exactness and readiness). The volume of the OM is described by a selectively chosen set of operators (FORGETTING and REMINISCENCE - kind of the material, level of the learning at proactive interference, similarity between the elements, reminiscence in time; ROLE OF THE EXERCISES - distribution of the exercises in time, level of learning; KIND OF THE MATERIAL - length of the row, degree of rationalization, place in the row, heterogeneity of the elements, independence of the stimuli, similarity between the elements; FORMULATION, MOTIVES, EMOTIONS - deliberate and undeliberate remembering weak and strong motivation, affect; ORGANIZATION OF THE ANSWERS MATERIAL - intellectual activity, associations between the stimuli, length of the group; DENSITY OF THE INFORMATION SYMBOLS; INFORMATION; INFORMATION POWER; and others), which are described mainly by interpolation with chosen incoming inner and exit variables. It is shown that there is not selfcontrol of the model in the terms of the statistical games. ;

SECOND: The algorithm and the programmes of BASIC for teaching of the OM model are simulated in the computer. The one, who teaches, alone formulates, controls, grounds his own bases of data by describing, juxtaposing and organizing of objects (depending on the individual motives) in a multi-graph according to the OM model. Adaptation is realized (as optimization in the conditions of insufficient a priori information for the environment of operative memory) and Teaching (as fixed-step of the

method for improving on the properties of the system teaching (OM) without strict delimitation. The Adaptation is in non-steady conditions and using the theory of stationary processes, the problem is defined for the minimizing the functional losses  $J(\omega) = E_x [Q(x, \omega)]$  with the help of the algorithm of type  $\omega[n] = \omega[n-1] - \delta[n] \cdot S(x[n], \omega[n-1])$  under the observation of the additional static and/or dynamic conditions. The mathematical base of the algorithm of the adaptation is the apparatus of stochastic approximation, bearing in mind that this method had already been used under certain circumstances of non-steadiness (with decisions depending on the problem itself-example, in adaptive control systems). For the adaptation of the process of teaching for each concrete human OM, it is necessary a correction depending on the individual parameters of the human being and the modal structure (depending on the effectiveness of memorizing the elementary portions teaching information) of the operative memory.

In conclusion the role of human OM in teaching is shown with the application of the reached results in psychology and pedagogy (in the teaching with computing technique). The role of human operative memory in teaching is determined by the model systems "TEACHING-MEMORY" and "MEMORY-TEACHING". The first of them describes the following structure of interaction (between the teaching and the human memory): selectivity during the informational transition between instantaneous and short-term memory; actuality of mnemonic information in the operative memory for a certain period of time; reliability of the given information from the semantic long-term memory. The following processes are secured: memorizing of the essential information; active and passive storing; reliability of the after following reproductiveness. The second system "MEMORY-TEACHING" defines the optimal mnemonic condition for creation of highly effective hierarchical associative-semantic structure of the meaning, stored in the long-term human memory.

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Modelling and numerical simulation of  
wind-generated circulation and matter  
transport in shallow lakes

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Permanently growing water demands and permanent decrease of water quality call for increasing investments in water quality management. Here the decision maker can no longer employ thumb rules but ought to make use of general computer models and program packages.

Circulation of water and distribution of pollutants in shallows lakes are significantly influenced by the wind. As a mathematical model of "medium" complexity, the Ekman model (derived from the Navier-Stokes equations by using the concept of turbulence viscosity and assuming hydrostatic pressure distribution and small circulation velocities) gives a globally good approximation to the three dimensional velocity field of circulation in shallow lakes. Horizontal velocity distributions in arbitrary depths can easily be calculated from a "stream function" which turns out to be the solution of a boundary value problem for a Poisson-like partial differential equation (with additional convection terms) in two space variables.

In the computer program package LAKE (developed by G.Stoyan and the author together with H. Baumert from the Institute of Water Management, Berlin, GDR, cf. [1]), this partial differential equation is solved numerically (on an arbitrary two dimensional domain) by a specially adapted finite difference scheme on an equidistant rectangular grid. From

the stream function, velocity vectors are computed by direct calculations. The obtained velocity field determines the coefficients in the ordinary differential equation of drifting body trajectories (solved numerically by a predictor-corrector method) and in the mass transport equation (a partial differential equation of diffusion-convection type, solved by a finite difference scheme closely related to the one mentioned above).

Besides, the system LAKE (which is applicable to lakes with arbitrary geometry, arbitrary depth distribution, arbitrary distribution of inflow and outflow links) contains a large number of modules for data manipulations (aggregation of input data, evaluation of output data, storing on external files, etc.) and for presentation of results (via printer or plotting machine).

Altogether, the package LAKE consists of about 35 modules (all in FORTRAN) realizing special tasks (each module contains a self-explanatory comment section). By means of a "main program" the modules needed in a specific situation can be combined. "Main programs" for some standard situations are available. The package is, in different versions, available by licence.

In the paper results of some real life calculation examples are presented and typical applications are indicated.

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RESULTS OF EVOLUTIONAL RANDOM SEARCH PROCEDURE FOR  
LOAD FLOW OPTIMIZATION IN ELECTRIC NETWORKS

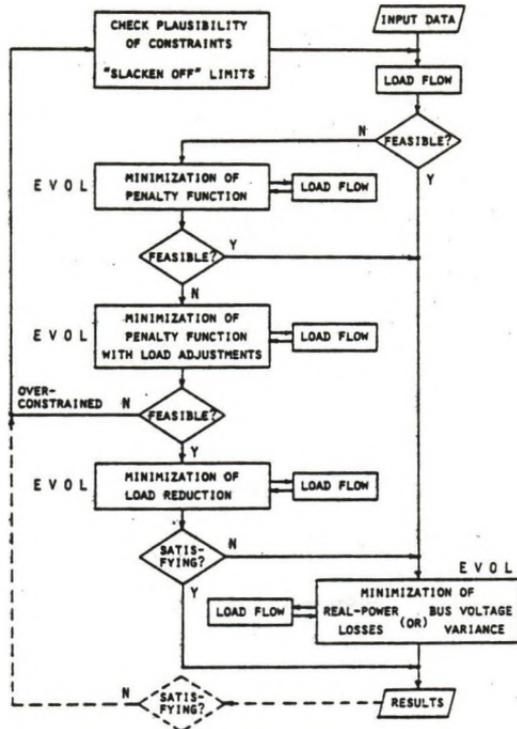
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When calculating the power flow in an electric transmission network it is the purpose of load flow optimization to adjust free control parameters (reactive power injections, voltage magnitude at reactive power sources, tap sets of transformers) in such a way, that given technical limits are satisfied and a global scalar objective function is minimized. Possible objective functions are transmission losses or the variance of bus voltages. Imposed constraints are upper and lower limits for power inputs at nodes, the allowable band for bus voltage magnitudes and maximum power or current ratings of transmission components.

To master the nonlinear and non-convex character of this multivariable optimization problem and to overcome difficulties originating e.g. from the mentioned constraints of different physical dimension (scaling problems!) and from the "fissured" solution space caused by them a derivation-free random search procedure following the principles of biological evolution was chosen to solve the constrained reactive optimal power flow problem. Starting from an arbitrary system state the solution process is decomposed into two steps, determining first a feasible and then optimal load flow (within the feasible range).

Beyond that the facility of load adjustments for security reasons was also included in the search process (see flow chart of solution procedure): Contingency analysis added to load flow analysis can be used to detect possible overloaded situations and such to provide the necessary security assessment within the framework of a preventive security concept. Cases may occur in the course of such outage simulations, where constraints are violated and where these violations cannot be eliminated completely by adjusting the normal

control parameters mentioned above. Therefore the proposed strategy was adapted to handle these cases. In a stepwise procedure in the first step load adjustments are determined in some prespecified nodes, which are additionally necessary to get the system state feasible. The necessary unavoidable amount of this load reduction is minimized then in a second step (in combination with the other control adjustments).



The performance of higher developed evolutionary strategies was investigated by means of a test series with some example networks and real systems. Experiences learned when looking for the proper choice of strategy parameters which provide best performance are reported and results obtained are presented - e.g. illustrating the behaviour of the algorithm with regard to different types and sets of controllable variables.

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On the optimal cooling of steel during continuous casting

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In continuous casting secondary cooling is required to accelerate steel solidification and to strengthen the solidified shell. Improper water quantity or distribution may form and enhance different kinds of surface and internal defects. So the strand is to be cooled down according to a pattern which depends on steel quality, product size, casting speed and machine design. To define the cooling strategy and spray system design the knowledge of heat flow and solidification rate must be known.

In this contribution, a mathematical model is presented which simulates the heat flow in the strand during the continuous casting of the steel billet and optimizes the casting conditions. Mathematically we are led to a boundary control problem, where the state is governed by a nonlinear parabolic-type equation (with phase changes). When the state is discretized by a finite element method in space and the finite difference method in time, and an appropriate numerical quadrature is used to compute the criterion functional the problem reduces to a nonlinear programming problem. Several numerical examples will be given.

Simulation of Waiting Lines in Series Under Special

Conditions

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A sequence of  $m$  service stations can be arranged in series such that a unit has to go through one station after another of the sequence before the whole service is finished.

The analytical discussion of the system in series is rather difficult. We confine ourselves to a particular case for which solution will be given by simulation.

The problem is what happens to a unit after it has completed service by the  $k^{\text{th}}$  station in the line. If the  $(k+1)^{\text{st}}$  station has already finished its previous unit, it is free and can take the next unit as it emerges from the  $k^{\text{th}}$  station.

Let us suppose that the  $(k+1)^{\text{st}}$  station is not free, all units in service have to stay in their respective stations until all stations complete service on the units in them, when all units move at once, each into the next station. The one in the  $m^{\text{th}}$  station leaving the line and one from the queue moving into the first station /if there is no unit in the queue, all units wait until another unit arrives, when they all move/.

It is assumed that each separate service station /process/ has its own service rate without specifying particular probability distributions for either the interarrival or the service times.

The whole simulation can be split up three parts, namely:

- /1/ starting period: from 0 time until the first unit starts its service at station m,
- /2/ operating period: the time that elapses between the end of the starting period and the time the last unit starts at the first station,
- /3/ finishing period: the rest of the simulation.

Waiting and idle times can be calculated for subsequent arrivals in the starting, operating and finishing periods - separately. However, uniform formulae can be created for all of the three periods, that is

$$WT_{i-l, k+l+1} = TSMAX - TS_{i-l, k+l}$$

and

$$IDT_{i-l, k+l+1} = TSMAX - TS_{i-l+1, k+l+1}$$

where for each subsequent  $i / i=2, \dots, m/l = 0, 1, \dots, m-n-1$  with the restriction that the value of  $n$  and  $k$  vary during the different periods of the wimulation as given in the table below

period \ variable	starting	operating	finishing
n	m-1	0	varies from 1 to m
k	0	0	n

During the simulation n means the number of service stations where there is no activity, while k indicates the number of stations where all of the services have been finished.

After the end of the simulated process total waiting and idle times can be calculated at each station.

Notations:

m: the number of service stations

$WT_{ih}$ : the amount of time the  $i^{th}$  unit spends waiting to enter the  $h^{th}$  station, where  $i=1, \dots, L$  and  $h=1, \dots, m$

$IDT_{ih}$ : the amount of time the  $h^{th}$  station remains idle while waiting for the  $i^{th}$  unit to arrive, where  $i=1, \dots, L$  and  $h=1, \dots, m$

$TS_{ih}$ : ending time of service of the  $i^{th}$  unit at the  $h^{th}$  station, where  $i=1, \dots, L$  and  $h=1, \dots, m$

$TSMAX: \max/TS_{i,0}, TS_{i-1,1}, \dots, TS_{1,i-1}/$ .

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## OPTIMAL METHODS FOR SMOOTH CONVEX FUNCTION MINIMIZATION

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The majority of methods dealing with the problem  $\inf_{x \in \mathbb{R}^n} (f(x))$  (1), where  $f(x)$  is a smooth convex function, are based on the idea of approximation of the cost function. It is the approximation approach which allows us to obtain asymptotic rate convergence estimations for these methods. But the total time we need to solve the problem (1) by method  $m$  depends mainly on the global rate convergence estimation of method  $m$ . Function

$\bar{F}_m^{\mathcal{F}}(s, k)$  is called the global rate convergence estimation for method  $m$ , if for an arbitrary positive number  $k$ , for an arbitrary start point  $x_0 \in \mathbb{R}^n$  and for an arbitrary cost function  $f \in \mathcal{F}$  the inequality  $f(x_k) - f^* \leq \bar{F}_m^{\mathcal{F}}(x_0 - x^*, k)$  is true. It is proved in [1] that if  $\mathcal{F} = \mathcal{F}_m^L = \{f(\cdot) \mid m \|x - y\|^2 \leq \langle f'(x) - f'(y), x - y \rangle \leq L \|x - y\|^2 \quad \forall x, y \in \mathbb{R}^n\}$

(2), then for an arbitrary method  $m$  dealing with problem (1)

$$\bar{F}_m^{\mathcal{F}}(s, k) \geq \text{const} \cdot L \|s\|^2 \cdot \min(\exp(-\sqrt{\frac{m}{L}}k), \frac{1}{k^2}). \quad (3)$$

The methods for which the inequality (3) may be replaced by the inverse are called  $\bar{F}_m^L$ -optimal methods. Nemirovsky and Yudin proved that for all the methods mentioned above (the gradient method, the conjugate gradient method, the variable metrics method)

$\bar{F}_m^{\mathcal{F}}(s, k) \leq \text{const} \cdot L \|s\|^2 \cdot \min(\exp(-\frac{m}{L}k), \frac{1}{k})$  and hence they are not  $\bar{F}_m^L$ -optimal.

The first  $\bar{F}_m^L$ -optimal methods were obtained in [1, 2]. However on each iteration of these methods it is necessary to solve subproblems of twodimensional [1] or onedimensional [2] minimization with great precision. The first  $\bar{F}_m^L$ -optimal method without great complexity of each iteration was proposed in [3]. In [4] the whole class of  $\bar{F}_m^L$ -optimal methods was developed from

the method [3]. The construction [3] was used in [5] to obtain optimal algorithms for more general cost function classes.

The structure and proofs of the method [1-5] are purely algebraistic. No geometrical ideas were used in these methods. It put obstacles in the way of further optimal method development. In this report we propose a simple geometrical approach for constructing  $\mathcal{F}_m^L$ -optimal algorithms. The main idea of this approach is to ensure the holding on inequality  $f(x_k) \leq \min(f(x) + \Psi_k(x) | x \in \mathbb{R}^n)$  (4) step by step. In (4) function  $\Psi_k(x) : \Psi_k(x) \rightarrow 0$  for each  $x \in \mathbb{R}^n$  when  $k \rightarrow \infty$ . The possibility of holding the inequality (4) follows from the global properties (2) of class  $\mathcal{F}_m^L$ . The global rate convergence estimation of such methods is the trivial consequence of inequality (4):  $f(x_k) - f^* \rightarrow 0$ . We present the simplest  $\mathcal{F}_m^L$ -optimal method of this type:

$$v_0 = x_0 = \bar{x} \in \mathbb{R}^n; A_0 = L;$$

$$k = \frac{1}{2} L^{-1} ((A_k^2 + 4A_k L)^{\frac{1}{2}} - A_k); y_k = (1 - \alpha_k)x_k + \alpha_k v_k;$$

$$x_{k+1} = y_k - L^{-1} f'(y_k); A_{k+1} = (1 - \alpha_k)A_k + \alpha_k m$$

$$v_{k+1} = (1 - \alpha_k) \frac{A_k}{A_{k+1}} v_k + \alpha_k \frac{m}{A_{k+1}} y_k - \frac{\alpha_k k}{A_{k+1}} f'(y_k), \quad k=0, 1, \dots$$

$$\text{For this method } f(x_k) - f^* \leq \frac{4m \| \bar{x} - x^* \|^2}{(e^{\sqrt{m}k} - 1)^2} \xrightarrow{m \rightarrow 0} \frac{4L \| \bar{x} - x^* \|^2}{k^2}$$

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## Good lattice points for quasi-random search methods

Harald Niederreiter

Abstract

Random search is a familiar method in nondifferentiable optimization. It is based on random sampling from the domain on which the target function is defined. If random samples are replaced by well-chosen deterministic point sets, we arrive at a so-called quasi-random search method. Quasi-random search methods were studied numerically by Niederreiter and McCurley [3] and Niederreiter and Peart [4], and a systematic theory of such methods was developed by Niederreiter [1]. Related investigations were carried out by Russian mathematicians (see e.g. Sobol' [6], [7]).

In this paper we study a special class of deterministic point sets and its application to quasi-random search methods. These point sets are easy to implement and with a proper choice of parameters they are well suited for quasi-random search methods. The structure of these point sets permits their use for continuous as well as for discrete optimization problems.

The basic construction for the normalized domain  $[0,1]^s$ , the  $s$ -dimensional unit cube, proceeds as follows. Let  $m$  be a large integer and let  $\underline{g} = (g_1, \dots, g_s)$  be an  $s$ -dimensional lattice point, i.e. a point with integer coordinates. Then we define the points

$$x_n = \left( \left\{ \frac{ng_1}{m} \right\}, \dots, \left\{ \frac{ng_s}{m} \right\} \right) \in [0,1]^s \quad \text{for } n = 1, 2, \dots, m,$$

where  $\{t\} = t - [t]$  and  $[t]$  is the greatest integer  $\leq t$ . The quality of this point set depends on the choice of the lattice point  $\underline{g}$ . "Good" lattice points modulo  $m$  are those  $\underline{g}$  for which for given  $m$  the corresponding point sets  $x_1, \dots, x_m$  are well suited for quasi-random search methods. A quantity measuring the suitability of lattice points is introduced. It is shown that for any given  $m$  there exist good lattice points modulo  $m$ . Effective constructions of such lattice points are discussed. We mention

also the use of good lattice points in refinements of quasi-random search methods such as localization of search (see Niederreiter [2] and Niederreiter and Peart [5]).

Good lattice points can also be used in discrete optimization problems. For instance, if we want to optimize a function defined on a discrete grid  $\{0, 1, \dots, m-1\}^s$ , we can use the points  $y_n = mx_n$  ( $1 \leq n \leq m$ ) of the grid, where  $x_n$  is as above. If  $g$  is a good lattice point modulo  $m$ , then the  $m$  points  $y_1, \dots, y_m$  provide an efficient way of sampling from the  $m^s$  points of the grid. If the rate of change of the target function on the grid is under control, we get an attractive method for approximating the solution of a discrete optimization problem.

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ON THE USE OF STOCHASTIC GRADIENTS TECHNIC  
FOR OPTIMIZATION PROBLEMS IN HILBERT SPACE

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The problem of search of gradients in Hilbert space is rather complicative. Thus it is difficult to use gradients methods for the numerical search of optimum under restricts. With the aim to construct an universal algorithm of gradient type for the solution of any problem of convex programming in Hilbert space we offer to use the combine method of penalty functions and stochastic gradients for the solving of limiting extremal problems. Its essence is the following.

The functional restricts are took off with the help of integral penalty function. The obtained problem of limiting (with the penalty constant increase to infinity) optimization in Hilbert space is substituted by the problem of limiting (with the increase of the penalty constant and the approximation order) finite-dimensional optimization, for example by the scheme of Ritz. For the numerical search of Fourier coefficients of the solution the iterative procedure of gradient type is constructed. There the penalty function and the approximation order are accordingly increased with the growth of the iteration number. The approximated penalty functional will be in the form of multivariable integral. Instead of exact computation of its gradients the stochastic quasi-gradients are of use, i.e. quasi-gradients of integrand in random points. Thus for the search of  $u(\cdot) \in L_2(X)$  which realises the minimum  $\min_{u \in K} \int_X \Phi(u(x)) dx$ ,  $K = \{u \in L_2(X) \mid G(u(x)) \leq 0 \quad \forall x \in Y \subseteq X\}$ ,  $X \subset \mathbb{R}^n$ ,

in the form  $u = a_j$  for orthonormal basis  $\xi = \{\xi_j\}$  in  $L_2(X)$ ,

$$a_j^{t+1} = a_j^t - \beta_t \frac{\partial}{\partial a_j} \left\{ \Phi \left( \sum_{j=1}^{l_t} a_j^t \xi_j(x^t) \right) + C_t \left[ G^+ \left( \sum_{j=1}^{l_t} a_j^t \xi_j(y^t) \right) \right]^2 \right\}$$

$\forall j=1, 2, \dots, l_t; \quad a_j^{t+1} = 0 \quad \forall j=l_t+1, \dots, l_{t+1}; \quad \forall a^t \in R^{l_t}$ .

Here  $\{x^t\}, \{y^t\}$  are sequences of independent realizations of of random variables equidistributed on  $X, Y$ ;  $C_t, l_t \rightarrow \infty, \beta_t \downarrow 0$  are controlling number sequences of the algorithm.

It is proved the possibility to choose constructively the controlling parameters for the convergence of the iterative procedure to the solution of minimization problem of strictly uniformly convex functional under convex restricts in strong metric of Hilbert space with the probability equals one. This approach may be extended for the numerical solving of minimization problem: for convex functionals (on the base of iterative regularization), for functionals of maximum, for the integrals over measures and weak distributions in Hilbert space, - and for the solving of variation equations.

With a view to investigate the practical convergence of the algorithm above the problem of elasticoplastical torsion of cylindrical rod was solved. The computations gave the first approximation for the solution Fourier coefficients rather quickly.

A VISUAL INTERACTIVE SIMULATION MODEL  
FOR THE DESIGN OF RAILWAY STATIONS

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ABSTRACT

The construction of new railway stations or the modification of the lay-out of existing ones represent very high investment costs and therefore intensive design studies must be carried out in order to find well-balanced solutions. However, this is a complex decision problem with multiple objectives to be satisfied, some contradictory, some ill-defined, and numerous constraints. Furthermore, as the lifetime of a station is very long, one faces a long-term planning problem, and therefore it is crucial that the planners have the possibility of evaluating alternative lay-out solutions under different traffic load scenarios. This raises many modelling difficulties, and a fully analytical model could hardly be suitable to cope with the complexity and requirements of the problem. Simulation, however, appears to be an attractive approach to be investigated.

This paper describes a simulation model developed to support design studies of Campanhã Railway Station, a major node in the Portuguese railway network. The present track and platform lay-out no longer meet the requirements of the traf-

fic load imposed on the station (more than 440 daily trains), and therefore a complete re-design has become necessary.

The model represents the station, the incoming lines and the interactions with the neighbouring stations and it enables the designers to evaluate different lay-out solutions under various traffic load conditions. With this purpose, a series of performance measures is provided after each simulation run (such as train delay statistics, platform occupation data, track circuit utilization, etc). In addition to this, the model depicts a dynamic representation of the station and all train movements, allowing for a qualitative assessment of the solution performance. The model is fully interactive, enabling the user to interrupt the simulation run to alter the state of the station (e.g., disabling a platform or forcing an early departure of a train), to move forward and backwards in simulation time, or ask for additional information.

A series of interactive facilities enable the designers to generate alternative lay-out solutions and to define different traffic load conditions, preparing the relevant data for the simulation model.

The paper also discusses the importance of the visual representation and the interactive facilities in improving communication with the Railway Company staff responsible for the design studies and in strengthening the confidence of the end users in the model.

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**SIMULTANEOUS SYNTHESIS AND ECONOMIC OPTIMIZATION OF THE  
POWER SYSTEM ON AN OIL/GAS PRODUCTION PLATFORM**

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A simultaneous synthesis and economic optimization method is described for the power system on an oil/gas production platform. The model described here include the influence of a variable production profile with a corresponding variable power requirement over the life of the oil/gas field. The multiperiod mixed integer linear programming (MILP) model is based on the minimal present-value total cost for the power system including investment, cost of weight on platform structure and deck as well as running costs over the lifetime of the platform.

The proposed model can select among several types of gas turbines for satisfying the various power requirements and also considers the possible assignment of electrical motors or gas turbine to rotary process equipment. The model also has the capability to decide how best to run the power system at each time period taking into account reduced efficiency with load for each individual gas turbine. This model can also be implemented as a computer tool for the optimal synthesis of power systems and can be further extended for the synthesis of the production process.

The application of this model is illustrated by using an example from an oil/gas production platform in the North Sea. The problem consists of selecting gas turbine type as

generator driver and to decide what kind of driver, gas turbine or motor, is going to be used at the compressors and pumps. It is possible to choose between four types of gas turbines for this problem.

The size of the MILP model is 55 binary variables, 135 continuous variables and 300 constraints. The optimal solution was obtained in approx 234 sec. of CPU time on a DEC-20 computer using the branch and bound algorithm of the LINDO computer code.

To compare the effect of selecting other types of gas turbines as generator driver, the MILP model was resolved using preselected values of the binary variables for different types of gas turbines. The difference in net present value of investment and fuel cost varies between 9-16 million dollars depending on the gas turbine type. In all solutions the compressors and pumps are driven by motors.

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## DC MOTOR DRIVE PARAMETERS OPTIMIZATION

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This paper presents the procedure for the optimum parameter adjustment of separately excited dc motor drive whose hybrid diagram is shown in Fig. 1. The essential parts of the drive are motor, thyristor converter and speed and current regulators. It will be assumed that the mode of speed regulation is known. Matrix representation of the state equations describing behaviour of the system may be written in the form:

$$\dot{Y} = AY + FP; \quad X = CY; \quad Y(t_0) = Y_0 \quad (1)$$

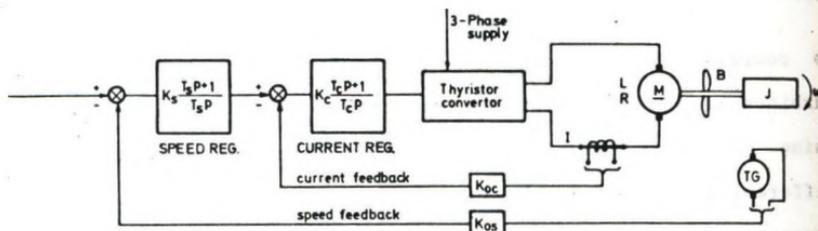


Fig. 1. Hybrid diagram of studied system

In developing procedure for optimization, we have to find variable system parameters  $q = \{K_s, T_s, K_c, T_c, K_{oc}, K_{os}\}$  which will minimize the performance index given by:

$$J = \int_{t_0}^{t_0+T} [Y(t, q) - Y_r(t)]^T [R] [Y(t, q) - Y_r(t)] dt \quad (2)$$

where  $Y(t, q)$  -  $n$  dimensional state vector as a function of the system parameters vector  $q$ ,  $Y_r(t)$  -  $n$  dimensional state vector of reference system,  $R$  -  $n \times n$  positive definite weighting matrix. Choosing appropriate value for  $R, T$  and reference system outputs, performance index becomes a function of the vector  $q$  only, i.e.  $J = J(q)$ . The partial derivatives of the criterion  $J$  with respect to the parameters  $q$  give the optimum conditions as  $\partial J / \partial q = 0$ , or

$$\frac{\partial J}{\partial q} = \int_{t_0}^{t_0+T} [Y(t, q) - Y_r(t)]^T [R + R^T] [V(t, q)] dt = 0, \quad (3)$$

where the vector function  $V(t, q) = \frac{\partial Y_1(t, q)}{\partial q_j}$ ,  $j=1, 2, \dots, n$ ,  $j=1, 2, \dots, l$  is sensitivity function. In order to obtain the gradient components of  $J$  it is necessary to have the system sensitivity function of all of the system outputs relative to the system parameters which are supposed to be optimized. In this case the equation which defines the sensitivity functions may be written as:

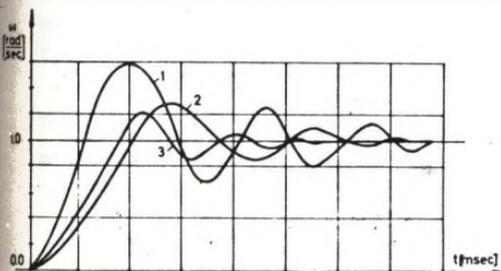
$$\dot{Y}_{qj} = AY_{qj} + \dot{A}_{qj} Y, \quad j=1, 2, \dots, l, \quad (4)$$

where

$$\dot{Y}_{qj} = \begin{bmatrix} \frac{\partial Y_1}{\partial q_j} & \dots & \frac{\partial Y_n}{\partial q_j} \end{bmatrix}^T \cdot \dot{A}_{qj} = \begin{bmatrix} \frac{\partial A_{11}}{\partial q_j} & \dots & \frac{\partial A_{1n}}{\partial q_j} \\ \dots & \dots & \dots \\ \frac{\partial A_{n1}}{\partial q_j} & \dots & \frac{\partial A_{nn}}{\partial q_j} \end{bmatrix} \quad (5)$$

It means that, in order to find first derivatives of the criterion J with respect to the system parameters, we have to solve  $n+(n+1)l + 1$  differential equations. Minimization of the criterion has been carried out using Rosen gradient and Fletcher-Powell methods which have been proved many times as very efficient in solving different minimization problems. Using foregoing procedure a numerical example has been carried out. The system is characterized by following parameters: 1100 kW; 500V; 2500A; 1040 rpm; 91,5%; 0.0041 Ohm; 0.0031 H; 0.411V/rpm/min.; 3.52Nm/A;  $K_t=97$ . Calculated suboptimal values of the parameters and criterion for different starting points are given in the Table. The responses of the optimized system to the unit step input is given in Fig.2. It should be noted that suboptimal solution can be improved if we use obtained suboptimal parameters using one procedure as starting point for another (curve 3).

	$T_s$	$T_c$	$K_s$	$K_c$	$K_{oc}$	$K_{os}$	J
Unst.str.Point.	0.0172	0.3024	38.623	0.4340	0.082	1	11943,98
Rosen	0.0942	0.0712	28.067	0.2715	0.017	1.01	3.6E-3
Fletch-Powell	0.0304	0.0738	28.077	0.6545	0.006	0.99	3.75E-3
Stable str.Point	0.0162	0.0756	28.067	0.4340	0.041	1	1.6
Rosen	0.02320	0.0736	28.067	0.3579	0.011	0.99	3.4E-3
Fletch-Powell	0.3783	0.0164	28.068	0.3740	0.011	1.03	3.2E-3



1. Fletcher-Powell: Unstable starting point.
2. Rosen gradient procedure: Unstable starting point
3. Rosen - Fletcher - Powell

Fig.2. The Responses of the system to the unit step input.

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Mogens Pedersen & Hans F. Ravn:

The Generalized Maximum Principle  
Applied to Production Planning

This paper describes the application of new theoretical results in discrete time optimal control theory applied to a classical - but difficult - production planning problem.

For the solution of discrete time optimal control problems in general there are three methods that can be used:

- dynamic programming: advantage: in principle capable of solving the problem by a stagewise approach  
disadvantage: curse of dimensionality
- maximum principle: advantage: a stagewise approach  
disadvantage: differentiability necessary for adjoint equations; normally only necessary conditions
- mathematical progr.: advantage: well-developed  
disadvantage: possibly many variables; normally necessary conditions.

We shall here consider the maximum principle. Recently the requirement of directional convexity has been done away with by a suitable generalization of the Hamiltonian (ref. 1). But also in this version the maximum principle yields only necessary conditions for optimality, at most.

In this paper we give a review and application of a new version of the discrete time maximum principle, which is capable of yielding sufficient conditions. Moreover, some of the usual obstacles are removed: there are no requirements of convexity, linearity or directional convexity, and there is no requirement of differentiability.

In this respect therefore the new sufficient maximum principle is as generally applicable as dynamic programming. The relationship between the two methods is discussed.

The application of the principle is to a production planning problem. We consider a two item, three machine problem, where the cost to be minimized consists of set-up costs, production costs and inventory costs. The restrictions are that the demand (assumed known for each period) is to be satisfied, and that further there are restrictions on production capacity and inventory levels.

The algorithm is described, and computation experience is given, including the solution to the dual problem which yields a bound on the optimal criterion value which is more tight than the usual one obtained by Lagrangean relaxation.

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GLOBALLY CONVERGENT EXACT PENALTY ALGORITHMS FOR CONSTRAINED  
OPTIMIZATION

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In recent years an increasing attention has been devoted to exact penalty methods for the solution of constrained problems by means of unconstrained minimization techniques (See, e.g. [1], [2], [3]).

A common feature of existing penalty methods is that "exactness" can only be established with reference to some compact set  $S$  containing the problem solutions, so that the threshold value of the penalty coefficient depends on  $S$ .

This causes, in principle, an inherent difficulty in the unconstrained minimization, since the level set corresponding to the penalty parameter and to some given initial point, even if compact, need not be contained in  $S$ .

As a consequence, global convergence results for exact penalty methods, employing automatic procedures for the adjustment of the penalty parameter, require the additional assumption that the sequence constructed by the algorithm is bounded.

In this paper we define two classes of algorithms which avoid these difficulties under mild regularity and compactness assumptions on the problem.

The first class is based on the continuously differentiable exact penalty function considered in [4], with the additional inclusion of a barrier term on the boundary of  $S$ . The second class is based on a similar improvement performed on the continuously differentiable exact augmented Lagrangian function proposed in [5]

For both functions, a complete equivalence can be established between the solution of the constrained problem and the unconstrained minimization of a continuously differentiable function whose global and local minimizers are contained in the interior of  $S$ .

On this basis globally convergent algorithms, employing an automatic adjustment rule for the penalty parameter, are defined which cannot produce unbounded sequences; thus overcoming the main drawback of existing exact penalty function methods.

For both classes, Newton-type and Quasi-Newton schemes are described, which allow to conciliate global convergence properties with an ultimate superlinear convergence rate.

Numerical results are reported.

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SAMPLING STRATEGY OPTIMIZATION  
FOR REGIONAL WATER QUALITY MONITORING

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The proper management of environmental resources has growing importance in many technico-economic projects. In concordance with this tendency, the rational design and operation of environmental monitoring systems recently has attracted increasing attention.

This presentation is devoted to a system of models, formulated to determine sampling strategy for regional water quality surveillance. In the considered lake-watershed system the lake water quality is affected mainly by external nutrient/pollutant loads, carried by tributaries to the lake. Monitoring is to be accomplished on tributaries of varying dynamics. The (discrete) measurement data are collected and transported to several laboratories, where they are analyzed. The main purpose of the monitoring network is to provide a statistically sound estimate of the annual/seasonal mean/total pollution load of the lake with minimum costs: the decision variables are the number of measurements to be carried out at each monitoring station and the number of different routes to be accomplished for

collecting/transporting samples to laboratories.

Modelling the arriving pollution loads as random variables, results from finite population sampling theory can be applied: this approach leads to a linear programming problem with a single nonlinear, but convex constraint. The model can be extended to cover the case of stratified sampling and of monitoring several water quality constituents. The selection of "quasioptimal" model parameters (variance limits in estimating mean values) and a sequencing/routing model is also briefly treated. Finally, an efficient solution procedure and some numerical results are summarized.

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MULTIEXTREMAL (GLOBAL) OPTIMIZATION  
ALGORITHMS, WITH ENGINEERING APPLICATIONS

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A class of methods for finding the (not necessarily unique) global optimizer of multiextremal mathematical programming problems has been recently investigated in an axiomatic framework by the first author. Several software realizations of the mentioned type of algorithms have been programmed and tested by the second author. This paper surveys the underlying theory, discussing also numerical aspects and some computational test results. Typical fields of applications, such as model calibration (parameter estimation) in forecasting, curve/surface fitting to experimental (measurement) data, design of complex engineering systems are proposed, with numerical examples.

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THE GREEDY ALGORITHM AND APPROXIMATE TRAVELLING SALESMAN  
METHODS

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The greedy algorithm solves the problem of determining the maximum-weight independent subset of a matroid. Greedy-type algorithms are often used for the approximate solution of other combinatorial optimization problems.

The Travelling Salesman Problem (TSP): Given a complete arc-weighted graph  $G = (V, E)$ , the minimum-weight Hamiltonian circuit is to be found.

A framework of a greedy-type algorithm for the TSP:

Let us define sets of partial solutions  $S_1, S_2, \dots, S_n$  where  $S_i$  represents the partial solutions consisting of  $i$  edges of  $G$ .

Step  $k$ . of the algorithm: Let us choose the maximum-weight edge which together with the already chosen edges forms a member of  $S_k$ .

The sets  $S_i$  of partial solutions can be differently defined. A greedy-type algorithm can be described by the sets of partial solutions.

The following questions may arise:

1. Which of the greedy-type algorithms can be expected to produce the best approximate solution?
2. How good is this solution?

The edges in  $G$  can be partitioned into  $n$  subsets,  $E = E_1 \cup \dots \cup E_n$ , where  $E_i = \{(i, j), j = 1, \dots, n, j \neq i\}$ .

It can be seen that  $\prod_{i=1}^n E_i$ , that is, any Hamiltonian circuit is an element of the Cartesian product of the sets  $E_i$ , and any partial solution consisting more than 1 edge is an element of the Cartesian product of some of the sets  $E_i$ . Choosing an edge into a partial solution excludes other edges from further consideration, that is, the expectation values and variances of the weights of the edges in the sets  $E_i$  will change.

The expectation value of the weights of the Hamiltonian circuits can be easily computed. Choosing the minimal element of a set  $E_i$  yields a reduction of the expectation value. A greedy-type algorithm can be regarded optimal - concerning the expectation value - if that edge from the minimum-weight elements of the sets  $E_i$  is chosen, for which the reduction of the expectation value is maximal.

We can formulate other algorithm-optimisation criteria too, concerning variance, sum of covariances, upper bound for worst case behaviour, etc.

We attempt to construct a greedy-type algorithm which takes into consideration the above optimisation criteria at the same time. This algorithm is based on cluster-analysis. The computational results are encouraging.

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SOME PROBLEMS OF THE DISEASE MATHEMATICAL MODELS APPLICATION

IN CLINICAL PRACTICE

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The main results of joint research of mathematicians, immunologists and physicians headed by Academicians G.I.Marchuk, R.V.Petrov, N.I.Nisevitch and others are presented. The investigations started in 1975. Their aim is the construction of mathematical models of the diseases and the development of methods of clinical-laboratory data processing for objective estimating the pathologic process gravity, its course forecast and basing on facts the recommendations for the most adequate treatment choice. The results of these investigations were discussed at the IFIP Conferences and are presented in /1,2,3,4/.

Solving the above problems we run into following difficulties:

1. "Our models, are they adequate?" Qualitatively different solutions obtained with corresponding values of model parameters correspond to different forms of the diseases observed in clinical practice. "Biological consequences" obtained within model investigations correspond to the statements of modern immunology and clinical experience. One of the most clear examples -- the treatment of chronic forms of the disease "by aggravation" -- was predicted with the aid of the model and tested in clinical practice /1,2/. All this permits considering the constructed models to describe (on the whole) correctly the process under investigation.

2. "Not those indices which are necessary for the model are measured". Laboratory and clinical indices of gravity plotted on the base of data on laboratory analysis and physician's estimates of clinical symptoms expression appeared to be effective for getting over this difficulty /1,4/. Having analysed experience of the above indices application we noted the following general relationships:

(a) When the disease takes the "smooth" course (i.e. without aggravations) and patient recovers, laboratory and clinical indices of gravity are

close to each other during the whole course of the disease, and they normalize with the highest intensity, almost the same for a number of illnesses. Their dynamics can be described by stochastic equation:

$$d\varphi_t = - (A - B^2/2) \varphi_t dt + B \varphi_t dW_t$$

where  $\varphi_t$  - index of gravity at the moment of time  $t$ ;  $d\varphi_t$  - stochastic Ito differential for index of gravity;  $dW_t$  - Winer's process differential;  $A$  - rate of functional rehabilitation;  $B$  - non-uniformity parameter.

(b) Deceleration of functional rehabilitation process is usually connected with unfavourable course of the disease.

(c) Indices of gravity can be assumed to be proportional to the part of organ damaged  $m$  in the simplest model of the disease /3/.

3. "What should we optimize?" One can show that for enough wide class of disease models the minimization of functional  $J_0(u) = \int_0^T m(u,t) dt \rightarrow \min_{u \in U}$ , ( $u$  - "permissible control") leads to the results which agree with clinical experience, and distinguishes (if possible) "smooth" disease course.

4. "How to process data?" It is shown in /3/ that the method of maximum likelihood is very convenient for statistical estimation of model parameters. This method easily allows to take into account the peculiarities of clinical-laboratory data reception. It is assumed that the disease model parameters can be presented in the form:  $\alpha = \bar{\alpha} + \xi t/\epsilon$ , where  $\bar{\alpha}$  is average over time value of parameter  $\alpha$ ,  $\epsilon > 0$  is a small parameter, and  $\xi t/\epsilon$  takes into account "fast" disturbances of parameter  $\alpha$ . It is shown that under enough common conditions the value  $[x(t,\alpha) - x(t,\bar{\alpha})] / \sqrt{\epsilon}$  ( $x(t,\alpha)$ ,  $x(t,\bar{\alpha})$  are the model solutions corresponding to parameters  $\alpha$ ,  $\bar{\alpha}$ ) at  $\epsilon \rightarrow 0$  weakly converges to Markov-Gaussian random process. This property permits effectively applying the method of maximum likelihood for estimation of the disease model parameters.

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Line search objective functions in recursive quadratic programming algorithms for constrained optimization

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In order to force convergence in algorithms for constrained optimization, it is usual to impose the condition that no iteration may increase a "line search objective function", that depends on the original objective function and on constraint violations. In particular, the differentiable exact penalty function that is proposed by Fletcher (Mathematical Programming, Vol. 5, 1973, pp 129-150) is very suitable, except that its value depends on first derivatives. We are concerned with making use of such functions in order to obtain reliable and fast convergence without calculating second derivatives. One has to ensure that sufficiently small trial changes to the variables will reduce the line search objective function, but it is difficult to satisfy this condition, because there is insufficient information to calculate all the terms of the gradient of the line search objective function. We consider this problem in the case when all constraints are equalities. A successful algorithm that combines line searches in the space of the variables with differences of Lagrange multiplier estimates is described. Also an algorithm is presented that makes use of trust regions. Both methods are supported by convergence theorems, and some numerical results are given.

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SOME RESULTS ON INFINITE DIMENSIONAL RICCATI EQUATIONS

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ABSTRACT

We give some results on infinite dimensional Riccati equations. We shall consider deterministic as well as stochastic control problems for distribute parameters systems.

We shall also consider boundary control problems and periodic systems.

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The Application of Systems Engineering in the Optimization  
of Agricultural Structure

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Abstract

Since 1980, our country has paid great attention to the application of systems engineering and operations research methods in macro-economic analysis so as to speed up the realization of scientific management in agriculture.

In August, 1980, we began with the problem of improving the saline-alkali soil in Yucheng County, Shandong Province and worked out a mathematical model for the optimization of agricultural structure in this area.

In a one-year period from 1982 to 1983, in cooperation with the Agriculture Division Office of Changqing County, Shandong Province, we finished research for an optimal production structure for planting and livestock breeding in this area. In working out the structure for planting, we took into consideration the fact that different crop-changing formulas are used in the field arrangements on different soil. In the structure for livestock breeding, attention has been paid to the different numbers of each type of animal retained for breeding purposes in different areas. The aim of this optimal structure is to achieve, under certain conditions, the greatest net profit in this county within the 2-year production cycle. The condition constraints take into consideration the following factors: social needs, national responsibilities, production capability, ecological environment, storage-capabilities, marketing situations, the natural growth of the agriculture and livestock breeding industry itself, their relationship etc.. Based upon different climatic conditions, 4 large-scale linear programming models were set up, each model having more than 3,000 variables and 100 or so constraints and each having an optimal solution. Sensitivity analysis was used to determine the influence of the fluctuation of the marketing prices of agricultural and livestock products on the structure. The whole computation was done on the UNIVAC 1100 computer. Game theory was employed to get the optimal counter-measures against the influence of climatic factors. The result of the computation was considered practicable by certain experts and the department of production management in Changqing County. By employing the new structure in arranging production, the net profit of planting (under different climatic conditions) could be increased by 13.1%-27.6% and that of livestock breeding by 80%. Satisfactory results have been achieved during the first stage of implementation in 1984. Statistics of this country have shown that the total output of this area has been increased 16.8% by this new production structure. Now the plan is undergoing its second year of implementation.

In 1984, in order to extend the application of this method, we also began to cooperate with the agriculture management departments of Weifang City, Huan Tai County, Jimo County and Zhanhua County, all in Shandong Province, to carry out research to find the optimal production structure for these districts. In addition to doing research in planting and livestock breeding for an optimal structure, we also comprehensively studied the forestry and aquatic products industry. In Huantai County we concentrated our research particularly on a

structure for aquatic breeding in lake areas. In Weifang we analysed the production structure of a three-county area with the emphasis on the opening up of the area along the coast, on developing an optimal plan for aquatic industry and on the improvement of the saline-alkali soil.

In our country, the application of systems engineering and operations research in the management of agriculture has received great attention from respected figures in various disciplines. The State Planning Committee and the Committed of the Agriculture Division of Shandong Province have entrusted us with the task of working out a prediction model for the readjustment of production structure in the countryside of the whole province in five fields: agriculture, forestry, husbandry, fishery and processing industry.

The project has so far gone through the work of making the model and of obtaining statistics and it is estimated that this project will be finished by September, 1985. To push forward the process of this work, scholars in the field of operations research are working hard to study actual problems. At the same time, more persons are being trained in the field of systems engineering and operations research in order to put the models into practice.

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COMPUTER AIDED EXPANSION PLANNING OF COMBINED  
ELECTRIC POWER AND DISTRICT HEATING SYSTEMS -  
A MIXED INTEGER PROGRAMMING MODEL

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The expansion planning problem in an electricity and heat supply system is characterized by the complex of questions arising from time, location and way of expanding or modification the existing system to satisfy the power and heat demand in the future considering technological, economical and last not least ecological restraints. Type and installed capacity of powerstations, way of heat decoupling, type and topological expansion of the power and the heat transmission network, reserve requirements and alternative contracts are some topics of interest to be handled during the planning process. This real problem represents a stochastic nonlinear optimization problem in the mathematical sense, including a great number of equations, inequations, variables and coefficients so that computer aided planning methods become very important and helpful.

The Mixed-Integer-Programming (MIP)-Model, described in this paper, is a long-term planning model for the solution of the above mentioned tasks. Not only global and strategic studies can be carried out with help of the model, but also investigations of individual projects are possible. The objective of the optimization problem has been formulated into a minimum function of costs containing the total amount of the present worth of fixed and variable costs (investment costs and costs of operation), which is to be minimized over the planning period under certain restraints. The extensive group of restraints includes demand constraints, expansion and long-term operation constraints for power-stations, contract restraints and in case of the expanded model version considering both the electric and heating transmission network topology also the corresponding constraints for expansion and long-term operation.

The presentation of the MIP-model within this paper is started with the discussion of some elementary mathematical components for the long-term digital simulation of the power and heat system as a whole. The first step is the evaluation of the objective as a monetary criterion for the quality of the solution. The mathematical formulation of the power and heat demand restraints will be explained as a next step. The planning or optimization horizon is subdivided into summer and winter half-years. The restraints for the long-term operation of typical combined power and heat stations as condensing, extraction and back-pressure power plants are shown afterwards, also for heating boilers to catch the peak heat demand. The extension of the one-node standard model to a more-node model considering both the power and heat transmission topology including a simplified loadflow simulation will be explained.

The application of the planning-model to a practical power and heat system is illustrated within the next chapter of the paper by some diagrams. Some computational aspects evaluating the optimal and suboptimal solutions for the practical example are also discussed starting with the data handling, the numerical solution phase to the interpretation and the sensitivity of the solution in conclusion of the paper.

Further aspects for the incorporation of the model into a computer-aided planning package with a hierarchical structure in the optimization horizon from long- to short-term optimization are mentioned.

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## Convex programming on differentiable manifolds

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The best feature of convex programming problems in non-linear programming is that any local minimum is at the more efficient than in other cases. But the convex and generalized convex functions are defined on convex sets and this fact limits the use of this notion.

In the lecture the purpose is to extend the convexity notion to differentiable manifold. First the arcwise-convex functions are defined and characterized on arcwise-connected sets, on surfaces and on differentiable manifolds.

After it is considered the relation between this kind of convexity and the classical convexity.

Finally it is demonstrated the arcwise convexity has some important topological properties.

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MATHEMATICAL MODELLING OF THE PERSONNEL  
DYNAMICS IN ORGANIZATIONS

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Two mathematical models of the personnel dynamics in organizations are constructed. In the first model the individual promotions within an organization are described by a homogeneous semi-Markov process with a finite number of states, the flow of recruits being represented by the nonstationary Poisson process. It is found that the number of individuals in various states are asymptotically mutually independent Poisson variables.

For the second (deterministic) model of personnel dynamics in variable environment a boundary value problem for the linear partial differential equations is formulated. Mathematical analysis of the solutions is carried out including the study of their qualitative behaviour. To establish the weak ergodicity of solutions the Hilbert projective metric is applied. The results of the application of both approaches to some particular manpower system are presented.

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**POWER SYSTEM CORRECTIVE SWITCHING :**  
**A NEW APPROACH USING NON LINEAR PROGRAMMING**

BY J. RYCKBOSCH - FRANCE

1 - WHAT IS CORRECTIVE SWITCHING ?

The power system operator can generally establish connections between nodes in a great number of ways ; through modifications in the topology, he can reduce quickly and economically the overloads which may have occurred after an incident : this is what is called "corrective switching".

2 - A DIFFICULT COMBINATORY PROBLEM

Any topological modification can always be modelled as the addition or removal of a certain number of lines ; so we have a enormous combinatory problem, which cannot be studied completely. Moreover, it is impossible to imagine an heuristic method a priori choosing the most probable manoeuvres, reducing significantly the number of possibilities we have to study : a power system is an indivisible entity, and a local modification can have very distant effects on the whole system.

3 - THE PROPOSED NEW APPROACH

In the direct current approximation, equations are grouped into Kirchhoff's first law, and Kirchhoff's second law (for any cycle  $\Gamma$  of the network, we have  $\sum v_i = 0$  where  $v_i$  is the voltage phase difference between the end-nodes of the line  $i$ ).

In the Kirchhoff's second law, we write the phase differences as  $v_i = x_i (t_{1i} + t_{2i})$  where  $x_i$  is the reactance of line  $i$ ,  $t_{1i}$  the actual power flow through line  $i$ ,  $t_{2i}$  a fictitious unbounded power flow.

To describe all the possible switching configurations, we impose for all  $i$  :  $t_{1i} \cdot t_{2i} = 0$ . If  $t_{2i} = 0$ , then we recognize in  $v_i = x_i t_{1i}$  the standard equations : the line  $i$  is connected to the network ; if  $t_{1i}$  is 0, the line  $i$  is switched off and the variable  $v_i$  is unbounded.

Solving "corrective switching" problem thus means verifying the following system (S) :

$$\left. \begin{array}{l} \text{Kirchhoff's law} \\ \text{exclusion constraints} \end{array} \right\} \begin{array}{l} A \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} = B \quad \text{linear equations} \\ t_{1i} \cdot t_{2i} = 0 \quad \text{non-linear equations} \\ |t_1| \leq \bar{t}_1 \quad \text{inequations} \\ |t_2| \leq +\infty \end{array}$$

Actually, we prefer to solve the following optimization problem :

$$(\mathcal{F}) \quad \text{Min } \Sigma t_{i1}^2 \cdot t_{i2}^2$$

$$At = B$$

$$|t| \leq \bar{t}$$

$$\text{With } t = \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

All the solutions of (S) are unconstrained local minima of (F). So we are not looking for a global minimum, but only a local one and we can deal with a non convex problem. On the other hand, some solutions of (F) are not solutions of (S), but these bad solutions are always constrained minima and they can be eliminated, by multiplying the objective function by penalty functions becoming very high if the constraints tend to be bending.

Lastly, it should be mentioned that (F) is solved by a reduced gradient method, very suitable for linear constraints.

#### 4 - MODEL PERFORMANCES

Our method has already been successfully tested, first on the IEEE test system and later on several situations which actually occurred on the French regional networks. The computational time is already quite satisfactory (~ 3 seconds on IBM 3081) and could be greatly improved if we write a specific optimization code for our problem.

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OPTIMAL CONTROL OF FREE-BOUNDARY PROBLEMS.

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An important class of Optimal Control problems for non linear distributed parameter systems is constituted by free boundary problems, in particular, when the state is solution of a variational inequality. Multiphase systems as solidification problems, continuous mechanics problems, as elasto-plasticity, filtration problems, ..., are some well-known examples of such systems.

A typical example is given by the following variational inequality which modelizes the one phase Stefan-problem :

$$(1) \quad \begin{cases} \left( \frac{\partial y}{\partial t}, \phi - y \right) + a(y, \phi - y) \geq (f + Bu, \phi - y) \\ y(0) = 0 \quad \forall \phi \in K = \{ \phi \in H_0^1(\Omega) \mid \phi \geq 0 \text{ a.e.} \} \end{cases}$$

(y is the state variable and u is the control variable).

For such state system, different criteria may be studied. We consider three of them, which are the most specific problems.

$$i) \quad J(u) = \|y - z_d\|_{L^2(Q)}^2$$

In this case, we control the variation of the state variable y.

$$ii) \quad J(u) = \|x_{F(u)} - x_d\|_{L^2(Q)}^2$$

where  $x_{F(u)}$  denotes the characteristic function of the set :

$$F(u) = \{(x, t) \mid y_u(x, t) = 0\}$$

and  $X_d$  the characteristic function of a given subset  $F_d$  of  $\mathbb{R}^n$ .

In this case, we control the evolution of the free-boundary.

$$\text{iii) } J(u) = \mathfrak{m}_\delta(I(u)) \quad (\text{or } \delta(I(u)))$$

$$\text{with } I(u) = \{t \in [0, T] \mid \exists y(x, t) \notin Z_{ad}\}$$

( $\mathfrak{m}_\delta$  Lebesgue measure and  $\delta$  diameter of  $I(u)$ )

In this case, we minimize the domain where the state constraint  $y \in Z_{ad}$  are not verified (i.e. for example domain where defaults can exist).

For such optimal control problems, two main difficulties are to be considered :

- + the existence of optimality conditions
- + the implementation of efficient numerical methods.

For that, different techniques have been developed. They use principally the two ideas :

- i) To transform the initial problem into an optimal control problem with linear state equation and nonconvex constraints on the state. For that, we introduce the multiplier associated with the constraint  $y \in K$ .
- ii) To use a penalty technique for the constraint  $y \in K$ . Then we obtain an optimal control problem for a non linear parabolic equation.

To illustrate these different problems and the associated algorithms, we present problems connected to the optimal control of a continuous casting process. In that case, the physical system corresponds to the solidification of the steel. The goal is the computation of the best cooling system to increase the productivity and to guaranty the quality of the steel. Different numerical results are presented.

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APPLICATIONS OF MULTIPLEX ALGORITHM TO SOLVE  
 QUADRATIC PROGRAMMING PROBLEMS

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Abstract: Among the class of non-linear programming problems an important and richly studied subclass of problems is that of quadratic programming problems. Since the Kuhn-Tucker conditions are a set of linear inequalities for this class of problems, methods based on the simplex algorithm have been found to be quite efficient in solving them. Simplex algorithm is a univariate search technique and is therefore slow in convergence. A multiplex algorithm which is found to be fifteen to twenty times faster than the simplex even for a medium size linear programming problem has been developed and implemented successfully. The proposed algorithm is a multivariate search technique which brings into the basis a group of  $k$  number of variables ( $1 \leq k \leq m$ , where  $m$  is the number of resource constraints of the problem) not only at start but also in between passes. This procedure enables to choose a set of  $k$  ( $k \leq m$ ) linearly independent vectors to construct a nonsingular basis matrix at the beginning of each pass and is not so dogmatic in preserving the property of feasibility. Two criteria are used to identify the promising variables and they are the maximum rate of change criterion and the maximum change criterion. A matrix of intercepts of the

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decision variables is constructed which enables to bring into the basis a group of variables preserving the property of linear independence. In this respect, the multiplex algorithm is similar to the simplex procedure. However, the multiplex algorithm does not always guarantee basic feasible solution in the course of computation as simplex procedure does.

Certain checks are instituted to guard against landing on infeasible basic solution and if it does it is removed then and there by invoking dual simplex procedure. The multiplex algorithm finds a near optimal solution at the end of first pass. Simplex algorithm is choosy in selecting a feasible solution among basic solutions whereas multiplex accepts any basic solution and converges rapidly to optimal solution. From computational experience, it was found that preserving the feasibility property is a drag on the rate of convergence.

This is extended to solve quadratic programming problems also. The multiplex algorithm enables the quadratic programming problem to converge rapidly. The entry of multiple number of variables cuts down the computational effort considerably and this works successfully for quadratic programming problems as well. This is expected because quadratic programming problem is solved primarily using simplex procedure and hence multiplex algorithm should also be applicable equally efficiently. Numerical examples have been solved to illustrate the working of the algorithm and the savings in computation are reported.

OPTIMAL CONTROL METHODS  
FOR LARGE POWER SYSTEM  
PLANNING AND OPERATION

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## 1 - POWER SYSTEM OPTIMIZATION

The objective of a power system is the following: to satisfy the demand for electricity in the best conditions of cost and reliability. To achieve this, a great number of actions are necessary, ranging from the expansion planning of the plant mix and the networks to real-time control, and including all the phases of operational planning. For each of these actions, the operator must make choices: whether or not to build a new generation unit or transmission line, or decide on the maintenance scheduling for power plants or lines, or choose the unit commitment at any given moment, etc.

In order to assist the operator in taking the optimal decisions, an effort is made to provide a mathematical formulation for each of the problems and to solve them using optimization methods.

The main difficulties in power system optimization problems are linked to their considerable size, future uncertainty and the discontinuous nature of some variables.

## 2 - OPTIMAL CONTROL METHODS : FIVE APPLICATIONS FOR THE POWER SYSTEM

Optimal control methods are concerned with the optimization of intertemporal trajectories of systems. Any problem of optimal control is thus stated in terms of state variables, control variables and transition equations. The latter may be deterministic or stochastic. With the stochastic equation, an attempt is made to optimize the control variables as a function, at all times, of the state variables of the power system: this is what is called a strategy control, or a closed-loop control.

Optimal control methods are linked, in the main, to Dynamic Programming and the Maximum's Principle. Dynamic Programming allows for the processing of non-convex problems, and the optimization of strategy control variables: in this case, the term "Stochastic Dynamic Programming" is used. But this is applicable only to small-scale systems, unless it is combined with various techniques of decomposition. On the other hand, the Maximum's Principle remains applicable to large-scale power systems but cannot handle non-convex problems, or random future problems. In the special case of a linear quadratic problem, the Maximum's Principle leads to Ricatti's equation, which optimizes strategy controls, even for a large-scale system.

Rather than proceeding with a theoretical presentation of these methods, five applications for the optimization of the electric power system will be described :

- daily unit commitment using decomposition-coordination,
- scheduling of a tidal power station using the Maximum's Principle,
- scheduling strategy for a hydraulic reservoir using Stochastic Dynamic Planning,
- scheduling strategy for a group of nuclear reactors using stochastic decomposition-relaxation,
- load-frequency control using Ricatti's equation.

### 3 - DAILY UNIT COMMITMENT USING DECOMPOSITION-COORDINATION

Each day, on account of a forecast of the demand for electricity for the following day, the operator of a group of thermal units must establish the operational unit commitment. In addition to the coupling constraints of generation-load equilibrium and of the spinning reserve, there are also technical constraints specific to each unit. And it is important to find a least cost schedule, a term which includes not only the proportional operating costs, but also fixed costs and start-up costs.

In order to solve this large-scale, non-convex problem, its separability, due to the decoupled dynamics of the units, is exploited. The decomposition-coordination method consists in matching Lagrange multipliers with the coupling constraints and introducing the former into the objective function. The algorithm is performed by iterations between two levels :

- a coordination level where the Lagrange multipliers are updated,
- a decomposition level where the local assignment problems of each unit are solved using dynamic programming.

The decisive advantage of this method is its  $\epsilon$ -optimality.

### 4 - SCHEDULING OF A TIDAL POWER STATION USING THE MAXIMUM'S PRINCIPLE

All tidal power equipment, which can be extremely varied in nature, run in more or less regular alternations of two phases :

- transformation of the tides energy into potential energy, via storage by means of dams and gates,

- recovery of energy by turbinng.

*Maximizing the energy generation of such an equipment is a problem of optimal control, where the state variables are the basin levels, and the control variables the position of the gates and the turbined release flow.*

The use of the Maximum's Principle makes it possible to construct iteratively the optimal trajectory based on any initial trajectory.

### 5 - SCHEDULING STRATEGY FOR A HYDRAULIC RESERVOIR USING STOCHASTIC DYNAMIC PROGRAMMING

The presence of a hydroelectric reservoir in an electric power plant mix appears as an appreciable asset for ensuring the generation-load balance throughout the year in the best conditions of cost and reliability. Indeed, this equipment provides energy on a yearly basis which can be used at the times when it is the most useful : peak hours of the year, as a substitute for expensive thermal power, or even to avoid load shedding. Yet, at any moment the proper arbitrage must be made between immediate plant discharge providing sure savings, and maintaining the stored energy which will provide probable savings in the future. The arbitrage is all the more difficult because the future is quite uncertain : randoms involving demand, the unavailability of equipment and hydraulic inflows. Stochastic Dynamic Programming provides the optimal solution in the form of closed-loop decisions strategies (feedback).

### 6 - SCHEDULING STRATEGY FOR A GROUP OF NUCLEAR REACTORS USING STOCHASTIC DECOMPOSITION RELAXATION.

The particular feature of pressurized water nuclear reactors is that they can only be refueled when they are shut down. This operation requires 4-8 weeks and must be carried out on roughly a yearly basis. The operator of a group of nuclear reactors must thus optimize not only the assignment of refueling shutdowns, but also the use of the fuel of each reactor between refuelings. For a group made up of several dozen units, Stochastic Dynamic Programming, the only type of programming which enables the operator to compute decision strategies (i.e., decisions as a function of the state of the system at a given moment), is inapplicable. The decoupling of the unit dynamics does, however, allows us to envision optimization in a restricted category of strategies : local feedback. The computation consists in optimizing successively the scheduling strategy of each reactor, while taking into account the probable scheduling of all the other reactors.

This method, called stochastic decomposition-relaxation, provides, along with excellent numerical performances, scheduling strategies in the random future which are close to optimal strategies.

### 7 - LOAD-FREQUENCY CONTROL USING RICATTI'S EQUATION

The role of the Load-Frequency Control is to ensure permanently the fine tuning between electric power generation and demand, so that the frequency remains constantly as close as possible to its set value. It is important to optimize this fine tuning, without, however, subjecting the generating units to excessively high variations.

With this problem of optimal control, the state of the system encompasses a great number of variables whose dynamics are linearly coupled :

- the instantaneous frequency deviation (or the integral thereof...),
- the instantaneous deviation of power exchanged with foreign systems,
- the instantaneous generation deviation of each unit, in respect to the operational program.

The use of Ricatti's equation provides an optimal closed-loop solution, along with numerical performances which are perfectly compatible with real-time requirements.

## A CLASS OF INTEGER PROGRAMMING MODELS FOR PRODUCTION SCHEDULING

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Integer programming formulations are presented for a class of the following production scheduling problems:  $n$  different operations  $j$  ( $j=1, \dots, n$ ) have to be performed many times (operation  $j$   $z_j$  times) on  $m$  nonidentical machines  $i$  ( $i=1, \dots, m$ ). For each operation  $j$  given are processing times  $p_{ij}$  that  $j$  has to spend on the various machines  $i \in I_j \subseteq I = \{1, \dots, m\}$  that are capable of performing it. The subsets of machines  $I_j$  ( $j=1, \dots, n$ ) do not need to be disjunctive. Each machine can process at most one operation at a time, however the operation of one type can be processed on a number of machines simultaneously. The operations are subject to general precedence relations represented by an arbitrary acyclic digraph  $G = (J, A)$  with vertex set  $J = \{1, \dots, n\}$ , Associated with each arc  $(j, k) \in A$  there is an integer  $c_{jk} \geq 1$  denoting the number of times the operation  $j$  must be performed before the operation  $k$  can be started and performed once. The problem objective is to determine an assignment of operations to machines and lot sizes of the scheduled operations over a scheduling horizon in such a manner as to complete the production order  $(z_1, \dots, z_n)$  and minimize some time or cost criterion  $\gamma$ . A conceptual formulation of the scheduling problem is

minimize  $\gamma$

subject to four groups of the constraints:

- 1) the assignment constraints
- 2) the precedence constraints
- 3) the nonpreemptibility constraints
- 4) the production order completion constraints.

Two types of the problem formulations are considered: the integer formulation with the integer assignment variables  $x_j^t$  denoting the number of machines assigned to operation  $j$  in period  $t$ , and the

binary formulation with the binary assignment variables  $x_{ij}^t$  denoting whether or not the operation  $j$  is performed on machine  $i$  in period  $t$ . The binary formulation is a general one whereas the integer formulation can be used only in the case of unit processing times  $P_{ij} = 1$  for all  $i \in I_j$ ,  $j \in J$ . In the case of the binary formulation the precedence and the nonpreemptibility constraints incorporate nonalgebraic expressions (the entier function  $[ \cdot ]$ ). The only exception is the case of unit production order  $z_j = 1$  for all  $j$ , and a treelike digraph  $G$  with  $c_{jk} = 1$  for all  $(j,k) \in A$ .

It follows from an analysis of admissible schedules that the assignment of machines is constant for a number of consecutive periods and hence the variables  $x_{ij}^t$  ( $x_j^t$ ) need not be defined for all  $t$ . To limit the number of variables which have to be explicitly evaluated one may aggregate a number of consecutive periods into one longer period and introduce new variable  $h_t$  denoting its time duration. Then the scheduling problem will be to determine the assignment of machines in each of such periods and their time durations. Such a reformulation of the scheduling problem is particularly useful in the case of batch-type production where inventory of semi-finished products and the assignments are updated only at the beginning of each aggregate period.

Heuristic approaches to solution of the integer programming optimization problems are discussed for the case of minimax optimality criterion.

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OPERATION PLANNING OF POWER SYSTEMS  
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ABSTRACT

This work deals with the economical optimization in electric utilities. After a short review about the optimization activities of the Styrian electric utility STEWEAG, a report is given about an attempt of IBM Austria and STEWEAG to practically implement a short term operation planning system, which is universally applicable to electric power systems and heating systems. Some of the highlights of this optimization model are:

- . Complex water reservoir systems including reservoir networks, time delays for the water flow and water contracts to downstream plants
- . Coupled energy systems with electric and heating power generation
- . Sophisticated interchange contracts taking into account several types of contracts
- . New acceleration strategies for solving the optimization problem

The solution of the optimization problem is based upon the mixed integer programming package MPSX/370 and MIP/370. The optimization kernel is embedded in an extended control program containing a number of new acceleration measures which were necessary to achieve practically acceptable results in reasonable computing times.

A paper giving an overview about this program package and dealing with the optimization technique has been already published by P.G. Harhammer /1/. In the present paper emphasis is given to the mathematical model and the optimization results. The application of this system and the results achieved with it will be discussed on the basis of real life problems of the Styrian electric utility STEWEAG. Finally reports will be given about the planned extension of this package for the mid term operation planning problem.

OPERATION PLANNING STRATEGY

The short term operation planning with a planning horizon of one up to seven days is executed in two steps

- . Weekly optimization
- . Daily optimization

Based upon the computer assisted short term forecast the unit commitment of the whole (or the remaining) week is optimized with time steps of four hours. It is the main goal of the weekly optimization to deal with the start-up and shut-down decisions of thermal plants taking into account the different load patterns of the days, the weekend influence of the water reservoirs and the fuel restrictions. The results are global unit commitment, storage management and fuel management schedules. The uncertainty in the weekly forecast justifies the rough time discretisation.

Embedded in the weekly optimization the daily optimization is calculated on a 48 time step basis in order to get smooth schedules for the components of the energy system. The boundary constraints for the daily models are automatically transferred from the weekly model into the daily one.

In addition the solution of the weekly model is used as a starting solution for the daily optimization.

#### Mathematical model:

The mathematical model of the energy system is automatically generated by an universally applicable matrix generator. A mathematical representation of the model system will be given (although somewhat simplified for the sake of clarity).

#### Results

Experiences with this system will be shown based upon different example problems of the STEWEAG power system. This system has a very complex nature because nearly all types of plants are present. The energy generation is characterized by a strong hydro generating contribution, a relatively complex interchange contract and combined thermal power plants for electric power and heating power generation.

/1/ P.G. HARHAMMER Optimization of large scale MIP models  
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Abstract

Optimal Energy Management

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This report presents the result of a joint development in Austria to optimally allocate the resources (primary energy, contracted secondary energy) for the operation of a given energy system. Single line based energy systems (electricity, district heating, steam) and complex mutually coupled ones (cross-operation of public energy systems, industrial energy households) can be taken into account considering a planning period of a day or a week in advance or a week combined with an underlying day respectively. The program is enduser oriented and is operated menu driven in an VM/CMS environment.

The program package consists of four modules. The first one covers the menu driven input of system and operation data. The second module (matrix generator) transforms the given data via the mathematical MIP model into the input data stream of the standard optimization program (MPSX/370 with MIP/370 extension) being the third module. The fourth module presents graphically

and numerically the optimization results via colour displays or colour printers respectively.

High attention was given to a relative simple mathematical model of the highly non-linear energy process to obtain both practice acceptable results and short execution times for the optimization run. Special measures (starting solutions, heuristic preoptimization, system analysis techniques) combined with the standard optimization program decreased the execution time by 77 % to 7,24 minutes (IBM System 3033) for a very complex dynamic model (48 time steps) with a matrix of 4000 by 9000 (including 1000 integer variables). Some 50 % of CPU-time could be saved when running the hierarchical model system (week with one underlying day).

Some percent savings in operation costs can be expected by optimally allocating the necessary resources (primary energy, contracted secondary energy) when applying the program package "Optimal Energy Management" to plan the operation of public or industrial energy systems.

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Design, analysis and implementation of thermodynamically motivated simulation for optimization of subgraphs

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Abstract

A general principle was elaborated to apply thermodynamically motivated strategies on NP-complete subgraph optimization problems with given placing requirements for subsets of vertices. Such problems arise for example in the area of automation equipment, integrated circuit and computer network layout where wiring problems are important. The thermodynamically motivated strategy is a stochastic optimization algorithm by simulated annealing of ideal gases. For fixed temperature  $T$  it is a finite homogeneous Markov chain, which converges to the Boltzmann distribution under certain assumption. It is proved, that the sequence of Boltzmann distributions for  $T \rightarrow 0$  converges to all optimal solutions equally distributed and all other feasible solutions are reached with probability 0. Beside, some remarks on the speed of convergence are given.

An interactiv FORTRAN programe - package for solving the Steiner Tree Problem and the optimization of interacting paths systems with help of thermodynamically motivated simulation was developed. Computing results for graphs up to 600 vertices are presented including a comparison with a Greedy algorithm.

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## MATHEMATICAL PROGRAMMING SOFTWARE

## - Abstract -

The development of efficient and reliable algorithms for solving practical optimization problems was one of the main research areas of the last 20 years in mathematical programming. In the present situation a significant amount of numerical experience and knowledge on the performance of computer programs is available, in particular in linear programming, quadratic programming, least squares optimization, and smooth nonlinear optimization. Only in linear programming, we observe major additional attempts to facilitate model building, program execution and report writing for those who have to solve practical application problems. But based on the previous comment, we have to be aware that also in other areas, mathematical programming codes will be in the hands of non-specialists or occasional users, who have difficulties (or do not possess the time) to find a suitable algorithm, to transform the problem into the required form, to understand the documentation and to execute the FORTRAN code.

The intention of the lecture is to present a few ideas on principal approaches and to give some information on a specific system with the name NLPSYS, that supports model building, numerical solution and data processing of nonlinear programming problems, i.e. of problems in the form

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & g_j(x) = 0 \quad , \quad j=1, \dots, l \\ & g_j(x) \geq 0 \quad , \quad j=1+1, \dots, m \\ & x_l \leq x \leq x_u \end{array}$$

where  $x$  is an  $n$ -dimensional vector and where all problem functions are continuously differentiable. In the remainder of this abstract, only some information on NLPSYS will be given to understand the underlying idea by means of an example.

Various options are available in NLPSYS to facilitate the formulation of problem functions. The objective function e.g. may be a linear or quadratic function, a linear or nonlinear least squares function, a parameter estimation function, a sum of functions or a general function without a structure that could be exploited. Only the problem relevant data need to be provided by a user in an interactive way. Functions must be defined by sequences of FORTRAN statements addressing a numerical value to a user-provided function name. All generated problems are stored in form of a data base system, so that they are easily retrieved, modified, or deleted on request. NLPSYS selects a suitable mathematical algorithm and writes a complete FORTRAN source program. The system executes this program and stores the numerical results in the data base, so that they are available for further processing. Since

individual names for functions and variables can be provided by a user, it is possible to get a problem dependable output of the achieved solution.

The user has the option to choose between two solution levels. The standard level will execute a sequential quadratic programming algorithm or a suitable variant to take advantage of special problem structures. If this method fails or if a suitable starting point is not available, a user could proceed to another solution level which is based on an implementation of the ellipsoid-method.

The user will be asked whether he wants to link the generated FORTRAN program with some of his own files or whether he wants to insert additional declaration and executable statements to formulate the problem. Moreover it is possible to provide keywords and to define 'experience fields', so that the system is capable to learn from previous solution ways. A failure analysis explains some reasons for possible false terminations and proposes remedies to overcome numerical difficulties.

All actions of NLPSYS are controlled by commands which are displayed in form of menus. Step by step the user will be informed how to supply new data. Whenever problem data are generated or altered, the corresponding information will be saved on a user-provided file. Besides commands to generate, solve or edit a problem, there are others to transfer data from one problem to another, to delete a problem, to sort problems, to update the known experience, to get a report on problem or solution data, to halt the system and to get some information on the system, the mathematical models and the available algorithms.

NLPSYS should be considered as an example for constructing a possible model building system. The underlying ideas could be exploited to other areas in mathematical programming or numerical analysis as well.

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A HEURISTIC ALGORITHM FOR INTEGER  
PROGRAMMING PROBLEMS

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The algorithm discussed can be referred to the class of the component algorithm of the possible integer directions. It is aimed at solving the following integer programming problem.

$$\begin{aligned} \max z &= \sum_{j=1}^n c_j x_j \\ \sum_{j=1}^n a_{ij} x_j &\leq b_i, \quad i = 1, 2, \dots, m \\ 0 &\leq x_j \leq d_j, \quad j = 1, 2, \dots, n \\ x_j &- \text{integer}, \quad j = 1, 2, \dots, n \end{aligned}$$

The coefficients of each constraint are normalized at every iteration of the algorithm, depending on the sign of the difference between the right side and the value of the left side of the respective constraint for the actually considered integer point.

The possible integer direction, and step length are defined on the base of the normalized coefficients, so that the total unfeasibility of non-satisfied constraints were decreased, if the current integer point is an unfeasible solution, or the objective function was improved, if the current integer point is a feasible solution.

The objective function improvement is in fact brought to the decrease of the unfeasibility of a constraint, the latter being constructed on the base of the objective function, and its value for the actually found possible solution.

Any integer point can be chosen as initial, if its components satisfy the constraint  $0 \leq x_j \leq d_j$  /  $j = 1, 2, \dots, n$ . The algorithm stops when no possible integer direction can be found.

Two procedures, determining a possible integer direction with one or two non-zero components have been proposed. For this case, a theoretical analysis of the algorithm has been performed

which has shown that the algorithm consists of a finite number of iterations. The number of the elementary (basic) operations at worst is limited by a third degree polynomial, multiplied by the parameter, determined from the initial unfeasibility and the optimal value of the objective function. The program realization has been run on an EC-1033 computer. 14 test-examples of low and middle dimension (up to 100 variables and constraints), found in the literature, have been used. A comparative analysis with the results, obtained on the same computer by means of the well-known program realizations of two exact and two approximate algorithms, has been carried out.

The program realization of the algorithm takes up comparatively little main memory. The memory needed by the algorithm when working depends on the initial matrix of constraints only.

Besides, the algorithm is very fast and precise enough in the obtained solutions. Because of that, it has been included in the software of a microcomputer system for industrial transport control. The system has already practical implementation.

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## QUADRATIC BOOLEAN PROBLEMS AND LOVASZ BOUNDS

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Quadratic Boolean problems are the natural generalization of linear Boolean problems. They are defined accordingly by

$$\min K_0(x) \quad (1)$$

subject to

$$x \in E^n$$

$$K_i(x) \leq 0; \quad i = \overline{1, m}, \quad (2)$$

$$x_j \in \{0, 1\}; \quad j = \overline{1, n}, \quad (3)$$

where  $K_i(x)$  are quadratic functions of  $x = (x_1, x_2, \dots, x_n)$ .

The problem (1) - (3) is NP-complete in general case.

Branch and bound methods can be used in order to obtain an exact solution. A new technique of evaluating bounds is suggested.

Booleaty of variables can be replaced by a constraint

$$x_j^2 - x_j = 0; \quad j = \overline{1, n}. \quad (3')$$

Let  $u = (u_1, u_2, \dots, u_m, \bar{u}_1, \dots, \bar{u}_n)$  be  $(n+m)$ -dimensional vector of dual variables. The Lagrange function of (1) - (3') is defined as

$$L(x, u) = K_0(x) + \sum_{i=1}^m u_i K_i(x) + \sum_{j=1}^n \bar{u}_j (x_j^2 - x_j) = \\ = (K(u)x, x) + \ell(x, u),$$

where the elements of  $K(u) = \{K_{ij}(u)\}_{i,j=1}^n$  are linear in  $u$ , and  $\ell(x, u)$  is linear in  $x$  and  $u$ . The dual bound is defined as follows:

$$\psi(u) = \inf_x L(x, u),$$

$$\psi^* = \sup_{u \in \bar{U}^+} \psi(u), \quad (4)$$

where  $\bar{U}^+ = \{u | u \in U, K(u) \text{ is nonnegative matrix}\}$ ,  $U$  is the subspace of  $(n+m)$ -dimensional space, cut by the constraints  $u_i \geq 0$  for  $i$ , corresponding to inequalities in (2).

Dual bounds (4) look like famous Lovasz bounds. That is why

it is quite natural to ask if the dual bounds equal Lovasz bounds for the maximum stable set problem.

Let  $G(V, E)$  be a graph where  $V$  is the vertex set,  $E$  is the edge set of  $G$  and  $V = \{v_1, \dots, v_n\}$ . Boolean variable  $x_i$  corresponds to every vertex  $v_i$  of  $G$ . Then the maximum stable set problem can be rewritten as the quadratic Boolean problem

$$\alpha(G) = \max_x \sum_{i=1}^n x_i \quad (5)$$

subject to

$$x_i \cdot x_j = 0; \{v_i, v_j\} \in E, \quad (6)$$

$$x_i^2 - x_i = 0; i = \overline{1, n}. \quad (7)$$

The proof of equality between Lovasz bounds and dual bounds for the maximum stable set problem is given in the report.

Let us consider additional constraints

$$x_i \cdot x_j \geq 0; \{v_i, v_j\} \in \overline{E}. \quad (8)$$

Then the dual bound of problem (5) - (8) equals the more precise bound of McEliece, Rodemich, Rumsey.

Adding of quadratic (linear) constraints easily matches the scheme of dual bounds computation.

Thus dual bounds represent generalization of Lovasz bounds for a wide class of combinatorial problems.

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A QUASI-SUBGRADIENT SCHEME  
FOR CALCULATING SURROGATE CONSTRAINTS

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The quasi-subgradient as a notion substituting for the well known subgradient in the class of quasiconvex functions emerged from a duality theory presented in [3]. Its definition uses a certain quasiconjugate which is somewhat similar to the conjugate of a convex function. In terms of supporting functionals the quasi-subgradient corresponds to a level set while subgradient concerns the epigraph set of a function. This is determined by the fact that epigraphs are not necessarily convex for quasiconvex functions while level sets are.

The surrogate constraint duality in the form presented in [2] or [1] leads to a constrained maximization problem with a quasiconcave objective function. It may be considered, from the operative point of view, as a rather difficult problem for it involves an objective function which may be discontinuous.

It is the aim of this paper to show that such a problem can be solved with algorithms based on a general recursive scheme modeled after [4]. Quasi-subgradients are used as main search directions and additional directions are defined to seek feasible surrogate multipliers when the maximization process yields infeasible ones. A support vector of a given constraint function with respect to the nonnegative orthant  $R_+^m$  is proposed to serve in that case. The main convergence

result is proved for the general formulation so it remains valid for any algorithm modeled after the described scheme.

Two quasi-subgradient algorithms for solving the surrogate dual program are presented for example as particular versions of the scheme. One was described in [1] while another is a new proposal. Both are endowed with a new effective stop test and their convergence is assured by the main theorem.

Finally these two algorithms are compared experimentally with several integer test problems. All test problems have nonlinear separable objective functions and linear inequality constraints generated randomly.

Taking all presented results into account, we find some algorithms based on the presented scheme useful for solving surrogate dual programs in practice. Since the surrogate dual offers a tighter bound than the Lagrangean dual on the primal optimal value this gives opportunity to improve branch-and-bound methods for integer programming problems.

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Linear programming for electric power distribution system expansion planning: A dynamic expansion model for loop structured networks

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In urban supply areas electric power is distributed frequently by cable networks with a special topology connecting few infeeding points with a great number of transformer substations in a spatially restricted area. The network topology is characterized by the fact, that each substation is connected with the other nodes by exact two branches. One example is the loop structured or ring network, often used in practice by its low investment costs and the clear operation conditions.

The expansion planning problem of such loop structured networks with the objective of minimum capitalized investment costs over the planning period is related to the symmetric travelling-salesman problem. But there raise the following additional difficulties: Reasonable investment decisions in power distribution systems require a planning period of at least ten years taking into account the dynamic load development. Therefore a linear optimization model must contain several time steps each covering one or some years. A fundamental problem is the concavity of the continuously expanded cost function for the simultaneous erection of several cable lines in the same trench similar to the fixed-cost transportation problem. A new additional load node is often supplied by cutting an existing cable and joining the ends to the new substation. Therefore the topological structure of the citizen street network must be modelled too, which enlarges considerably the number of variables and constraints.

The solution method is based on time discretization and the application of a large linear optimization model avoiding as far as possible integer variables for modelling discrete investment decisions. This is achieved by the special property of transportation problems, that basic solutions are pure integer under special conditions. This method

presupposes first of all a sophisticated constraint system to prevent the degeneration of the optimal solution. The classical system of restrictions defines generally upper bounds for the number of branches within all subsets of nodes up to a certain size. But in this new approach to be presented constraints for minimal flows within selected groups of nodes are used. In contrast to the classical method the number of this constraints does not grow exponentially but only quadratically with the overall number of nodes.

A second problem is an adequate formulation for the transportation capacity of the cables. The usual constraints for mixed problems would destroy the integrity of the decision variables in the optimal solution and thus they cannot be applied here. It is necessary to use a separate flow for each demand node in connexion with suitable flow restrictions. Last but not least the non-convex transportation cost function caused by the multiple use of the same cable trench must be included in the continuous linear model.

Powerful standard software (IBM/MPSX 370) is used in search of an optimal solution for the described problem. A FORTRAN coded matrix generator has been established, which enables planning engineers in power utilities to apply this optimization model for many variants of practical problems in this field. Characteristic upper bounds for the model size implied by computing time are about 50 substations, 150 street crossings and three time steps. A practical example demonstrates the application of the presented planning method.

So far there have been published only heuristic procedures for static problems of practical size. The presented model is the first dynamic expansion model for loop structured power distribution networks in urban supply areas. It is hoped that in future such optimization methods will be widely accepted as useful tools for power engineers.

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## FORCED INTEGRO DIFFERENTIAL MODELS WITH DELAYS

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The aim of the paper is the study of an optimal control problem for a Volterra integro-differential system with delays which models the competition between two species. In fact let consider the forced prey-predator model with response delay to resource limitations:

$$\begin{aligned} N_1'(t) &= [b_1 - a_{12}N_2 - (b_1/c) \int_{-\infty}^t N_1(z)k_1(t-z)dz] N_1(t) + u_1 \\ N_2'(t) &= [-b_2 + a_{21} \int_{-\infty}^t N_1(z)k_2(t-z)dz] N_2(t) + u_2 \end{aligned} \quad (1)$$

where  $a_{12}, a_{21}, b_1, b_2, c \in \mathbb{R}_+$ , the kernels  $k_1(t) = T_1^{-2} \exp(-t/T_1)$ ,  $T_1 > 0$ ,  $k_2(t) = T_2^{-1} \exp(-t/T_2)$ ,  $T_2 > 0$ ,  $N_1$  = number of prey,  $N_2$  = number of predators,  $u_1, u_2$  = the rates at which prey and predators are released. The system has a positive equilibrium  $e_1 = b_2/a_{21}$ ,  $e_2 = b_1(c - b_2/a_{21})/ca_{12}$ , provided  $c > b_2/a_{21}$ , and one knows that at least for some ranges of parameter values, the equilibrium is stable.

Usually one desires a biological equilibrium, which suggests to take as target set  $P(e_1, e_2)$ ,  $e_1 = N_1(T)$ ,  $e_2 = N_2(T)$ ,  $T$  - specified or not, or being asked minimal.

Let take as performance index the functional

$$\begin{aligned} J(\varphi_1, \varphi_2, u_1, u_2) &= c_0(T - t_0) + \frac{1}{2} \int_{t_0}^T [N_i(T) - e_i]^2 + \\ &+ \int_{t_0}^T [c_1 u_1(t) + c_2 u_2(t) + c_3 N_1(t)] dt \end{aligned} \quad (2)$$

where  $\varphi_i(t) = N_i(t)$ ,  $i=1,2$ ,  $t \in [t_0, T]$  - which means that on the interval  $[t_0, T]$  the populations can be observed,  $\varphi_i$  being the results of measurements and  $c_0, c_1, c_2, c_3$  are real nonnegative constants.

Denote by  $\Phi$  - the set of admissible initial functions  $\varphi = (\varphi_1, \varphi_2)$  and  $U_{ad}$  the set of admissible controls  $u = (u_1, u_2)$

We state the following:

Optimization Problem : If the state is given by the solution of system (1), find an admissible policy  $\{\varphi, u\} \in \Phi \times U_{ad}$ , such that the control  $u$  transfers the initial state  $\varphi = (\varphi_1, \varphi_2)$  to the target  $P(e_1, e_2)$  and minimizes the performance index (2).

Regarding the existence of such an optimal policy one can apply an extension of a theorem due to Neustadt.

In order to determine it, we approximate system (1) by its linearization about the equilibrium and construct the optimal policy for the new optimization problem by means of certain necessary conditions using dynamic programming techniques.

The results are suitable to be applied to various biological problems regarding harvested populations.

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## NEW APPROACH TO THE TRAVELING SALESMAN PROBLEM

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In spite of existence of many different algorithms for the traveling salesman problem (TSP), its solution remains a difficult task. For this reason the aim of this paper is to propose a new mathematical formulation for the time-dependent traveling salesman problem (TDTSP). The idea of the proposed formulation is to introduce new variables, which allow to reformulate TDTSP as the known problem of minimum cost flow in a gain network and with additional constraints on variables to be integer. Such reformulation can lead to new algorithms based on the branch and bound scheme. In particular, a new method for calculation of lower bounds can be constructed by solving the reformulated problem with real valued variables.

More precisely, the proposed formulation of TDTSP is as follows:

$$\begin{aligned} \text{minimise} \quad & \sum_{j=1}^n c_{0j}^0 x_{0j}^0 + \sum_{t=1}^{n-1} \sum_{i=1}^n \sum_{j=1, j \neq i}^n c_{ij}^t x_{ij}^t + \\ & + \sum_{j=1}^n c_{jn+1}^n x_{jn+1}^n \end{aligned} \quad (1)$$

subject to

$$\sum_{j=1}^n x_{0j}^0 = 1 \quad (2)$$

$$2 \cdot x_{0j}^0 = f_j^1 + \sum_{k=1, k \neq j}^n x_{jk}^1, \quad j=1, 2, \dots, n \quad (3)$$

$$2 \cdot \sum_{i=1, i \neq j}^n x_{ij}^t = f_j^{t+1} + \sum_{k=1, k \neq j}^n x_{jk}^{t+1}, \quad \begin{matrix} t=1, 2, \dots, n-2 \\ j=1, 2, \dots, n \end{matrix} \quad (4)$$

$$2 \cdot \sum_{i=1, i \neq j}^n x_{ij}^{n-1} = f_j^n + x_{jn+1}^n, \quad j=1, 2, \dots, n \quad (5)$$

$$\sum_{t=1}^n f_j^t = 1, \quad j=1,2,\dots,n \quad (6)$$

$$x \geq 0, \quad x \text{ integer} \quad (7)$$

$$f \geq 0, \quad f \text{ integer} \quad (8)$$

where  $x_{ij}^t$  is a decision variable, which equals to 1 if the salesman travels from  $i$ -th city to  $j$ -th one at  $t$ -th stage of his tour and 0 otherwise; while  $c_{ij}^t$  is the associated cost. In the above,  $f_i^t$  is also a decision variable, which equals 1 when  $i$ -th city is visited at  $t$ -th stage of the tour and 0 otherwise.

It can be shown that the problem (1) - (8) is equivalent to the known TDTSP formulation (see [1]). The above problem, without integrality constraints, gives convenient lower bound which can be used in branch and bound algorithms. Let us note that this relaxation can be intuitively interpreted as a collection of routes passing through  $n$  cities. In every route a certain weight  $f_i^t$  is attached to each city in such a way that (6) holds.

Numerical investigations of an algorithm based on the above ideas are in progress.

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ON A CLASS OF MULTICRITERIA OPTIMAL CONTROL PROBLEMS

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This paper presents an approach to solving a class of multicriteria optimal control problems. Specifically, a closed and convex valued multifunction will define certain preference structure in the goal space of a dynamical vector minimum problem. These preferences are in compliance with all additional information used for selecting a control from the set of nondominated controls.

In the first part of the paper we recall some earlier results (cf. Skulimowski [2]) concerning the use of a closed reference set  $Q$ . It has been proved that, under assumptions similar to those in the classical distance-minimization, the solutions obtained while minimizing the distance from the set  $Q$  are nondominated.

The above concept has demonstrated its usefulness in analysis of numerous vector optimization problems. However, it may turn out to be insufficient for solving some types of dynamic problems, for instance in optimizing dynamical systems with open or uncertain time of termination. Considering such a vector optimization problem with a preference structure of the minimal distance type leads us to introduce the multifunction  $F$  describing the demand set of vector-objective values for each moment of possible termination of the process.

The more precise statement of the problem may be presented as follows. The subordinated system  $S_1$  described by the controlled ordinary differential equation on the interval  $[t_0, T]$  minimizes the set of scalar-valued objectives  $J(u, r) = (J_1(u, r), \dots, J_N(u, r))$ , while  $u$  is a control for  $r$  belonging to  $[t_1, T]$ ,  $t_0 \leq t_1 \leq T$ . The criteria  $J_1 \dots J_N$  are absolutely continuous functions of  $r$ . Next, it is assumed that the system  $S_1$  is not autonomous, in the sense that it must follow certain

demands imposed by a superior decision center  $S_2$ . This additional information about preferences takes the form of a requirement, namely that the values of  $J(u,t)$  belong to the set  $F(t)$  for  $t_1 \leq t \leq T$ , where  $F$  is a Lipschitz-continuous multivalued function with values in the family of closed and convex subsets of the goal space.

In the applications of the above dynamical decisional model the constraints defining  $F(t)$  may be implied for instance, by cost, energy technology or social limitations.

Posing demands  $F(t)$  it is not always possible to foresee if there does exist at least one  $t$  belonging to the interval  $[t_1, T]$  such that  $J(u,t)$  is an element of  $F(t)$  for a nondominated control  $u$ . When the superior center demands cannot be fulfilled for all  $t$ , it is necessary to regularize the problem in a way which would allow their minimal /in some sense/ satisfaction. In this case, auxiliary integral objectives applying the distance in the goal space may be introduced. We prove several theorems which warrant that the solution methods proposed are well-defined. We also give a characterization of the situation when the goals of systems  $S_1$  and  $S_2$  are conflicting.

Some applications of the methods described are presented in the final part of the paper.

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## OPTIMIZATION PROBLEMS WITH RANDOM OBJECTIVE FUNCTION

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We consider the following optimization problem

$$(F_{\eta} : V \rightarrow \mathbb{R}) \rightarrow \min \quad (1)$$

where  $F$  is an upper semi-continuous function defined on a Banach space  $U$  with scalar values and  $V$  is an element of a family  $S$  of closed, convex and totally bounded domains of optimization contained in  $U$ . We assume that there is a probability distribution  $p$  in the space of  $\mathbb{R}$ -valued upper semi-continuous functions on  $U$  and that  $F_{\eta}$  depends on the  $p$ -random factor  $\eta$ . There also exists a quasi-measure  $\nu$  defined on the smallest algebra of subsets of  $U$  containing  $S$ .

$S$  and  $\nu$  form the additional information structure which will be applied to the analysis of the problem (1). We present some examples of such structures and give their further properties. Then we recall the ideal point method for solving the problem (1), i.e. the method of finding  $u_{\text{opt}} \in V$  such that

$$J(u_{\text{opt}}) := \int_{\mathbb{E}} [f(u_{\text{opt}}) - \min \{ f(u) : u \in V \}]^r dp \quad (2)$$

is minimal for given  $r \geq 1$ . We propose some methods of applying the structure  $(S, \nu)$  to solve the problem (1), e.g. by minimizing the generalized functional

$$J_1(V, u, r) := \varphi(\nu(V), \int_{\mathbb{E}} [f(u) - \inf \{ f(v) : v \in V \}]^r dp) \quad (3)$$

We assume that  $\varphi$  is a continuous real function,  $r \geq 1$ , and  $u \in V$ . Thus the problem (1) may be transformed into a problem of finding the optimal pair  $(u_{\text{opt}}, V_{\text{opt}})$  with  $u_{\text{opt}} \in V_{\text{opt}}$ . We prove several existence theorems and point out the connections with vector optimization. In particular, we investigate the interdependence between the set

$V_{opt}$  and the set of all non-dominated solutions to the problem (1) with the set of criteria being the support of the probability distribution  $p$ . We also study the dynamic case with random termination time and a time-parametrized family of random objective functions described by certain E-valued stochastic process  $\Phi$ .

Next, we consider the problem (1) with random domains of optimization  $V$  assuming that a probability distribution  $P$  on the family  $S$  is given. We assume that the distributions  $p$  and  $P$  fulfil conditions which makes possible their uniform treatment according to the hitherto presented methods.

Examples of applications of the proposed methods are shown in the final part of the paper, one of them referring to the well-known jet propulsion design problem.

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MODELLING THE DECISION MAKER'S ASPIRATIONS  
IN MULTICRITERIA ANALYSIS

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Multifarious methods solving multiobjective optimization problems require some knowledge about a system taken under consideration and usually base on a certain preference structure introduced in the objective space. The nature of additional information while solving real-life multicriteria problems brings about that appropriate modelling and formulation of the decision maker's /DM's/ demands and aspirations are of great importance to the adequacy of the solution selecting procedures.

This paper presents an application of aspiration levels /AL-s/ to the problem of selecting a satisficing solution in the process of multicriteria decision making. The assumed preference structure for solving the multicriteria optimization problem implies making use of DM's AL-s, the idea which has been studied e.g. by Wierzbicki, Górecki and others. Each AL is a point in the objective space which represents desired values of criteria. Besides of the desired values, in this paper we also consider antipodal AL-s /or "forbidden levels"/, achievement of which may be regarded as the DM's failure.

AL-s as well as forbidden ones come as results of experts' judgments involved in decision making. We assume that each forbidden level is dominated by at least one attainable

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point while each desired point dominates at least one nondominated attainable point.

Evaluations of the experts are assumed to be independent and indifferent from the DM's point of view. The separate consideration of each level with any solution choice method would lead to specifying a set of non-comparable results, each of them being evaluated by making use of only a part of all the information available. In order to apply the information coming from all the experts, in an aggregated form, appropriate for solution choice methods, we propose to consider all desired levels as well as forbidden ones jointly. The sets thus obtained will be denoted  $A_d$  and  $A_f$  respectively. A few aggregation methods of constructing  $A_d$  and  $A_f$  considering the individual expert judgments are also given and discussed.

Thus we here come to the model of the DM's preference consisting of the /possibly fuzzy/ sets  $A_d$  and  $A_f$ , and two seminorm-like functions  $g_d$  and  $g_f$  modelling the DM's wish to reach the set  $A_d$  and to avoid  $A_f$ , respectively.

To select a solution, taking into account all above preference elements, we use the following procedure :

First we find the set  $D$  which contains all solutions nondominated in the sense of the functions  $g_d$  and  $g_f$ . Then we confine our search to the intersection of  $D$  and of the set of nondominated points  $P_f$ . This intersection is called  $D_f$ . Finally, we apply a generalization of the skeleton set method in order to select the only solution.

It may be proved that the solution thus obtained maximizes an utility function which can be interpreted as the characteristic function of a certain fuzzy set generated by  $A_d$ ,  $A_f$ ,  $g_d$  and  $g_f$ . We also prove a sensitivity property of this solution.

EXTENDED QUADRATIC AND EXTENDED CONIC FUNCTIONS  
IN OPTIMIZATION

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The paper describes some recent results on the area of unconstrained minimization

$$\min f(x) = f(x^+), \quad f: R_n \rightarrow R_1$$

where  $x^+$  is the optimal solution.

Theory of A-orthogonality, where A is a symmetric, positive definite  $n \times n$  matrix, has made possible to develop attractive algorithms for solving linear algebraic systems, eigenvalue and optimization problems. Several conjugate direction algorithms were proposed for minimization of a quadratic function

$$F = 1/2 (Ax, x) - (b, x)$$

later for an extended quadratic function [1]

$$f = h(F(x))$$

where  $h: R_1 \rightarrow R_1$  and  $F: R_n \rightarrow R_1$  is a strictly convex quadratic function. Recently conjugate direction algorithms have been suggested for minimization of a conic function [2,3]

$$f = F(x)/l^2(x)$$

where  $l: R_n \rightarrow R_1$  is a linear function. A generalization of a conic function was introduced in [4]. An extended conic function  $f: R_n \rightarrow R_1$  has the form

$$f = \Psi(F(x), l(x))$$

where  $\gamma: R_2 \rightarrow R_1$ ,  $F: R_n \rightarrow R_1$  is a strictly convex quadratic function and  $l: R_n \rightarrow R_1$  is a linear function with a constant gradient vector  $c$ . An conjugate direction algorithm which minimizes such a function has the form [4]

$$x_{i+1} = x_i - \alpha_i h_i \quad i=0,1,2,\dots,n-1$$

where

$$(Ah_i, h_j) = 0 \quad i < j < n \quad (g_{i+1}, h_i) = 0$$

and  $g$  is the gradient vector of  $f$  and

$$(h_i, c) = 0 \quad i < n-1 .$$

In this paper it is shown that the above mentioned theory can be applied on minimization of a extended conic function

$$f = \psi(h(F(x)), l(x)) .$$

Further a conjugate gradient algorithm for minimization of a conic function and an extended conic function in the form

$$f = h(F)/l^2$$

where  $h: R_1 \rightarrow R_1$  will be described.

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ON EXISTENCE OF ASSIGNMENTS  
IN ZERO-ONE MATRICES

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Let us consider an  $n \times n$ , zero-one matrix  $A$ , and let us denote:  $a_i = \sum_j a_{ij}$ , for all  $j$ ;  $a_j = \sum_i a_{ij}$ , for all  $i$ , and  $p = \sum_{i,j} a_{ij}$ . We assume there is no zero row or column in the matrix  $A$ . Some sufficient conditions for existing of assignment in an  $A_{n \times n}$  matrix, of cost  $O(p)$ , are given.

The conditions we have proved are:

- $a_{j_1} \neq a_{j_2}$ , for all  $j_1 \neq j_2$ ; ( $a_{i_1} \neq a_{i_2}$ , for all  $i_1 \neq i_2$ );
- $a_i \geq \lceil n/2 \rceil$  for all  $i$ , and  $a_j \geq \lceil n/2 \rceil$  for all  $j$ .

We have also proved that there is no assignment, if the cardinality of any partial assignment, obtained by a simple greedy procedure, is less than  $\lceil n/2 \rceil$ .

The assignment problem we have studied appears as a feasibility test in the threshold method applied for solving the bottleneck assignment problem. Known necessary and sufficient conditions for existing of an assignment require  $O(n^{5/2})$  operations.

Cheaper sufficient conditions are useful in the case of matrices of very large dimensions.

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Differential Stability of Solutions  
to Boundary Optimal Control Problems  
for Parabolic Systems

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Abstract

The paper is devoted to the sensitivity analysis of constrained optimal control problems for parabolic systems with respect to the deformations of the domain of integration. The new results obtained for convex, boundary optimal control problems with constraints on control are presented.

The state equation for the control problems considered in the paper is defined in the domain  $Q = \Omega \times (0, \tau)$  where  $\tau > 0$  is a given number and  $\Omega \subset \mathbb{R}^n$ ,  $n \geq 2$ , is a given domain with smooth boundary  $\partial\Omega$ . An example of such a control problem can be formulated in the following way:

Problem (P<sub>0</sub>):

minimize the cost functional

$$J(u) = \frac{1}{2} \int_Q (y(u; x, t) - z_d(x, t))^2 dQ + \frac{\alpha}{2} \int_0^\tau \int_{\partial\Omega} (u(x, t))^2 dr dt$$

subject to constraints

$$0 \leq u(x, t) \leq M, \text{ for a.a. } (x, t) \in \partial\Omega \times (0, T)$$

where  $\alpha > 0$ ,  $M$  are given numbers,  $z_d \in L^2(Q)$  is a given element and  $y = y(u; \dots)$  satisfies the state equation:

$$\frac{\partial y}{\partial t}(u; x, t) - \Delta y(u; x, t) = 0 \quad \text{in } Q$$

$$y(u; x, t) = u(x, t) \quad \text{on } \partial\Omega \times (0, T)$$

$$y(u; x, 0) = y_0(x) \quad \text{in } \Omega$$

here  $y_0(\cdot) \in L^2(\Omega)$  is a given element.

We assume that there is given a family of deformations  $\Omega_\epsilon = T_\epsilon(\Omega) \subset \mathbb{R}^n$ ,  $\epsilon \in [0, \delta)$ ,  $\delta > 0$ , of the domain  $\Omega$ , defined [2] by a vector field  $V(\dots) \in C^1([0, \delta); C^2(\mathbb{R}^n; \mathbb{R}^n))$ . We introduce a family of optimal control problems  $(P_\epsilon)$ ,  $\epsilon \in [0, \delta)$  for which the state equation is defined in  $Q_\epsilon = \Omega_\epsilon \times (0, \tau)$ . We denote by  $u_\epsilon$  a unique optimal control for the problem  $(P_\epsilon)$ . We prove that the element  $u_\epsilon \in L^2(\partial\Omega_\epsilon \times (0, \tau))$  is differentiable, in the appropriate way, with respect to the parameter  $\epsilon$ , at  $\epsilon=0^+$ . Actually we prove the existence of the so-called material derivative  $\dot{u} \in L^2(\partial\Omega \times (0, \tau))$  and Lagrange derivative  $u' \in L^2(\partial\Omega \times (0, \tau))$  of the optimal control  $u_0 \in L^2(\partial\Omega \times (0, \tau))$  for problem  $(P_0)$  in the direction of a vector field  $V(\dots)$ . We show that the elements  $\dot{u}$ ,  $u'$  are given by a unique solutions of the auxiliary control problems for parabolic systems. Related results for the elliptic systems are given in [2].

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vector field  $V(\dots)$ , which depends actually on the normal component on  $\partial\Omega$  of the vector field  $V(0, \dots)$ . We derive the form of the directional derivative of an arbitrary integral functional depending on the solution of problem (1), in the direction of a vector field  $V(\dots)$ .

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ANALYSIS OF THE COMPLEXITY AND ROBUSTNESS OF SOME NEW,  
GLOBAL ALGORITHMS FOR CONVEX PROGRAMMING

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We propose and compare some new algorithms for solving the convex programming problem (of approximating - within prescribed accuracy  $\epsilon$  - the value of)

$$f_0^* := \inf\{f_0(x) \mid f_i(x) \leq 0, i=1, \dots, m, x \in R^n\}, \quad (1)$$

where the functions  $f_i, i=0, \dots, m$  are assumed (if not stated otherwise) to be convex only. Besides of its independent interest, this problem is important when we try to "globalize" the convergence of locally fast, say, second order methods devised for similar but more smooth problems: e.g. the author has shown (1984, in a forthcoming IIASA publication) that a modified, "graph" ellipsoid method can be used - instead of line searches - to globalize a second order method, achieving thus a global convergence rate depending only on  $n$  but not on the conditioning (or Lipschitz constant) of the Hesse matrixes. Here we propose a new imbedding, i.e. continuation method for globalization (especially suited for the "smooth" case).

We assume that - at each point  $x$  (or "step" of the algorithm) - the value and one of the subgradients of (at least one of) the functions  $f_i, i=0, \dots, m$  can be computed within a specified accuracy  $\epsilon_0$ . The proposed methods can be regarded broadly as recursive algorithms for approximating - by ellipsoids - the set of possible minimumpoints  $x^*$  (or minimumpairs  $(x^*, f_0^*)$ ) compatible with the information gained at the previous steps, thus they yield more than just approximations of  $(x^*, f_0^*)$ .

These approximations are constructed - for several classes of problems (1) - to meet (i.e. be globally "optimal" with respect to) various criteria: minimize the number of arithmetical operations and (or) function (subgradient) evaluations needed to reduce the uncertainty in the value of  $f_0^*$  by a fixed factor (say 1/2), by taking into account memory restrictions or requirements for the stability ratios  $\gamma_0 \leq \epsilon_0/\epsilon, \gamma_1 \leq \epsilon_1/\epsilon$ , where  $\epsilon_1$  is the (machine) error of the elementary arithmetical operations. Payoff relations between these criteria will be presented together with corresponding, constructive upper and lower bounds. Special attention will be given to the two case  $m=0$

and  $f_0(x) = \langle c, x \rangle$ ,  $f_i(x) = \langle a_i, x \rangle$ ,  $i=1, \dots, m$ , where e.g. a simple operation count shows that if  $m \geq \text{const.} \cdot n^{4/3}$ , then the ellipsoid method has a faster (globally guaranteed) convergence rate (per arithmetical operation) than Karmarkar's method.

A new method - recommended for the cases  $\text{const.} \cdot n \leq m \leq \text{const.} \cdot n$  and linear (or smooth)  $f_i$  - is obtained from the following observations, providing an affine invariant realization of an idea of Karmarkar (and a new look at maximum entropy spectral estimation). It is based on fast and robust computability - for an arbitrary intersection of halfspaces  $P$  - of a point  $z(P) \in P$ , - analytically depending on the parameters of  $P$ , i.e. its defining halfspaces.

Theorem. Each, nonempty, bounded, convex polyhedron

$$P = P(a^m, b^m) := \{x \mid \langle a_i, x \rangle \leq b_i, i=1, \dots, m, x \in \mathbb{R}^n\}, m > n$$

can be (uniquely) represented in  $\mathbb{R}^m$  as the intersection of a simplex  $S = \text{convex hull} \{ \alpha_i e_i, i=1, \dots, m \}$  - where  $\alpha_i > 0$ ,  $e_i$ ,  $i=1, \dots, m$  are the orthonormal basis vectors in  $\mathbb{R}^m$  - with a linear subspace  $L$  containing the center of  $S$ . In the  $x$  space  $L$  this center  $z(P)$  is the solution point of the following "convex" problem

$$\max_{x \in P} g(x), \quad g(x) := \prod_{i=1}^m (b_i - \langle a_i, x \rangle)^{1/m}, \quad F(z) := \sum_{i=1}^m \frac{\alpha_i}{b_i - \langle a_i, z \rangle} = 0.$$

The ellipsoids of smallest (largest) volume contained in (containing)  $S$ , when intersected with  $L$  yield ellipsoids  $E_1(P)$ , resp.  $E_2(P)$  with similarity ratio  $m$  contained in (containing)  $P$ .

Now we can obtain the solution point  $x^*$  of (1) (for the chosen  $f_0, \dots, f_m$  assuming that it is unique) as  $z(P(a^m, c, b^m, f_0^*)) = P(a^m, c, b^m, f_0^*)$ , i.e. as the endpoint of a regular homotopy curve  $z(h) = z(P(a^m, c, b^m(h), h))$ ,  $f_0^* \leq h \leq h_0$  assuming that a good approximation of  $z(P(h_0))$  can be computed explicitly (e.g. when choosing  $b_i(h_0)$  large enough except for  $n$  indexes  $i$ , for which we set  $b_i(h) \equiv b_i$ ). For  $(h - f_0^*)$  small, we should switch to the equivalent system (which is under the usual assumptions of strict complementarity - singularity free)  $c + \sum_{i=1}^m \lambda_i a_i = 0$ ,  $\lambda_i (b_i(h) - \langle a_i, x \rangle) = h - \langle c, x \rangle$ ,  $i=1, \dots, m$  for  $(x^*, \lambda^*)$ . Special stepsize rules and update methods are devised.

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## STRUCTURAL OPTIMIZATION WITH RELIABILITY CONSTRAINTS

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During the last 10 years a number of effective methods have been developed to determine the reliability of structural systems, e.g. offshore jacket structures. The strength of the structural members and the loads are modelled as random variables and the significant failure modes are modelled as elements in a series system. The method used in this paper to determine the significant failure modes is the  $\beta$ -unzipping method developed by the authors.

In classical deterministic structural optimization all variables are assumed to be deterministic, and the design variables are usually the cross-sectional sizes of the elements in the structure. The objective function is most often chosen as the cost of the structure or as the weight of the structure. The constraints signify that the stresses should everywhere be smaller than some prescribed values and/or that the displacements should be smaller than some prescribed values. Further that all cross-sectional areas are greater than or equal to zero.

In this paper a probabilistic structural optimization problem is formulated. The objective function and the design variables are the same as for the classical problem. The strength of the cross-sections and the loads are modelled as stochastic variables. The constraints are now altered so that they express that the reliability of the structure has to be greater than some prescribed value.

In the paper the choice of reliability constraints is discussed. The constraints can be related to the reliability of the individual elements in the structural system or they can be related to the reliability of the whole system. If the latter type of constraints is chosen then the  $\beta$ -unzipping method or an equivalent method has to be used for evaluating the constraint.

For the above-mentioned reliability optimization problem a new optimization algorithm has been developed especially considering the difficulties caused by the last type of constraint. It is generally far too expensive to identify all the significant failure modes and the corresponding systems reliability at every step in an optimization algo-

rithm. Therefore, only the most significant failure modes are identified when evaluation of the constraint is needed. When the design variables change, also the set of significant failure modes changes and therefore, the function describing the constraint corresponding to systems reliability changes, i.e. the constraint is not stationary during the optimization process.

A discussion of a number of optimization methods relevant for the new optimization algorithm is also given in the paper.

A computer program for the proposed algorithm is developed, and some new numerical results are presented.

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ABSTRACT

IAEA'S ACTIVITIES IN ELECTRIC SYSTEM EXPANSION PLANNING

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Economic assessments of the role of nuclear energy as part of developing Member States' national energy and electricity planning are an important part of the Agency's comprehensive programme in nuclear power planning and implementation. Such assessments involve three major types of interdependent and closely related activities: developing methodologies specifically adapted to developing countries; undertaking training courses on energy and nuclear power planning techniques, including the use of methodologies developed by the Agency; and carrying out energy and nuclear power planning studies in co-operation with requesting Member States. Close co-operation has been established with other international organizations, particularly the World Bank (IBRD) in joint IAEA/IBRD electric power sector assessment missions to developing countries.

The IAEA has developed two computer models for use in energy and nuclear power studies. One is the MAED (Model for Analysis of Electricity Demand), which is a "demand model" using the scenario simulation approach to estimate medium- and long-term demands for energy in a country (or a region), including a detailed treatment of the electrical sector. The second model is the WASP (Wien Automatic System Planning) package, which is a system of computer programs using dynamic programming techniques for economic optimization in electric system expansion planning. This "supply model" is structured in a flexible, modular system which can treat the following interactive parameters in an evaluation: load forecast characteristics (electric energy forecast, power generation system development); power plant operating and fuel costs; power plant capital costs; power plant technical parameters; power supply reliability criteria; and power generation system operation practices.

For training in energy planning, the IAEA offers three interregional courses. The first course is "Energy Planning in Developing Countries with Special Attention to Nuclear Energy" which familiarizes energy specialists in developing countries with the fundamental elements of comprehensive national energy planning. The second is "Electricity Demand Forecasting in Nuclear Power Planning", which aims at improving estimates of future electrical energy needs and focuses on the use of the MAED model. The third course is on "Electric System Expansion Planning", also known as the WASP Course, which trains senior planners, electrical engineers and economists in the techniques of planning the expansion of electricity generation systems. This course emphasizes the use of the WASP model.

The IAEA, upon official request, also renders direct assistance to Member States in carrying out energy and nuclear power planning. The objective of these joint studies is to assist the Member States in detailed economic analyses and planning studies in order to determine the need for nuclear energy and its role within their national energy plans.

MONOTONE SEQUENCES OF FEASIBLE SOLUTIONS  
FOR QUADRATIC PROGRAMMING PROBLEMS WITH  
M-MATRICES AND BOX CONSTRAINTS;

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This paper considers algorithms for an important class of quadratic programming problems with a very special structure. The Hessian is an M-matrix and the constraints are bounds on the variables. Such problems arise after approximation of some elliptic type problems with free boundary. They include various types of Dirichlet problems with obstacles and models of the journal bearing and of the application of torsion to a bar.

Some special properties of this class of quadratic programming problems are presented. They form a theoretical background allowing to construct many algorithms. Each of them constructs a monotone sequence of feasible solutions starting either from the lower bound /the lower case/ or from the upper bound /the upper case/. The corresponding sequences of gradients in the lower case /upper case/ should have nonpositive /nonnegative/ components for any variable released in the computational process from its lower /upper/ bound. It is proved by the author that at each iteration the current feasible solution  $x^k$  satisfies in the lower case /upper case/ the following vector inequality  $x^k \leq \hat{x} / x^k \geq \hat{x}$ , where  $\hat{x}$  is the optimal solution of the considered problem. The third possibility is to generate two monotone sequences of points - one starting from the lower bound and the second starting from the upper bound. It will be shown that Scarpini's algorithm is a realization example of this third possibility.

Two other new algorithms making use of the above presented ideas and the pre-processing scheme of Cottle and Goheen are introduced. The first one starts from the lower bound on variables and is the direct generalization of the Chandrasekaran's algorithm /designed originally for the problems with lower bounds only/. The upper bounds are treated directly. In our approach only the solution of one unconstrained quadratic subproblem is required at each step. It is very attractive since the

Pang's algorithm /known as the best one for this class of problems/ needs to solve at each step a quadratic programming sub-problem with lower bounds. The number of iterations does not exceed the number of bounds. Our first algorithm makes use of the Cottle and Goheen pre-processing scheme only, at the beginning:

The second one generates two sequences of points /the first /the second/ one starting from the lower /upper/ bound and makes use of Cottle and Goheen's pre-processing scheme either at each iteration or after a *à priori* given number of steps. The generated sequences form consecutively tighter bounds on the optimal solution. The way of the sequences generation is the same as in the first algorithm mentioned above.

The results of numerical comparison of the introduced algorithms and this of Pang will be also presented.

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ABSTRACT

CLUSTERING THEORY IN THE DESIGN OF INFORMATION SYSTEMS

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In a typical information system design problem the system is eventually broken into subsystems and the design of each of these accomplished by a different team or group of analysts. The subsystems are then joined and the system is formed.

There are two apparent effects upon the ease of the work which result from the way the partition is realized. The greater the number of interfaces with other subsystems the greater the effort in coordinating the design of the subsystem with the other design work. Also, the more homogeneous is a subsystem, the easier is it to be designed. Consequently an objective function can be expressed, and a reasonable question is how to partition the set of components into subsystems in a way that is best for the design effort.

For such a problem it is appropriate to consider deterministic clustering methods, since they seek to partition optimally the elements of a finite set. An obstacle to application of such methods is that they generally involve the solution of complicated integer programming problems, and heuristics are usually required.

In the specific case of a hierarchical system structure, it has been possible to devise an algorithm for solving the problem exactly. The initial step is to employ a valid distance measure, an ultrametric, which allows the components to be considered

collinear objects, a situation in which the clustering problem is relatively easy to solve. If  $W$  = the number of within cluster distances is fixed, the objective function becomes linear, so the idea is to solve or account for the clustering problem corresponding to each value of  $W$ . By utilizing Lagrangian relaxation and incorporating the linear  $W$  constraint into the linear objective function, varying the multiplier will generate all the required solutions. For each value of the multiplier, the clustering problem becomes a shortest route problem, which can be solved expeditiously.

It is unnecessary to solve the large number of problems for the range of  $W$  because the optimal objective function values form a sequence that is nearly unimodal. Therefore, the problem of interest is solved via an efficient search over the one - dimensional multiplier space, where each function evaluation is given by the solution of a shortest route problem.

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Suppose we are given a finite family of standard Markov (or diffusion) processes  $X^\alpha$ ,  $\alpha \in A$ . The real process, which we observe is  $X^{\alpha^0}$ , where  $\alpha^0 \in A$  is an unknown parameter. Basing on our observation of the process  $X^{\alpha^0}$ , we control with the use of impulses. This means at Markov time  $\tau$  we are able to shift the process  $X^{\alpha^0}$  to  $\zeta$ , which is  $F_\tau$  measurable random variable. With such impulse we associate the cost  $c(x_\tau^{\alpha^0}) + d(\zeta)$ , where  $c, d$  are nonnegative continuous functions. Also, some running cost  $f(x_t)$  is incurred. Since our optimal strategy depends on  $\alpha$ , we estimate the unknown parameter, and then adopt the control to the value of our estimation. The estimator, we apply, is of the cost biased maximum likelihood type, introduced first in [1]. The paper is an attempt to generalize the discrete time case investigated in [2]. Two kinds of results are proved. Namely, if  $\alpha^* \in A$  is a Cesaro frequent point of the estimation, then the strategy which is optimal for the process  $X^{\alpha^*}$  gives the same value of the long run average cost functional for the true process  $X^{\alpha^0}$ . Moreover, for diffusion processes we have the value consistency of our estimator i.e., we can find the adaptive strategy for which the functional is the same as in the case if  $\alpha^0$  is known and we adapt the optimal strategy.

The impulsive control models are investigated with the use of technics of [3]. Also, some results concerning the adaptive control of diffusion processes of [4] are applied.

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Some observations on sequential quadratic programming methods

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Sequential quadratic programming (SQP) methods for solving a nonlinear program

$$(P) \quad \min \{ f(x) \mid g(x) = 0 \}$$

generate a sequence  $\{x_k\}$  by means of an iteration

$x_{k+1} = \phi_k(x_k) := x_k + s_k$  where the correction  $s_k$  is the optimal solution of the quadratic program

$$(P) \quad \min DF(x_k)s + \frac{1}{2} s^T B_k s \\ s: g(x_k) + Dg(x_k)s = 0,$$

depending on  $x_k$  and a symmetric matrix  $B_k$ . The Kuhn-Tucker points  $x^*$  of (P) are fixed points of the iteration,  $\phi_k(x^*) = x^*$ .

The convergence analysis of SQP-methods near K.-T. points  $x^*$  is simplified by analyzing an easily derived explicit expression for the Jacobian  $D\phi_k(x^*)$  of  $\phi_k$ . In this way it is found that for positive definite  $B_k = B_k^T$ , the K.-T. points  $x^*$  satisfying the 2nd order sufficiency conditions for a minimum of (P) are attractive fixed points of the iteration, whereas the K.-T. points  $x^*$  violating the necessary 2nd order conditions for a minimum are repulsive fixed points. Moreover, one obtains rather simple proofs for the Boggs-Tolle-Wang criterion for the

Q-superlinear convergence of the  $\{x_k\}$  towards  $x^*$  (i.e.  $\|x_{k+1} - x^*\| / \|x_k - x^*\| \rightarrow 0$ ) and for the Powell-criterion for the 2-step Q-superlinear convergence of the  $x_k$  (i.e.  $\|x_{k+1} - x^*\| / \|x_{k-1} - x^*\| \rightarrow 0$ ).

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Neuronal networks modelled by PETRI type  
nets with controllers

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In a first phase, models were constructed for four types of neurons: motoneurons, automatic neurons, Renshaw interneurons and receptor neurons; based mainly on works of: Hodgkin and Huxley (1952) Eccles (1957), Rall (1962), Schwindt and Calvin (1972), Baldissere and Gustafsson (1972), Traub (1977), Werner and Mountcastle (1965), Albus (1981). Modelling neuronal networks lead us to the conclusion that the information theory, probability theory, mathematical logics etc. should be completed by a set of principles of the nervous system, at its base:

1. The principle of "symmetry": practically all types of nervous systems are symmetrical. Symmetry is a basic condition of homeostatic equilibrium of the brain.

2. The principle of reciprocal inhibition (Sherrington, 1897) formulated originally to explain the movements of the limbs, proved itself to be a much more general principle, applicable even to the retina (Hertline, 1956).

3. The principle of re- and re-representation of functions (Jackson, 1931) or the principle of "hierarchical organisation" (Albus, 1981).

4. The principle of "neurohomeostasis": in the context of hierarchical organisation, the feed-back loops of a higher level are responsible for keeping constant the frequency of signals emitted by the neurons of lower levels.

5. The principle of "automatic neurons": the signals emitted periodically by automatic neurons are a basic condition for the neuronal networks specifically functioning.

6. The principle of phased modulation of signals by automatic neurons: under the influence of a signal from other neurons,

automatic neurons modify their firing phase; that process represents a form of plasticity (memory) (Frazier et al. 1967).

7. The "neuroholographic principle": in every interconnecting neuron of the feedback loops, composition phenomena, corresponding to the laws of electrophysiology may appear.

We exemplify the functioning of the above presented principles by a neuronal network model, composed by multiple hierarchically placed feedback loops, functioning as an Ashby (1970) type homeostat. The network model is conceived to discretely simulate the analogue functioning of neurons. That permits determination of every neuron's and the whole network's state at times  $t, 2t, 3t, \dots$  where  $t$  is the basic time unit considered ( $t=1\mu s$ ). The neuronal network is modeled by a PETRI type net, noted  $NN: NN=(N, S, F)$  where

$N$  is the set of locations that represents neurons

$S$  is the set of transitions that represents synapses

$R \subseteq (N \times S) \cup (S \times N)$  represents the flow relations between  $N$  and  $S$

To set  $N$  mark function:  $M: N \rightarrow [a, b]$  is attached, where  $a, b$  is the interval where the main parameter, characteristic of a neuron's state (the membrane potential), can vary. We also considered the function  $W: F \rightarrow [a', b']$  that defines each neuron's firing threshold, so that a neuron emits a spike if and only if  $M(n) > W(n, t)$ .

Any modification of a neuron's membrane potential is based on the initial value corresponding to a certain electrochemical equilibrium and on the specific functioning of each type of neurons. This particular functioning of the network is controlled by algorithms for each type of neurons, preceding the mechanism of checking and starting of transitions by acting upon each location. These algorithms we called "controllers". The simulations programs are written in Fortran and were runned on a FELIX-C 256 computer.

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THE USE OF FLYING HORIZON METHOD TO TRAFFIC  
OPTIMIZATION IN PROCESS CONTROL COMMUNICATION SYSTEMS

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A communication system comprising  $N$  main stations and  $K$  slave stations linked over a highway is considered. The two types of stations are different sources of traffic for the system. The information exchange between the stations is message-oriented and is performed in "command - response" sequences. There is a coordinator - station in the system, which transfers the highway control to the other stations and optimizes the traffic between them. The token-passing access method is used, as the control is transferred by a special MTF command. During one working cycle of the coordinator, all the  $K+N$  stations receive once the control over the highway. On the basis of the defined dynamical model, the coordinator solves the problem of determining the optimal strategy of control, minimizing the following cost functional of the communication system

$$Q = \sum_{i=1}^N Q_i = \sum_{i=1}^N \int_{t_0}^{t_f} q_i(x_i(t), u_i(t), v_i(t)) dt$$

where  $q_i$  are scalar functions, and the time interval  $[t_0, t_f]$  is large enough ( $t_f \rightarrow \infty$  included). The following notations are used:  
 $x_i(t)$  - denotes the length of the main station  $i$ 's queue at time  $t$ ,  
 $v_i(t)$  - the incoming information message flow into station  $i$  at time  $t$ ,

$u_i(t)$  - the control of station  $i$  at time  $t$ .

The model is described by the equations:

$$\frac{dx_i(t)}{dt} = v_i(t) - u_i(t), \quad x_i(t_0) = 0,$$

$$u_i(t) = \begin{cases} 1, & \text{when station } i \text{ has the control over the highway and} \\ & \text{transmits information messages at time } t \\ 0, & \text{otherwise} \end{cases}$$

The problem thus stated cannot be solved off-line. For this reason, the flying horizon method is applied, using frequently enough a feedback from the communication process actual state.

The coordinator strategy has three basic stages:

- choosing of the station, to which the control over the highway

- should be transferred;
- determination of the static optimal interval of control of this station;
- dynamic correction of the determined interval.

Receiving from the coordinator, at time  $t_i^k$ , the right to control the highway over the  $[t_i^k, \bar{\tau}_i^k]$  interval, the main station  $i$  transmits informational messages till the time  $\tau_i^k$  ( $\tau_i^k = \bar{\tau}_i^k$  or  $\tau_i^k < \bar{\tau}_i^k$ , but no messages are available in station  $i$ 's queue). Returning the control to the coordinator, the station transmits information about its current state  $x_i(\tau_i^k)$  and the predicted value of the incoming flow  $v_i(t)$  during the  $[\tau_i^k, \tau_i^k + \delta]$  interval, as well as the flow's mean value for this interval. The station does not control the highway until receiving the MTF command during the next cycle, i.e.  $u_i[\tau_i^k, t_i^{k+1}] = 0$ .

The flying horizon method discussed in the paper, is implemented in the SLINK-4 communication system, in which the cost functional is of the type:

$$Q = \sum_{i=1}^N \int_{t_0}^{t_f} x_i^2(t) dt$$

An analytical solution of the coordinator's task has been found out for SLINK-4 system.

The coordinator is not necessarily a separate hardware unit. In the SLINK-4 system, the coordinator's program is stored in each main station storage. In case the coordinator drops off, its functions are performed by another main station, which executes its own habitual functions at the same time. Thus, the system preserves the decentralized control principle, ensuring the necessary reliability. The pros of SLINK-4 are obvious, when comparatively intensive traffic exists, while if the traffic intensity is low, the system behaves as an ordinary one, using the token passing access method.

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A PROCEDURE FOR COMPUTING ALL INTEGER  
PROGRAMMING PROBLEMS AND ITS CONNECTION  
WITH MATROIDS

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Key words: discrete optimization, integer linear programming,  
matroids, greedy algorithm, computational complexity.

Consider an integer programming problem with all problem parameters being integers such that it would be linear if it were not for the fact that all variables are restricted to integral values and the optimization criterion is nonlinear but separable function.

The assumption that problem parameters are integers is not really any restriction because to an arbitrary degree of accuracy this can always be made to be the case. Such restriction only limits the selection of the physical dimensions used to measure parameters. In general, the set of all feasible solutions to such problem could be regarded as a lattice of integer vectors, Boolean lattice if the variables are restricted to zero-one values only, and optimal solutions could be obtained under condition that all computations are executed on integral entries. The problem of optimization is to find such lattice points for which the objective function is either maximum or minimum. All lattice points are mapped onto numbers and in this way the order structure of the real line is therefore an instrument of the perceiving apparatus. Through observations of its points only two directions are characterized by two relations: greater and lower. For the case with all zero-one variables an optimization problem can be reformulated as a problem of finding a proper ordering of the columns of the constraint matrix  $A$ . A "proper ordering" can be defined as such an ordering of  $k$ -tuple of integers, called the "sizes" of column vectors, the sum

of which is equal to given value and for which another sum, associated with the tuple, the sum of cost coefficients, is as large as possible. A zero-one programming problem can be considered as follows.

Given set of vectors  $A = \{a_1, \dots, a_n\}$ , find such a subset  $A'$  /simultaneously  $\bar{A}' = A - A'$  / for which

$$\sum_{a_j \in A'} s/a_j = b \quad /1/$$

and 
$$\sum_{a_j \in \bar{A}'} s/a_j = B - b, \quad /2/$$

where 
$$B = \sum_{a_j \in A} s/a_j.$$

All possible subsets  $A'$  determine a feasible region of problem /3/:

$$Z_1 = \text{Max} \sum_{j=1}^n c/a_j/x_j,$$

subject to 
$$\sum_{j=1}^n s/a_j/x_j = b, \quad /3/$$

$$x_j = 0 \text{ or } 1, \quad j=1, \dots, n,$$

and their complements  $\bar{A}'$  determine a feasible region of problem /4/:

$$Z_2 = \text{Min} \sum_{j=1}^n c/a_j/y_j,$$

subject to 
$$\sum_{j=1}^n s/a_j/y_j = B - b, \quad /4/$$

$$y_j = 0 \text{ or } 1, \quad j=1, \dots, n.$$

We have  $Z_1 + Z_2 = C$ , where  $C = \sum_{a_j \in A} c/a_j$ , and  $\sum_{a_j \in A'} c/a_j = Z_1$

if and only if  $\sum_{a_j \in \bar{A}'} c/a_j = Z_2$ .

The computational procedure works as follows. In step  $k$  we are seeking for such partition of  $A$  into  $A' \cup \bar{A}'$  in order to satisfy /1/ under condition

$$\sum_{a_j \in A'} c/a_j \geq z_1^{(k)}, \quad /5/$$

and, on the other hand, to satisfy /2/ under condition,

$$\sum_{a_j \in \bar{A}'} c/a_j \leq z_2^{(k)}, \quad /6/$$

while  $z_1^{(k)} + z_2^{(k)} = C$ . The successive values of  $z_1^{(k)}$  and  $z_2^{(k)}$  have to be ordered like that  $z_1^{(1)} < z_1^{(2)} < \dots < z_1^{(k)} < \dots$ , and  $z_2^{(1)} > z_2^{(2)} > \dots > z_2^{(k)} > \dots$ . The procedure terminates at the step  $k$  if neither  $z_1^{(k+1)} > z_1^{(k)}$  nor  $z_2^{(k+1)} < z_2^{(k)}$ .

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OPTIMIZATION OF THE FAULTED SECTION LOCATION PROCESS

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## 1. Introduction

The paper presents the problem of optimization of the faulted network section location and restitution of power supply to consumers connected to medium voltage distribution networks. To solve the problem an original algorithm based on the branch and bound has been elaborated.

## 2. The mathematical model

The decision process is based on a determination of an optimal decision and it may be shown as an graph which is called a decision tree.

Nodes of option in the decision tree represent states  $x_i$  of the network in the process of faulted section location,

The state  $X_i$  is determined as a vector  $X_i^T = [x_i^T, \theta_i^T$

$\tau_i^T, b_i]$  Components of the vector  $X_i^T$  are determined as follows  $Z_i^T = [\varphi_i^{(1)}, \dots, \varphi_k^{(1)}, \dots, \varphi_M^{(1)}]$  component  $i$  determines state

of switch  $k$  in the networks state  $X_i \theta_i^T = [\tau_i^{(1)}, \dots, \tau_j^{(1)}, \dots, \tau_w^{(1)}]$

coordinate  $\tau_j^{(1)}$  determines a time which has passed from a last moment of switching off a supply to the node  $j$  to a moment of finding of the network in state  $X_i$   $P_i^T = [p_1^{(1)}, \dots, p_1^{(1)}, \dots, p_N^{(1)}]$  is a vector of probabilities of faults of the particular section.

The parameter  $b_i$  means a repairing repairing staff during location of a damage in the network.

The optimization problem means a determination of a strategy  $\theta^*$  /a series of decisions/ of the faulted section location at minimum expectation value of a total cost of the economical losses  $K$  suffered by consumers due to the break down in supply, caused by a failure in the network plus a cost of location of the faulted section

$$E [K(\theta^*)] = \min_{\theta \in C} \{ E [K(\theta)] \}$$

/1/

where  $C$  - all permissible strategies.

## 3. Solution of the problem

## 3.1. Partition of set of solutions

In case when a set of solutions  $G$  is a set of paths in a decision graph than a certain arc  $u$  may be singularized and assumed as a criterion for a partition of the set of solutions and all paths in the graph may be split into two categories: a set which includes the arc  $u$  and other one which excludes the section  $u$ .

In this way we may obtain two sub-sets of solutions : a set of strategies  $\Gamma$  including decision  $u$  and a set of strategies  $\Lambda$  excluding decision  $u$ . A feature of a split  $u_j^i$  of sub-set  $G_{i,j}$  is determined when a maximum increase of a value of lower boundary of the criterion function is obtained for one of the sub-sets described above.

### 3.2. Calculation of lower boundaries of the criterion function

The lower boundaries of a function of expected costs of losses in the process of the section location are determined as follows

$$\inf(G_0) = \sum_{i=1}^N p_i w^i(\delta_i^*) \quad /2/$$

$$\inf(\Gamma_{k,1}) = \sum_{i=1}^N p_i w^i(\delta_i^* \Gamma_{k,1}) \quad /3/$$

$$\inf(\Lambda_{k-1,1}) = \inf(G_{k-1,1}) + \Delta(\Lambda_{k-1,1}) \quad /4/$$

where :

$p_i$  - probability of fault of section  $i$ ,

$\delta_i^*$  - optimal partial strategy that means a strategy at minimum cost of location of section  $i$ ,

$w^i(\delta_i^*)$  - a cost of location when a strategy  $\delta_i^*$  is applied,

$\Delta(\Lambda_{k-1,1})$  - an increment of the value of the criterion function.

### 4. Algorithms

Three different algorithms of optimal strategy of location of a faulted section in a network based on the above mentioned method have been prepared for the distribution networks : an accurate method, an approximate method, a method for on-line control location process.

All algorithms are based on branch and bound methods.

Numerical solution for optimal control  
of a distributed parameter system

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Consider the control system described by the operator equation

$$u(x, t) = Kf = \int_0^t K(x, t, s) f(s) ds, \quad a \leq x \leq b \quad 1)$$

where  $f$  is the control function.

Let  $F$  be the set of admissible controls

$$F = \{ f(t) \mid f(t) \in C^{(2)}, \quad a(t) \leq f(t) \leq b(t), \quad 2)$$

$$a(t), b(t) \in C^{(2)} \text{ are given functions } \}$$

The performance index is given by

$$J(f) = \int_0^T [u(x_0, t) - g(t)]^2 \quad 3)$$

where  $x_0 \in [a, b]$  is a given point,  $g(t) \in C^{(2)}$  is a given function.

The problem is: find  $f^* \in F$  such that

$$J(f^*) = \min_{f \in F} J(f) \quad 4)$$

In this paper a numerical solution for this problem is given.

Let  $t_i = ih$  be  $i=1, 2, \dots, n$  where  $h = \frac{T}{n}$ . Instead of 3) let us consider the functional

$$J_n(f^n) = \sum_{i=1}^n c_i^{(1)} \left( \sum_{j=1}^i c_j^{(2)} K(t_i, t_j) f_j - g_i \right)^2$$

where  $g_i = g(t_i)$ ,  $f_j = f(t_j)$  and  $c_i^{(1)}$ ,  $c_j^{(2)}$  are some constants.

Let us define the set  $F_n$  as follows

$$F_n = \left\{ \begin{array}{l} a(t_1) \leq f_1 \leq b(t_1) \\ a(t_2) \leq f_2 \leq b(t_2) \\ \vdots \\ a(t_n) \leq f_n \leq b(t_n) \end{array} \right\}$$

Instead of 4) we solve the problem

$$\min_{f_1, f_2, \dots, f_n \in F_n} J_n$$

It is proved that

$$\lim_{h \rightarrow 0} (K p_n^* - K f^*) = 0$$

and if

$$K p_n^* - K f^* = O(h^\alpha)$$

then

$$p_n^*(t_i) - f^*(t_i) = \begin{cases} O(h^{\alpha-1}) & \text{if } \alpha \leq 2 \\ O(h) & \text{if } \alpha > 2 \end{cases}$$

where  $p_n^*(t)$  is interpolational polinom on values  $f_1^*, f_2^*, \dots, f_n^*$

As an example a system governed by a partial differential equation is considered.

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Some new aspects of applying achievements of system theoretical approach in static or quasistatic design and control of civil engineering structures based on modern 'fiabilité' theory

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We must be rather cautious in applying the achievements of modern system-theory in the structural mechanics and dimensioning of civil engineering structures. We must choose the appropriate definition suitable to our aims from the rather rich quarry of information of general system's and circuit's theory. We do not agree with definition of "system" and its state characteristics under uncertain conditions respectively, which is described in the book of W. Bojarski. /1984., Warsaw/. To be able to use an another for us more adequate Polish definition of systems /after that of L. von Bertalaffy from 1947/ we must ensure the clear-cut discretization of generally in space and time continuous structural and loading properties.

Space discretizations of a structure must be checked whether they are identical with the original structure from point of view of rigidity, of grade of outer and inner or global static determinacy as well as of point of view of stability, we have sought to find new definitions on all of these fields in case of stochastic properties since the year 1974.

We shall use stochastic modelling on digital computer [1] i.e. outer loading effects and material characteristics can be combinations of very slowly changing continuous and of point-wise stochastic processes respectively. The grade of complexity of the stochastic modell must be suited to the nature of the problem as it was pointed out in some not yet published works of the author about behaviour of random elastic subgrade.

By use of the concept of design for serviceability-time

/initialized by V.V. Bolotin about 1979/, which is a good and useful reduction of a very complex multidimensional problem to the investigation of a simple random variable and by use of eventsequencing stochastic modelling on high-level programming languages of digital computers [2], proposed in [3] by the author, we get at a presumedly economically also very efficient tool of design of structures.

So in our case it can be omitted the use of dynamic programming generally used for time-dependent problems, more variants of structures can be taken into account in realistic comparison with eventual use of "fuzzy set theory" of Mr. A. Zadeh too and new advantages are given in step by step reconstruction or at maintenance processes of civil engineering structures.

Budapest, 1985-01-29.

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Linear and Nonlinear Oligopoly Models  
Ferenc Szidarovszky and Koji Okuguchi  
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A new general version of the Cournot equilibrium model is introduced, where each firm simultaneously produces several kinds of items, and it is assumed that the price vectors and cost functions depend on the total and own production levels of the producers, respectively. A new existence theorem is first proven and the relation of the equilibrium problem and fixed point problems of low dimensional point-to-set mappings is outlined.

Linear and nonlinear complementarity problems, and quadratic programming problems are constructed for the determination of the equilibrium points. On the basis of these methods a new uniqueness theorem is verified.

Dynamic models are examined with continuous and discrete time scales. Conditions for stability of the dynamic models are given in both cases.

The results presented in the lecture are generalizations and extensions of earlier results of the oligopoly theory.

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## Solving Scheduling Problems by Computer

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We have already made computer aided production control for several firms. In all these cases we had to solve job shop scheduling problems too.

All of the problems were multigoal optimization problems. We gave compromise solutions by multistep algorithms for them. The two most important goals were:

- to give an optimal makespan for the production flow, which minimizes the average lateness of the production,
- to minimize the setup time on a dispatched machine of the production.

We have made use of the following assumptions:

- If a setup time between the production of two items is less than the setup time from the idle state to the conditions necessary to process the second one as the first job in the sequence, there are some common features in their production. By the help of the production technology we can divide the set of the jobs into subsets which are sequence independent.

Let  $p_1, p_2, \dots, p_n$  be the processing times,

$a_{ij}$  /  $i=1, 2, \dots, n$ ;  $j=1, 2, \dots, n$ / the setup time between the jobs  $i$  and  $j$ .

Let the set of the jobs denoted by  $S$ , then  $S = \bigcup_{i=1}^m S_i$ ,

$q_i = \sum_{j \in S} p_j + \sum_{j, k \in S_i} a_{ij}$  the processing time of  $S_i$ .

This way the  $n$  jobs problem can be reduced to  $n/2$  or sometimes  $n/3$  jobs problem.

-Every production is a flow. Neither can be assumed that jobs arrive simultaneously in a shop that is idle and immediately available for work, /because it'd indicate too high inventory level./ nor that the jobs arrive intermittently according to stochastic process. /it'd indicate too high waiting time on the machines/. These scheduling problems could be considered as a 3 machines flow shop problem.

These assumptions and some special other one made possible to construct multistep algorithms and on-line computer programs based on them.

The scheduling problems came from very different firms. The first one was to minimize the setup time on CNC machines in a factory manufacturing telecommunication equipments. The second arised in the textile industry. The third problem came up in the paper industry. The scheduling programs were written for COMMODORE 64 personal computer.

The algorithm seems to be quick: 100 job can be soheduled in 5-10 minutes. The programs are user oriented.

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MULTIMODAL PRODUCTION-TRANSPORTATION PROBLEM  
WITH CAPACITY CONSTRAINTS

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The multimodal production-transportation problem with capacity constraints is a generalization of the production-transportation problem of the continuous production industry [1]. The problem we consider can be described in the following way. There are several plants at different locations and large number of customers of their products. Each producer can operate in several modes, characterized by different quantities of products and variable production costs. In addition there are several modes of transportation of products from plants to customers. The customers' demand for each product in a short-time planning period, the unit transportation costs for each product from plants to customers for every mode of transportation and capacity constraints on each route for every mode are known. The multimodal production-transportation problem with capacity constraints involves the determination of the producers' production programme and the modes of transportation of products from plants to customers which minimizes the production and transportation costs subject to the material balance constraints.

This problem is formulated as a large-scale linear programming problem and an algorithm based on Benders decomposition is suggested for solving it. In each iteration, there are two steps:

(i) The minimum cost network flow algorithm is used to solve multimodal transportation problem with capacity constraints for each product.

(ii) The solution of the muster problem which is the linear programming problem is obtained.

The algorithm is coded in Fortran on a UNIVAC 1110 computer system. Computational results and comparison with solutions obtained by simplex method for a sequence of constructed examples are given. The application in the petroleum industry is presented.

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Multiple-criteria analysis and evaluation in decision making of transport planning

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1. Introduction

Transportation -one of the most important parts of infrastructure- plays a significant role in the development of production; consumption and in the formation of foreign trade balance and so in the development of the whole national economy as well. The decisions connected with planning, organising or developing the different transport systems effect in almost every territory of the social-economical life. The fast development in the fields of decision making methods and computer technique has made favourable conditions to the investigation of complicated systems, like the transport systems. The paper deals with the methods of multiple-criteria analysis and evaluation in decision making of transport planning, organisation and development.

2. A weighted multi-dimensional evaluation of the efficiency in transportation

The method of measuring the efficiency -worked out previously by our department- enabled the complex qualification of efficiency level of differently aggregated transport systems. Developing this method it became possible to determine and analyse the most important facts influencing the efficiency. The matrix method -developed from the cross-impact technique- takes into consideration all the criterias influencing the efficiency and so it is possible to deal with different kinds of information.

The structure of the matrix is:

	determinant factors of the criterias				vectors characterising the impossibility of the criterias			
	1	2...	j...	n	$\gamma$	$\delta$		
criteria of the efficiency	1	$a_{11}$	$\dots$	$a_{1j}$	$\dots$	$a_{1n}$	$\gamma_1$	$\delta_1$
	i	$a_{i1}$	$\dots$	$a_{ij}$	$\dots$	$a_{in}$	$\gamma_i$	$\delta_i$
	m	$a_{m1}$	$\dots$	$a_{mj}$	$\dots$	$a_{mn}$	$\gamma_m$	$\delta_m$
vectors characterising the influential role of determinant factors	$\alpha$	$\alpha_1$	$\dots$	$\alpha_j$	$\dots$	$\alpha_n$		
	$\beta$	$\beta_1$	$\dots$	$\beta_j$	$\dots$	$\beta_n$		

The application of the method consists of the next steps:

- a./ exploration of the determinant factors of the criterias /with the application of brainstorming/;
- b./ determination of the closeness of the connections between the criterias and their determinant factors /by experts' estimates/;
- c./ counting the vectors characterising the influential role of determinant factors /  $\alpha$  and  $\beta$  /:

$$\alpha_j = \frac{\sum_{i=1}^m i}{\sum_{i=1}^m i} \quad j=1, \dots, n; \quad \beta_j = \frac{\sum_{i=1}^m a_{ij}}{\sum_{i=1}^m i} \quad j=1, \dots, n$$

/ i=0, if  $a_{ij} \neq 3$  and  
i=1, if  $a_{ij} = 3$  /

/ i=0, if  $a_{ij} = 0$  and  
i=1, if  $a_{ij} \neq 0$  /

where  $j=1, \dots, n$ : the number of determinant factors;

$i=1, \dots, m$ : the number of criterias;

$a_{ij}$  : the evaluated closeness between the  $i$ -th criteria and the  $j$ -th determinant factor;

- d./ counting the vectors characterising the impressibility of the criterias /  $\gamma$ ,  $\delta$  /:

$$\gamma_i = \frac{\sum_{j=1}^n j}{\sum_{j=1}^n j} \quad i=1, \dots, m; \quad \delta_i = \frac{\sum_{j=1}^n a_{ij}}{\sum_{j=1}^n j} \quad i=1, \dots, m.$$

/ j=0, if  $a_{ij} \neq 3$  and  
j=1, if  $a_{ij} = 3$  /

/ j=0, if  $a_{ij} = 0$  and  
j=1, if  $a_{ij} \neq 0$  /

- e./ choosing the most important determinant factors of the criterias on the basis of evaluation  $\beta_j \geq \beta$  critical;
- f./ decision about the operation on the basis of the multi-dimensional evaluation method of the efficiency.

The paper gives the results of the macroeconomic and microeconomic level investigations in the railway and road transport systems.

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$\epsilon$  - OPTIMALITY IN SLIDING PLANNING

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Consider the following control problem of minimizing

$$I(u) = M^u \sum_{t=1}^N (X_t - C)^2 \quad (I)$$

subject to system of equations

$$X_t = u_{t-1}^T \beta_1(t) + \beta_0(t), \quad Y_t = X_t + \xi_t,$$

where  $(y_t)$  is an observable process,  $\xi_t$  is an independent Gaussian variable,  $\beta_t = \{\beta_0(t), \beta_1(t)\}$  is an unknown parameters vector,  $u_t$  is a feedback control from the observations, i.e. nonanticipative function

$$u_t = u(t, y_1, \dots, y_t).$$

1. For the case  $N = 3$ ,

when  $(\beta_t)$  is a first order autoregressive time series, the upper and the lower bounds for value function

$$I^* = \inf_u I(u)$$

have been found in explicit form.

2. For the cases,

when  $N$  is finite or when

$$I(u) = M^u \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=1}^N (X_t - C)^2 \quad (?)$$

and  $(\beta_t)$  is stationary autoregressive moving-average time series, such that  $\{\beta_1(t)\}$  is nondegenerate, it is shown, that control strategy of one-step rolling schedules  $u^+$  is  $\epsilon$ -optimal, i.e.

$$I(u^+) \leq I^* + \epsilon .$$

3. For the case (2), when  $\beta_1(t) = \beta_1$  and  $\{\beta_0(t)\}$  is stationary time series, it is shown, that strategy  $u^+$  could be randomized so that  $\epsilon$  will be arbitrary small.

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Invariance theorems for variational problems with nonsmooth integrands

Wolfgang Thämelt, GDR

Consider the following one-dimensional variational problem

$$(1) \inf_u \int_{x_0}^{x_1} L(x, u(x), u'(x)) dx, \quad u(x_0) = u_0, \quad u(x_1) = u_1.$$

If  $L$  and  $u$  are sufficiently smooth and if  $L$  is invariant relative to the one-parametric transformation group  $h^s$  then

$$(2) \left. \frac{\partial L}{\partial u'} \cdot \frac{\partial h^s}{\partial s} \right|_{s=0} = \text{const.}$$

for any extremal solution of (1) (Noether's invariance theorem).

Rvačev studied problem (1) under the assumption that  $L$  is not smooth but convex with respect to its third argument. He proved that there is a  $l(x) \in \partial_u L(x, u(x), u'(x))$  for every  $x$  such that

$$(3) \left. l(x) \cdot \frac{\partial h^s}{\partial s} \right|_{s=0} = \text{const.}$$

for any extremal solution of (1). In the proof of this subdifferential version of (2) typically one-dimensional ideas are used.

In this paper it is shown that a similar result is valid in the case of the  $k$ -dimensional variational problem

$$(4) \inf_u \int_X L(x, u(x), Du(x)) dx$$

where  $X \subset \mathbb{R}^k$  is an open domain and  $u(x) = u_0(x)$  on  $\partial X$ .

Theorem 1: If

1.  $L$  is invariant relative to the one-parametric transformation group  $h^s$ ,

2.  $(L \circ k_u)(x, \cdot)$  is convex for all  $x \in X$  where

$$k_u(x, w) = (x, u(x), Du(x) + D_s h^s(u(x))) \Big|_{s=0} \circ Dw(x)$$

and where  $w \in C^1(X, \mathbb{R})$ ,

then  $u$  is an extremal solution of (4) only if

$$0 \in \int_X \partial_w (L \circ k_u)(x, 0) dx.$$

An essential part of the proof of theorem 1 is the application of a theorem by Ioffe/Tichomirov on the subdifferential of a parameter integral.

If  $L$  itself is convex with respect to the third argument then further calculation gives

**Theorem 2:** If

1.  $L$  is invariant relative to the one-parametric transformation group  $h^s$ .
2.  $L$  is convex with respect to the third argument, then  $u$  is an extremal solution of (4) only if for every  $x$  there is an element  $l(x)$  from the subdifferential of  $L$  relative to the third argument such that

$$(5) \quad 0 = \int_X Dw(x) l^T(x) D_s h^s(u(x)) \Big|_{s=0} dx$$

for all  $w \in C^1(X, R)$  with  $w(x)=0$  on  $\partial X$ .

The integral condition can be replaced by the following assumption on the integrand:

The surface integral

$$\int_G l^T(x) D_s h^s(u(x)) \Big|_{s=0} d\vec{F}$$

equals zero for almost every sphere  $G \in X$ .

Generalisations of these theorems in physically interesting directions (additional divergence terms in the invariance condition) can be derived.

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## RELIABILITY MODELLING OF STRUCTURAL SYSTEMS

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### ABSTRACT

It has of course for centuries been realized that structures are not completely safe. A number of serious failures have occurred and unfortunately, loss of life and injury are often involved. Further, it is characteristic that structural failures have great economic consequences. Consider for example failure of an offshore platform. For such a structure failure will usually occur during extreme weather conditions so that rescue operations are difficult or impossible. Therefore, the risk of loss of life is high. Further, apart from having enormous economic consequences, this kind of structural failure can result in very serious pollution problems from outslip of oil. On the other hand, it is a fact that at least for typical West European countries the risk to life from structural failures is negligible compared with e.g. travel by car or by air, coal mining, etc. The average annual risk per person is as low as  $10^{-7}$ . Therefore, the economic consequences of collapse and non-serviceability will usually dominate all other consequences, since the marginal returns in terms of lives saved for each additional million \$ invested in improving the safety of structures may be small in comparison with the benefits of investing the same sum in, say, road safety or public health.

In this lecture some of the latest research progress with regard to structural reliability will be presented. The following topics will be treated

- deterministic contra stochastic analysis and design
- reliability of single structural elements
- reliability of structural systems

These topics will be briefly discussed from the following points of view

- what do we know to-day?
- what can we do to-day?
- what kind of unsolved problems do we have?

Until recently structural engineering has been almost completely dominated by *deterministic thinking* characterized in design calculations by the use of specified minimum values for material strength properties, specified maximum values for load intensities and by prescribed codified procedures for computing stresses and deflections. However, it has been recognized for many decades that for example specifications of minimum material properties involves a high degree of uncertainty. Likewise, specification of reasonable load intensities is difficult and uncertain. These types of uncertainty combined with several similar forms of uncertainty (e.g. model uncertainty) result in an uncontrolled risk. It is a fact that total safety of a structure cannot be achieved even when one is ready to spend a lot of money on the structure. Further, it is a serious problem with the deterministic approach that no measure of the safety or reliability of the structure is obtained.

In modern structural reliability theory it is clearly recognized that some risk of structural failure must be accepted. To obtain a measure of this risk a probabilistic approach seems to be the most obvious approach. The intention behind this approach is to help the structural engineer to design a structure in such a way that it will not, at an acceptable level of probability, fail at any time during the specified design life.

Structural reliability modelling is first of all concerned with the rational treatment of uncertainties in structural engineering and the associated problems of rational decision making. All quantities that currently enter into engineering calculations are in reality associated with some uncertainty. For the purpose of quantifying such uncertainties and for the subsequent analysis it is necessary to define a *set of basic variables*, namely the set of basic quantities governing the static and dynamic behaviour of the structure. Basic variables are quantities such as mechanical properties of materials, dimensions, weight densities, environmental loads, etc. For the purpose of structural reliability it is necessary to distinguish between at least three types of uncertainty, *physical uncertainty*, *statistical uncertainty*, and *model uncertainty*. The physical uncertainty is taken into account by modelling the basic variables as stochastic processes or random variables. However, physical variability can be quantified only by examining sample data; but since sample data are limited by practical and economic considerations, some uncertainty must remain, namely the statistical uncertainty. Statistical uncertainty arises solely as a result of lack of information. Model uncertainty occurs as a result of simplifying assumptions, unknown boundary conditions, and as a result of the unknown effects of other variables and their interactions, which are not included in the model.

In the most general case *reliability* of a structure is its ability to fulfil its design purpose for some specified period of time. In a more narrow sense it is the *probability* that a structure will not attain each specified state of failure (ultimate or serviceability) during a specified reference period. This type of probability does not have a relative frequency interpretation and is called a subjective probability. The associated reliability

can be called subjective or Bayesian reliability.

Methods of structural reliability analysis are divided into three classes. *Level 3-methods* are methods where the probability of failure for a structure or a structural element is calculated on the basis of the full probabilistic description of the joint occurrence of the various quantities that effect the response of the structure and taking into account the true nature of the failure domain. *Level 2-methods* are methods where the joint probabilistic description of the basic variables is simplified and where the failure condition is idealized. Usually, level 3 calculations cannot be performed due to lack of data. There seems to be a general agreement that level 2 methods are sufficient at least to give a relative measure of the reliability of a structure. *Level 1-methods* are really not reliability methods but methods of design or safety checking. They are based on a deterministic thinking although the parameters used may be chosen on the basis of a probability distribution.

In this paper a level 2-method called the *reliability index method* will be described and it will be shown how this method can be used in evaluating the reliability of structural elements and structural systems. The method will be illustrated by an example, namely an offshore structure.

## ON DYNAMIC GAMES

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The theory of two-person zero-sum games in the different types of the control systems had been worked out in the works [I-4]. Lately the general theorems on the information of players had been proved, the idea of the informational system had been introduced, the evolutionary systems had been studied. Let's bring two statements about program iterations.

Let the dynamical system  $\Sigma = ([t_0, T], X, D_1, D_2, \mathcal{X})$  is given, where  $[t_0, T]$  - a segment of time,  $X$  - a set of states,  $D_1$  ( $D_2$ ) - a set of controls of the first (second) player,  $\mathcal{X}$  - a function of state. Second player strives to maximize the payoff  $H(x(T))$ , where  $H: X \rightarrow R^1$ . Let's consider the operator of the program iteration

$$\Phi_+(g)(t, x) = \inf_{\tau \in [t, T]} \inf_{u \in D_1} \sup_{v \in D_2} g(\tau, \mathcal{X}(\tau, t, x, u, v))$$

and corresponding iterations

$$V_1(t, x) = \inf_{u \in D_1} \sup_{v \in D_2} H(\mathcal{X}(T, t, x, u, v)), \quad V_2 = \Phi_+(V_1), \dots$$

If  $\alpha = \beta + 1$ , then  $V_\alpha = \Phi_+(V_\beta)$ , if  $\alpha = \sup \{ \beta \mid \beta < \alpha \}$ , then  $V_\alpha = \inf_{\beta < \alpha} V_\beta$ .

If the zero-sum dynamic games without discrimination  $\Gamma(t, x)$  have the decision [1-3], then  $\text{val } \Gamma(t, x) = \lim_{n \rightarrow \infty} V_n(t, x)$ . The criteria of the existing of the decision may be formulated in terms of the functions of Lyapunov. If the game  $\Gamma(t, x)$  has no value, then there is such ordinal number  $\Lambda$ , that  $V_\Lambda = \Phi_+(V_\Lambda)$ .

The quantity  $V_\lambda$  is a function of the value of the games in the class  $\epsilon$ -strategies [5].

Similar statements are also true for other operators of the program iterations and for other games with more general payoff.

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**COMPARISON OF DIFFERENT METHODS FOR SOLVING  
A NON-LINEAR, LARGE SCALE, STOCHASTIC DYNAMIC PROBLEM**

**ABSTRACT :**

The problem of optimizing the long-term (yearly) operation of a multistorage power system has received considerable attention in recent research, but has not yet been completely solved.

The choice of methods for solving such problems is indeed limited to dynamic programming which is usually not feasible for more than three state variables due to the exponential growth of the computing time and storage requirements.

Since there is not alternative to obtain an optimal global feedback, compromises have been made in the past by using techniques such as aggregation, relaxation... An aggregation-decomposition method using dynamic programming has been since developed at Electricité de France to solve the problem of the yearly operation of the electricity supply-demand system.

This paper first recalls the principles of this method, then presents, for the first time to our best knowledge, a comparison to the results obtained with the classical available methods, especially relaxation, that finally shows that the aggregation-decomposition method, when applied to the test problem mentioned above, leads to the nearest global feedback to the optimum.

Eigenvalues assignment for hyperbolic damped equations via a boundary feedback of finite rank.

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Abstract

We discuss wave equations with viscous (or, alternatively, structural) damping defined on an open bounded domain  $\Omega \subset \mathbb{R}^n$ , and we apply (physically implementable) feedbacks on the boundary  $\Gamma$  of  $\Omega$  to improve stability properties of the original free (i.e. homogeneous) system.

One canonical example is:

(1)  $x_{tt}(t, \xi) = \Delta x(t, \xi) - 2kx_t(t, \xi)$  , in  $Q$

(2) I.C.  $x_0(\xi)$  and  $x_1(\xi)$  , in  $\Omega$

(3)  $x(t, \sigma) = [\langle w_1, x(t, \cdot) \rangle_{\Omega} + \langle w_2, x_t(t, \cdot) \rangle_{\Omega}] g$  , in  $\Sigma$

$Q = (0, \infty) \times \Omega$  ,  $\Sigma = (0, \infty) \times \Gamma$ ,  $w_1 \in L_2(\Omega)$  ,  $g \in L_2(\Omega)$  ,

$\langle \cdot, \cdot \rangle_{\Omega} = L_2(\Omega)$  - inner product,  $k > 0$ . The right hand side of (3)

is the boundary feedback (of one dimensional range) consisting of the "interior observation" term [ ] of the full state  $x, x_t$  and of the actuator function  $g$ .

Another canonical example - mathematically more challenging and physically more appealing - consists of (1), (2) along with the new boundary condition

(4)  $\frac{\partial x}{\partial n}(t, \sigma) = [(\psi_1, x(t, \cdot) |_{\Gamma})_{\Gamma} + (\psi_2, x_t(t, \cdot) |_{\Gamma})_{\Gamma}] g$

$\psi_1 \in L_2(\Gamma)$ ,  $g \in L_2(\Gamma)$ ,  $(\cdot, \cdot)_{\Gamma} = L_2(\Gamma)$  - inner product, where now the

right hand side of (4) is defined only in terms of the boundary values (traces on  $\Gamma$ ) of the full state. Problem (1), (2), (4) is a fully boundary problem, with "boundary observation" term [ ] in (4) and boundary actuator  $g$ .

If  $g = 0$ , (free system) and  $k$  "small", then all free solutions decay in the energy norm with uniform upper bound  $e^{-kt}$  (exponential stability of the corresponding group, describing the solution, in the uniform energy-norm).

Then, given  $g \in L_2(\Gamma)$ , subject to certain mild assumptions (of approximate controllability type), we seek suitable vectors  $w_i \in L_2(\Omega)$  for (1), (2), (3) [or, respectively, suitable  $\tilde{w}_i \in L_2(\Gamma)$  for (1), (2), (4)], such that the following desirable properties are achieved:

(i) (spectrum assignment) an arbitrary finite number of eigenvalues of the closed loop system is preassigned at will, as to improve the corresponding "margin of stability" while the remaining eigenvalues approach from the left the vertical line  $\text{Re } \lambda = -k < 0$ , asymptotically (That the eigenvalues of the closed loop system must approach the vertical line  $\text{Re } \lambda = -k$  asymptotically, is a consequence of the finite range nature of the feedback, and we then impose that such approaching is achieved from the left of  $\text{Re } \lambda = -k$ );

(ii) (Riesz basis assignment) the corresponding eigenvectors of the closed loop system form a Riesz basis in an appropriate space ( $L_2(\Omega) \times H^{-1}(\Omega)$  for (1), (2), (3)).

Thus, a fortiori, the overall decay of all closed loop solutions is preserved to have uniform upper bound  $e^{-kt}$ , but now an arbitrarily finite number of "dominant modes" of the free system have their margin of stability increased at will.

These problems arise from, and are of interest to, the control of flexible structures.

SOME NEW RESULTS ON THE ASYMPTOTIC BEHAVIOR OF SOLUTIONS  
TO PROBLEMS OF DYNAMIC OPTIMIZATION

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It is widely known that some problems in dynamic optimization possess the so-called turnpike property. It means, roughly speaking, that extremal solutions tend to a limit (in a sense) when time-horizon of planning tends to infinity. Here are presented some new results, based on Hamiltonian approach proposed by Cass, Shell etc. [1].

We will consider controllable system of Pontrjagin form defined on the non-negative orthant

$$\dot{x} = f(x, u), \quad x \in \mathbb{R}_+^n, \quad u \in U, \quad t \in [0, T] \quad (1)$$

with the criterion of optimization in terminal form:

$$\langle a, x(T) \rangle \rightarrow \max, \quad a \in \mathbb{R}_+^{n*}. \quad (2)$$

We suppose that Hamiltonian  $H(x, p) = \sup \{ \langle p, f(x, u) \rangle, u \in U \}$  is a smooth function except on  $\{p=0\}$ , phase space  $\mathbb{R}_+^n$  is invariant under admissible trajectories of inclusion (1) and system is homogeneous, i.e.  $f(\lambda x, u) = \lambda f(x, u)$  for all  $x, u, \lambda \geq 0$ .

Theorem. Let us define function  $L(x, v)$  for  $\{(x, v): x^1 + \dots + x^n = 1, x^k \geq 0, v^1 + \dots + v^n = 0\}$  by formula

$$L(x, v) = \sup \{ \alpha : \exists u \in U : \alpha x + v = f(x, u) \}.$$

If  $L$  is concave, and  $l(x) = L(x, 0)$  reaches a maximum in the interior point  $x_*$  of the set  $\{x : x^1 + \dots + x^n = 1, x^k \geq 0\}$ , then the ray  $0x_*$  is a strong limit to solutions of extremal problem (1,2), which means that for  $T \rightarrow +\infty$  correspondent solution  $x_T(t)$  satisfies following condition:

$$\forall \varepsilon > 0 \quad \exists C_\varepsilon : \forall T > 2C_\varepsilon \quad \left\| \frac{x_T(t)}{\|x_T(t)\|} - \frac{x_*}{\|x_*\|} \right\| \geq \varepsilon \Rightarrow$$

$$\Rightarrow \quad t \leq C_\varepsilon \quad \text{or} \quad t \geq T - C_\varepsilon.$$

This theorem is a generalization of results [1] for homogeneous systems.

In dimension two, the lowest possible for homogeneous systems, one can cast off condition of concavity.

Theorem. In the case of two-dimensional systems with a Hamiltonian satisfying some weak additional properties (but not necessarily concave on  $\mathcal{X}$ ), having a natural economic interpretation, there exists a finite number of rays which are weak limits of extremal arcs. (It means that measure of time when extremal arc is far from one of rays, is bounded when  $T \rightarrow +\infty$ ).

In higher dimensions the extremals can be more complicated.

Theorem. For  $n \geq 3$  the system (1,2) can typically have quasiperiodic attracting solution, i.e.  $x(t+\tau) = \mu x(t)$  for some  $\mu, \tau > 0$  and all  $t$ . (The property is called typical, if systems having this property form some open subset in the space of all systems of the given class with a natural topology).

Some results are also obtained concerning destruction of these periodic attractors when discount rate is small positive.

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A METHOD OF A CHOICE OF AN EXPANSION STRATEGY OF THE WATER  
SUPPLY SYSTEM WITH RANDOM PARAMETERS

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The paper deals with the problem of choosing an expansion strategy of the water supply and wastewater treatment systems under uncertainty. The problem is formulated in term of a stochastic linear programming problem, in which the coefficients of an objective function and the vector of the right-hand side of the constraints are random vectors. These ones are described by the aid of an approximate probability density functions. Thus, we assume that a given random vector  $\underline{b}$  takes the values from the interval  $(\underline{b}_R^I, \underline{b}_R^M)$  with probability  $p^R$ . In the paper, we present the different formulations of the stochastic programming problem taking into account their usability for the proposed model of the system. We choose the formulation based on Dantzig and Madansky's formulation of the problem, which takes into account the specification of our problem and can be solved in the finite time. Using this formulation, we can transform our problem to the linear programming problem.

The expansion strategy model of the water supply system consists of costs criteria (the capital and operating costs of system objects, the cost of an environmental protection) and constraints which concern the water demands for users, the maximum capacities of water intakes, treatment plants and main pipelines. Taking into account  $R$  realizations of the random vector  $\underline{b}_R$ ,  $r=1,2,\dots,R$ , we formulate the extended set of constraints. Thus, we have  $R$ -times more relations of the constraints. For the coefficients of the objective function, we assume the  $S$

cost scenarios. Thus, we have  $S$  objective functions  $F^s$ ,  $s=1,2, \dots, S$ , of the problem. Additionally, we introduce the objective function  $F_v$  to penalize the sum of violations of the constraints. For the given scenario  $s$ , we obtain the bicriterion problem with cost function  $F^s$  and the penalty function  $F_v$ . Finally, we have the set of  $S$  problems which can be formulated as following:

$$\text{Min: } (F^s, F_v)$$

subject to extended set of the constraints,

for  $s=1,2, \dots, S$ .

These problems can be solved by the aid of the parametric linear programming method which gives the sensibility analysis of the solutions.

For the estimation of the solutions, we propose the coefficients which present the mean values of the object and the confidence level of the solution.

At the end, we present the illustrative example and formulate final remarks for next researches.

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SCHEDULING CARGO ARRIVALS FOR INVENTORY

REPLENISHMENT

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ABSTRACT

A fertilizers factory located at the Pajaritos Harbour in the Gulf of México uses two kinds of phosphoric rock as main raw materials. Both kinds of rock share the factory warehouse. Although some phosphoric rock is home produced and supplied to the factory warehouse by train, most of the rocks is imported and transported by cargos.

Some cargos are working permanently with the company and their schedules are determined in advance. Additional one-trip cargos have to be hired in order to fulfill inventory needs. These cargos are all of equal size and transport only one product at a time.

The problem then consists in scheduling the one-trip cargos for up to one year length periods and should consider: factory production plans, inventory policies, and operating constraints of the only available dock which is shared with other vessels.

Inventory policies require to maintain the warehouse as full as possible, being mandatory to satisfy the safety stock for each kind of rock.

For this scheduling problem, a deterministic model is proposed. The solution method, using a combination of deterministic simulation and a back-track technique where the search branches have been suitable pruned, has been satisfactorily implemented in a computer. Inventory behavior and dock utilization are conveniently plotted, making the output very readable and descriptive.

The algorithm could be easily generalized in order to handle more than two products, more than one warehouse, and more than one dock. Furthermore, some refinements could be implemented in case of large instances.

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MODELLING OF METROLOGICAL SYSTEMS

by

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An essential problem in natural sciences is the mathematical description of investigated materials using extensive and intensive properties, quantity, homogeneity or distribution. Some set theoretical models have been introduced in physics and system theory, though the problem remains unsolved till this day.

A measure theoretical model for the description of materials will be introduced in this paper.

The proposed measure theoretical model is built on four basic concepts: element, element property, component, component quantity. Using these concepts the distribution of material and of material quantity according to given element property can be interpreted, which can be used to model the inhomogeneity of materials.

This mathematical model is appropriate for the mathematical treatment of problems occurring in metrology /chemometrics, biometrics, psychometrics, econometrics, etc/.

The model can be used for describing the operation of indirect measurement systems like diagnostical systems or quality control systems. In indirect measurement systems two types of properties are assigned to the constituent elements, the observed property and the inferred property. Likely two types of quantities, the observed and the inferred quantities are assigned to material. As a consequence of this interpretation indirect measurement is actually the resolution of mixture distribution of material or of material quantity.

The general applicability of the proposed measure theoretical model will be illustrated by examples giving the unified description of analytical chemical measurements, describing the relationship between chemical structure and biological activity, and realizing qualification of foods. Medical diagnostical and psychometrical examples will also be presented.

In the lecture the advantages and disadvantages of the proposed model, further the limits and possibilities of application will be outlined. We believe that the ideas stimulated by the model will enhelp the mathematical description of chemometrical, biometrical, psychometrical etc. and qualification problems and thus enable the wider application of computing technique in these fields.

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DETERMINATION AND UTILIZATION OF CUSTOMER ELECTRIC SERVICE  
INTERRUPTION COSTS FOR POWER SYSTEM PLANNING AND OPERATION

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Functional planning and operation of a large power system is intended to provide a reliable electric energy supply as economically as possible. Reliability levels of large power systems have generally increased over the past few decades, but the cost of providing that reliability has increased at a higher rate. Therefore, the whole issue of reliability cost relative to the worth or benefit of having that reliability has become an area of major importance to most utilities in their concern for responsible planning and operation.

The determination of reliability cost has become a reasonably well established procedure, but in contrast, the establishment of the societal worth of having that reliability is an immature and imprecise process. Among the various methods which have been used to determine reliability worth, perhaps the method which has gained most support in North America is that of assessing the customer losses associated with power interruptions, i.e., the cost of unreliability. Customer surveys are often used to obtain such information.

This paper presents summary results of a mail survey of some 19,000 electric utility customers across Canada conducted by the Power System Research Group at the University of Saskatchewan, Canada, during 1980-82. The principal focus of this work was to obtain residential customer interruption costs and to make improvements in survey methodology; however, commercial (retail), small industrial and large users were also surveyed. An overview of the availability of interruption cost data and its applicability is presented.

Cost of interruption data are usually obtained and compiled according to major customer sectors, ie. residential, commercial, industrial, farm, etc. comprising the utility load. In general, costs tend to be a function of customer category and of interruption characteristics (frequency, duration, and time of occurrence) as well as other variables such as climate, etc. As more data on customer losses are collected and compiled, it becomes possible to generate a composite customer damage function for a particular utility service area with a known customer composition.

The utilization of customer cost data in the development of methods applicable to reliability considerations in power system planning and operation are discussed. In particular, the preparation of a composite customer damage function is described, which involves the combination of losses of the various customer categories weighted in proportion to their respective energy utilization within the service area in question. The sensitivities of the resulting composite customer damage function with variations of load mix and other parameters are explored. The paper describes the application of the composite customer damage function and other relevant criteria or indices derived from the above analysis in the assessment of reliability cost/reliability worth.

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A GENERALIZATION OF THE M-TRAVELLING SALESMEN PROBLEM

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The optimization problem under investigation is defined in a finite graph in which both edges and nodes have the given weights. This graph can be used as a model for such applications :

- the optimization of the electrical power distribution network of medium voltages,
- the optimization of one transportation problem.

In the first case it is assumed that the electrical power distribution network consists of lines which are one-sidedly supplied at the normal work state. The optimization problem of the distribution network consists in definition of the number of lines as well as assignment of the electrical power consumers to the lines and settlement of the sequences of the power consumers connections in the lines in such a way that the annual cost of the distribution network is minimal. The annual cost of the distribution network consists of both the annual construction cost of the network and the annual operating cost which is defined by the loss of power and energy in the network.

In the second case the paths of transport goods from stores to customers are calculated. The optimization problem of the transport consists in definition of the number of paths

as well as assignation the customers to the lines and settlement the sequences of transport the goods this way that the summary transport cost is minimal. The summary transport cost depends not only from the length of the paths but also from the weight of the transported goods in each segment of the paths.

These problems are formulated as a combinatorial optimization problem. The  $\hat{m} \leq m$  -travelling salesmen problem is a particular case of it (in case when the weight of all nodes of the graph are equal zero).

The branch-and-bound method is used for solving this combinatorial problem. In order to improve the efficiency of the algorithm we employ both the mixed branching strategy of the search tree nodes and the combination of branch-and-bound method with heuristic method this way that in each node of the search tree the heuristic solution is calculated. This approach makes the frequent improvement of the upper bound possible.

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Shape Optimization of Fillets of Machine Tool Components

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The shape optimization method presented here for minimizing stress peaks at the edge of open fillets caused by notch effect is applicable to plates with bending action - or shell-like structures with linear-elastic, homogeneous, isotropic material behaviour. The area of the edge which has to be optimized must be unloaded.

First, starting from the hypotheses that, if there is a completely constant tangential stress distribution  $/\sigma_t/$  on a notch surface, the occurring notch stress will be minimal, the problem of stress minimization is formulated in the form of a functional, the integrand of which contains the difference between the stress at the edge of the fillet and the optimal stress. This functional is now approximated by a series of purely geometric substitute functionals, the integrands of which contain the deviation of the curvature of the actually existing curve of the fillet edge from the curvature of an optimized fillet shape, that is a prescribed curvature correction.

Furthermore a variation area is defined, in which the required optimized curve of the fillet edge must lie. The optimization problem defined in this way, is now discretized by the introduction of splines. If the curve of the fillet edge which has to be optimized is described by means of cubic, parametric splines, the curvature corrections under consideration will be obtained as a function of the spline sampling points which are defined as optimization parameters. The now discretized optimization problem is solved by an algorithm of the non-linear constrained programming (Powell's variable metric algorithm for constrained optimization).

After its start the iterative optimization procedure goes on automatically without any further interventions of the user. Based on a starting geometry the stresses at the fillet edge necessary for the calculation of the curvature corrections, are calculated by means of the finite-element method.

As a solution to the discretized optimization problem the new spline sampling points of the curve of the fillet edge are obtained. In the case of convergence the procedure is stopped, in the other case a new FE-mesh is produced automatically with this new geometry and the next iteration is carried out. The application of the method to practical problems of mechanical engineering is shown with some examples and the special efficiency of this method is demonstrated.

Thus the shape optimization of a C-frame-press is presented for example and the quality of the results is verified by means of photoelastic tests.

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EMERGENCY SERVICE OPTIMIZATION PROBLEMS

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One of the most important factors affecting the possibility of removing or softening the consequences of an accident which has taken place is the minimization of the time of arrival of an emergency service transport unit at the accident point. As the man's life is often the price to be paid, the need for any possible optimization of the emergency service transport system is beyond any discussion.

Among system parameters and elements having direct influence on the arrival time there are purely technical factors (e.g. car parameters, route system, communication system) as well as organization ones (approach route choice, allocation of emergency service transport units). The paper deals with the problem of optimal initial allocation of emergency service transport units assuring the achievement of a minimal value of some objective function characterizing the system performance. A mathematical model of the real system, based on Queueing Systems Theory, is introduced and several aspects of the system optimization are discussed.

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It is assumed that in the area under consideration there is given a set of possible initial location points; the whole area is presumed to be divided into a number of regions centered around the given points. A number of emergency service transport units is given. The problem consists in finding an optimal initial allocation of them at the given initial location points.

The proposed approach to the problem is as follows. Regarding the emergency service transport system as a number of queueing systems a simultaneous analysis of them, with respect to the imposed by the problem statement constraints, is performed. The number of service channels of each of the queueing systems is equal to the number of emergency service transport units allocated at the point corresponding to the queueing system; its value is to be determined for each of the location points. Several objective functions, such as the expected waiting for service time or the expected number of calls waiting for service, can be taken into account.

A method for solving the presented problem is proposed. Moreover, an adaptive approach to the allocation problem (in case of changing conditions or insufficient initial information) is described. The paper is completed with comments on various aspects of possible extensions and the problem statement modifications. An outline for further research is given. A few remarks concerning the implementation of the presented approach are enclosed.

## The Analysis of I/O Configurations

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Abstract

In this paper we look at the problem of modelling and analysing CPU-disk communication. One of the most simple models of this system is the central server model. In this queueing network model the computer system is represented by two types of stations, the CPU and the disk units. All stations are considered to be single server stations with a FCFS service discipline and exponential service times. The jobs in the system are alternately processed by the CPU and a disk unit. The model yields a product form solution and can be analysed e.g. by using the mean value algorithm ([2]).

Reality is far more complex. In particular the disk service time depends on the number of clients in the system and the utilization of the communication network which connects the CPU with the disk units. In figure 1 this is shown by introducing a channel into the model. As there is just one channel in this model, to be used by both the disk units, a kind of blocking may occur. The model has no product form solution and therefore it is hard to analyse.

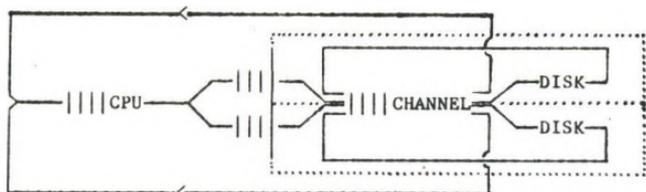


Figure 1 : CPU-Disk Model with Channel-Blocking

In the literature, e.g. [1], several iterative solution methods to this problem are suggested. There, a simultaneous use is made of aggregation and decomposition techniques. The approach suggested in

this paper just uses the aggregation of the channel and the disk unit. This can be done by using an heuristic extension of the mean value algorithm. Again the computer system is modelled as the central server model. The extension of the mean value algorithm lies in the recursive estimation of the aggregated disk service time. In each recursion step the information about channel blocking and throughput, calculated in the preceding recursion, is used to update the disk service time approximation. The mean value algorithm shows very robust against this extension.

The use of recursion is in particular advantageous when the system is to be imbedded in a computer terminal model. There, on the higher level of aggregation, the computer system can be seen as a single station with a population dependent work rate. As the throughputs for the various populations up to the maximum population are generated automatically, this gives a considerable reduction of the number of calculations, as compared to the iterative approach.

The solution method is illustrated by looking at an example of an IBM/MVS like computer configuration. Validation against simulation results shows that our method yields very accurate results.

The method can be extended to distinguish between different types of clients, as well as the case of multiple CPUs.

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The stopped distributions of a controlled Markov chain with continuous time.

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We are interested in characterizing the possible stopped distributions (by randomized stopping time), of a controlled temporarily homogeneous Markov process.

Let  $X^1, X^2, \dots, X^n$  will be Markov processes defined on the same probability space with value in the state space  $(E, \epsilon)$ . Let  $P^1, P^2, \dots, P^n$  denote Markov kernels (Markov transition functions) of  $X_1, X_2, \dots, X_n$  respectively.

Definition 1 - [3]

A measurable function  $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_n) : E \rightarrow [0, 1]$  is a control ( $\sigma \in \Sigma$ ) if  $\forall e \in E \quad \sigma_1(e) + \sigma_2(e) + \dots + \sigma_n(e) = 1$

Definition 2 - [3] - discrete time

A Markov process  $X^\sigma$  with a Markov kernel  $P^\sigma$  we will call a Markov controlled process with a control  $\sigma$  if  $\sigma \in \Sigma$  and  $\forall e \in E \quad P^\sigma(e, \cdot) = \sigma_1(e)P^1(e, \cdot) + \sigma_2(e)P^2(e, \cdot) + \dots + \sigma_n(e)P^n(e, \cdot)$ .

Definition 3 - continuous time

A Markov process  $X^\sigma$  with a Markov transition function  $P^\sigma$  we will call a Markov controlled process with a control  $\sigma$  if  $\sigma \in \Sigma$  and  $\forall f \in \bigcap_{i=1}^n D_{A^i} \quad \forall e \in E \quad A^\sigma f(e) = \sum_{i=1}^n \sigma_i(e) A^i f(e)$

where  $A^\sigma, A^i, 1 \leq i \leq n$  are infinitesimal generators of semigroups of transition functions  $P^\sigma, P^i, 1 \leq i \leq n$  respectively.

In [3] is given a characterization of stopped distributions of a controlled Markov chain with discrete time.

Theorem 1 -[3]

If  $X_1, X_2, \dots, X_n$  are discrete time processes and  $E$  is a countable set then:

$$\exists \sigma \in \Sigma \quad \exists T \in \text{rst} \quad X_0^\sigma \sim \mu \Rightarrow X_T^\sigma \sim \nu \Leftrightarrow \forall f \in \bigcap_{i=1}^n S_{P^i} \quad \langle \mu, f \rangle \geq \langle \nu, f \rangle$$

where  $S_P$  denote P-excessive functions.

Theorem 1 is a partial generalization of the Rost [1] result for uncontrolled case. In [2] Rost gave us the characterization of stopped distributions of a continuous time Markov process. In that paper we characterize stopped distributions of a controlled Markov chain with continuous time.

Theorem 2

If  $X_1, X_2, \dots, X_n$  are continuous time processes and  $E$  is a countable set then:

$$\exists \sigma \in \Sigma \quad \exists T \in \text{rst} \quad X_0^\sigma \sim \mu \Rightarrow X_T^\sigma \sim \nu \Leftrightarrow \forall f \in \bigcap_{i=1}^n S_{P^i} \quad \langle \mu, f \rangle \geq \langle \nu, f \rangle$$

Theorem 2 is proved in the paper by Rost [2] theorem and some geometric considerations in Banach space similar to those considered in the paper [3].

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ON THE GENERALIZATION OF MORRISON'S METHOD FOR  
SOLVING CONSTRAINED MULTICRITERIA PROBLEMS

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Consider the multicriteria minimization problem

$$\min_{x \in X} f(x), \quad X = \{x \in \mathbb{R}^n \mid g(x) \leq 0\}, \quad (1)$$

where the mappings  $f: \mathbb{R}^n \rightarrow \mathbb{R}^r$  and  $g: \mathbb{R}^n \rightarrow \mathbb{R}^m$  are continuous. The values of the mapping  $f(x)$  define the set of attainable objectives  $Y = \{y \in \mathbb{R}^r \mid y = f(x), x \in X\}$ . We consider the weak Pareto-optimal objectives set

$$Y_* = \{y \in Y \mid \max_{1 \leq i \leq r} (y^i - y^i) \geq 0 \forall y \in Y\}$$

and the corresponding set  $X_* = f^{-1}(Y_*)$  as the solution of the problem (1).

A. Wierzbicki proposed to use the penalty functions method for solving the problem (1). Here the Morrison's method generalization is proposed.

We denote the positive and the negative orthants of  $\mathbb{R}^l$ , where  $l=r+m$ , by  $\mathbb{R}_+^l$  and  $\mathbb{R}_-^l$ , respectively. Also we introduce an auxiliary nonnegative positively homogeneous function  $Q(z)$  that equals zero if  $z \in \mathbb{R}_-^l$ . The simplest example of such a function is  $l_p$ -norm of vector  $z_+$ .

Let vector-valued function  $h(x,y)$ , where  $y \in \mathbb{R}^r$ , be such that its first  $r$  components are  $f^i(x) - y^i$  and the other  $m$  components are  $g^i(x)$ . In this case the function  $H(x,y) = Q(h(x,y))$  can be constructed.

Suppose that the initial vector  $y_0 \in \mathbb{R}^r$  is given and  $y_0 \notin Y_+$ , where  $Y_+ = Y + \mathbb{R}_+^r$ . Suppose also that an arbitrary direc-

tion  $e \in \text{int } R_+^r$  is chosen. The following iterative process is some generalization of the first version of Morrison's method for solving the problem (1)

$$x_k \in \text{Arg } \min_{x \in R^n} H(x, y_k), \quad y_{k+1} = y_k + c(x_k, y_k, e)e, \quad (2)$$

where  $c(x, y, e) = H(x, y)/Q(e, 0)$ . It is proved that at each step the vector  $y_k \in \text{int } Y_+$  and all accumulation points of the sequence  $x_k$  belong to the set  $X_*$ .

Supposing in addition that the function  $Q(z)$  is differentiable on  $R^1$  except the boundary of the orthant  $R_+^1$  we can construct a generalization of the second version of the Morrison's method, where

$$c(x, y, e) = \langle Q_z(h(x, y)), h(x, y) \rangle / \langle Q_u(h(x, y)), e \rangle. \quad (3)$$

Here  $Q_u(z)$  denotes the first  $r$  components of the gradient  $Q_z(z)$  of the function  $Q(z)$ . The process (2) with the coefficient choice according to (3) can be applied for convex multicriteria problems only.

In the proposed methods the vector criterion and constraints are convoluted jointly in one common criterion. The initial vector  $y_0$  can be considered as a parameter. Different choice of  $y_0$  define different weak Pareto-optimal objectives from the set  $Y_*$ . The other way of obtaining different weak Pareto-optimal objectives consists in fixing the vector  $y_0$  and changing the direction  $e$ .

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## Computer Aided Management System for Construction Projects

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The address discusses the model of a management control system, called " HOPIR " and designed for housing and public construction projects based on prefabrication technology. The development of the system has been sponsored by the Central Research Development Program of the Hungarian Government.

Housing construction is one of the high priority areas of the Government's welfare programs.

The management of housing projects involves the solution of complex engineering and financial planning problems, and the efficiency and cost effectiveness of such solutions are of the greatest importance for the national economy.

### The Structure and Functions of the System

The system includes the following subsystems: Engineering Subsystem / or Technical Planning Subsystem - based on the opinions and requirements of home owners /; Precasting Production Control Subsystem; Storage And Transportation Control Subsystem; Subsystem; Resource Control Subsystem.

### Mathematical Methods Applied in the System

The system has been developed for the ES 1011 Computer produced by VIDEOTON and adapted for operating the DMS 600 data base management system. The data base has a logical structure suited to the peculiar traits of the management process and it contains the various data - such as technical and performance standards - used by the several subsystems.

The data-base management system has a CODASYL-type networking structure, and is suitable for handling complicated interrelations required by some routines.

The operations performed by the various subsystems are described in CPM-type networks, with the exception of the Site Management Subsystem, for which - taking into account the stable manning

levels and the requirements which usually apply to assembly-line type operations - a special line balancing algorithm has been developed by the Construction Department of the Technical University of Budapest.

Precasting take place on permanent sites, and it involves a number of recurrent processes the optimization of which is achieved by the application of simulation techniques.

The fundamental objectives of the system are: shorter lead times; cost reductions in terms of material, energy, transportation, and labour costs; improved labour performance. The individual subsystems are centered around these objectives.

As the winner of a system development and application competition jointly sponsored by the National Committee for Technical Development, the Central Bureau of Statistics, and the Ministry of Building and Urban Development - the "DELEP" Construction and Civil Engineering Company has received a grant to control and coordinate the development of the above described management system, in cooperation with Hungarian universities and research organizations.

The application of the system offers the following advantages to construction companies:

- provides comprehensive, computer aided management support for the entire project cycle from concept to turn-key commissioning,
- helps to reduce construction costs,
- enhances effective management at all levels by providing real-time interactive information accessing which is a significant new development in the field of industrial computer applications in Hungary.

Construction companies in Hungary and in some other COMECON countries have also shown substantial interest in the system.

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Some Applications of a Duality Concept  
in Totally Ordered Sets

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During the last years we investigated duality concepts in totally ordered sets. The approach was motivated by some observations about algorithms for solving sharing problems, i.e. problems of the form

$$\min_{x \in P} \max_j f_j(x_j)$$

where  $P$  denotes the set of optimal solutions and  $f_j, j=1..n$ , denote functions defining cost or time criteria. The concept leads to efficient methods for solving problems with different types of constraints, e.g. linear, combinatorial and convex constraints.

A further motivation arose from the discussion of balanced network flow problems. A network flow in the classical one-source-one-sink model is called balanced, if the flow value in every arc is bounded by some proportion of the total flow from source to sink. Application of the general dual approach leads to a genuinely polynomial method for maximizing total flow on the set of all balanced network flows. Recently, we generalized balanced network flows to balanced submodular flows. Again, the dual method applies and leads to a method for solving balanced submodular flow problems. Here, the method is of genuinely polynomial complexity modulo the complexity of submodular function minimization.

Similar, as in the case of sharing problems<sup>4</sup> balanced problems may be considered for quite different types of constraints. Here, additional bounds depending on a single parameter are introduced in

order to balance feasible solutions, e.g. we may consider

$$\max\{z \mid x \in P, l(z) \leq x \leq u(z)\}.$$

Again, we can apply the general dual method in order to derive efficient methods either for maximizing the parameter or for checking the existence of balanced feasible solutions.

The efficiency of the resulting method is mainly due to the fact that the dual approach exploits the special structure of the respective constraints. In fact, the main effort is usually spent for constructing a feasible solution for certain fixed values of the parameter. Then, complexity bounds are derived from the complexity of the feasibility procedure times the number of occurring fixed values. Although the latter number can often easily be shown to be finite, some intricate arguments are necessary to derive efficiency. Since feasibility procedures are quite well-developed and efficient implementations are often known, we suppose that the resulting procedures will lead to fast codes for many of the considered problems. Some preliminary numerical experiences on sharing problems support our claim.

Finally, it turned out that the general dual approach yields surprisingly simple proofs of classical duality results, e.g. for linear and convex programming problems.

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## Shape Optimization and Free Boundary Problems

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These ten last years the shape optimization problem was simply understood as the best shape problem for a physical system [1],... - a criteria function  $J(\Omega, y)$  being minimized by selecting the shape  $\Omega$  ( $y$  is the solution of a well posed boundary value problem on the domain  $\Omega$ ). The solution  $y$  is then controlled by the domain  $\Omega$ . To get the shape gradient  $G$  ([5],[6]) of the criteria function two steps are necessary: the shape sensitivity analysis of  $y$  and the introduction of an adjoint equation. For numerical computation three technics have been developed to get an approximation by finite element method of the gradient  $G$ .

The next developments are now in several directions :

a) For some particular problems it turns out that the shape sensitivity analysis of  $y$  is not necessary and then many results are extended to non smooth problems.

b) On the other hand for some unilateral problem (e.g. The control of a contact set [10],[3]), and for incremental methods (when following a large deformation in elastic or viscoplastic non Newtonian fluid) just the shape derivative  $y'$  ([5],[6]) is necessary in particular when the state equation solution  $y$  is itself a physical flow vector speed and  $V$  a virtual deformation speed of the shape (as introduced in [5],[6] and [11]) the shape derivative  $y'(V)$  of  $y$  in the direction  $V$  is used with  $V=y$ . For a large class of elliptic second order unilateral problems  $y'$  has been characterized [3].

d) Free boundary problems for stationary fluid flow. Two new methods are being developed numerically as well as theoretically. We shall present three examples :

d-1) Non linearized water waves in a two dimensional canal, using a gradient method for an auxiliary criteria function [10] and using a direct shape variational formulation of the Bernoulli free boundary condition [8],[9].

d-2) A Newton shape method (second order method) to solve a free boundary in a non Newtonian flow (Norton-Hoff flow) [12].

d-3) Finally some older results will be presented relatively to the shape variational formulation of a free boundary arising in adiabatic plasmas physic (H. Grad equations).

e) Non differentiable problems including unilateral constraint problem such as obstacle problems and actuators

on sensors best location problem [12].

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