Ms 5702/6-7. Eötvös Loránd jegyzetei: Mágnesség

2 kötet fol. 2 bor.

KÉZIRATTÁR VEDŐNAPLÓ
1972 17 SZ

19

1916 Júniusban a

Néplapból összegyűjtött kérdések

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

9/2015 M

0,933393 −1

| $\lg \ell^2$ | $\lg \ell^4$ | $\lg \ell^6$ | $\lg \ell^8$ |
|---|---|---|---|
| 0,866786 −1 | 0,733572 −1 | 0,600358 −1 | 0,467144 −1 |
| 0,291642 | 0,229749 −1 | 0,288249 −3 | 0,352183 −4 |
| 0,158428 | 0,963321 −2 | 0,888607 −4 | 0,819327 −5 |
| 1,440220 | 0,091901 | 0,000774 | 0,000066 |

477060
239004
083768
0,799832
2,062826
2,262994

~~1,441060~~
919
91967
1,532187
774
1,531413

0,733572 −1
0,342496 −1
0,076068 −1

0,600358 −1
0,546543 −4
0,146901 −4

0,467144 −1
0,792392 −5
0,259536 −5

0,119143
000140
0,119283
0,000018
0,119265

0,600358 −1
0,999791 −2
0,600149 −2

0,039824
0,000010

0,467144 −1
0,544068 −5
0,011212 −5

0,467144 −1
0,552851 −2
0,019995 −2
0,0

$\ell = 0,857814$

+ 8,601160
1,772936
10,374096

4,073412
25,388659
+ 23,895247
1,491418

− 0,571293
+ 0,233575
− 0,337718

$$\frac{\partial^8 L}{\partial x^2 \partial y^6} = +7.860496 \left\{ +18,586575 + 0 + 1,548045 + 2,322780 + 0,297330 - 0,902826 \cdot 0,205689 + 0,451454 \cdot 0,705689 \right\}$$

$$\begin{array}{r}
+18,586575 \\
+1,548045 \\
2,322780 \\
0,297330 \\
\hline
+22,754730 \\
0,318586 \\
\hline
+23,073316 \\
185701 \\
\hline
+22,887615
\end{array}$$

$$\frac{\partial^8 L}{\partial x^2 \partial y^6} = +179,908006$$

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

$$\frac{\partial^8 L}{\partial y^8} = -7.860496 \left\{ +18,586575 + 0 + 1,548045 + 2,322780 + 0,297330 + 0 + 0,225727 \right\} = -7.860496 \cdot 22,980457$$

$$\frac{\partial^8 L}{\partial y^8} = -180,637790$$

$$\begin{array}{r}
179\,908\,006 \\
\hline
-0,729784
\end{array}$$

$$7.860496 \left\{ \begin{array}{r} -0,902826 \; 0.205689 \\ +451454 \\ \hline -0,451372 \end{array} \right.$$

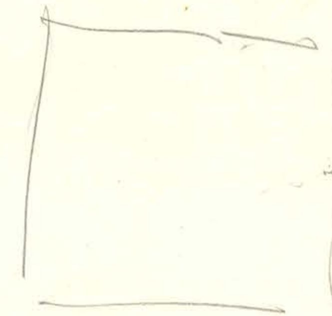
$$-\cancel{\ldots}\left[a^2+b^2+c^2+l^2-2l(a\cos\alpha+b\sin\alpha)\right](-a\cos\alpha-b\sin\alpha+l)$$

$$-(b-l\sin\alpha)(a-l\cos\alpha)(a\sin\alpha-b\cos\alpha)$$

$$\frac{cl}{c^2(b-l\sin\alpha)^2+(a-l\cos\alpha)^2 S^2}\cdot\frac{1}{S}$$

$$\frac{1}{2}\lg\frac{a^2+(l-b)^2}{a^2+(l+b)^2}-\lg\frac{c+\sqrt{a^2+(l-b)^2+c^2}}{c+\sqrt{a^2+(l+b)^2+c^2}}\qquad\frac{\pi}{2}$$

$$\frac{a^2+(l+b)^2}{\quad}-\lg\frac{c+V}{\quad}$$



$$\left(\frac{\partial F}{\partial\alpha}\right)_{\alpha=0}+\left(\frac{\partial F}{\partial\alpha}\right)_{\alpha=\pi}=l\left(\left(\frac{\partial Y}{\partial\alpha}\right)_0-\left(\frac{\partial Y}{\partial\alpha}\right)_\pi\right)-l\left(X_{\alpha=0}-X_{\alpha=\pi}\right)$$

$$\left(\frac{\partial F}{\partial\alpha}\right)_{\alpha=\frac{\pi}{2}}+\left(\frac{\partial F}{\partial\alpha}\right)_{\alpha=-\frac{\pi}{2}}=+l\left(Y_{\alpha=-\frac{\pi}{2}}-Y_{\alpha=+\frac{\pi}{2}}\right)+l\left(\left(\frac{\partial X}{\partial\alpha}\right)_{\alpha=-\frac{\pi}{2}}-\left(\frac{\partial X}{\partial\alpha}\right)_{\alpha=+\frac{\pi}{2}}\right)$$

$\cos\alpha\,(a-l\cos\alpha)$

$\frac{cl}{\quad}$

0,00133    0,00183

89    184

228    237

$\frac{120}{240}\,9000+10000$

$$X=\frac{c^2(b\mp l\sin\alpha)^2+(a-l\cos\alpha)^2 S^2}{(a-l\cos\alpha)^2 S^2}\left(\frac{\pm c\,l\cos\alpha}{(a-l\cos\alpha)S}-\frac{c(b\mp l\sin\alpha)}{(a-l\cos\alpha)^2S^2}(\pm l\sin\alpha\,S+\ldots)\right)$$

$$S=\sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha+b\sin\alpha)}$$

$S'$

0,293389 = log c
0,305556−1

0,987833

0,293389
0,637258

0,656131−1

0,152857−1
0,152699

0,305556−1

0,269001
0,368257

84° 7′ 42″

2,22484
−1,96512

4,29996
1,42134

3,38646

1,4660766
20362
2036

1,4683164
5,8732656

24° 22′ 20″

0,4188790
63995
970

0,4253755
1,7015020

2 { ¾ log √1,02022 / √4,45147 . 4,29996 / 3,38646

0,008694
0,648503

~~0,043~~

0,004347
0,633464

0,637811
0,853998

0,783813−1

0,324252
0,529746

− 0,216187 . 0,497789
− 0,995578

0,142186 . 5,881929

0,078072 + 4,881929
.78072

4,960001

3,451473
3,861712
1

8,313185

0,020217
3,861712
2

5,881929

1,857874 . 9,313185

13,328595 + 8,313185
8,313185

21,641780

0,152857−1
0,769512

0,922369−1
0,695482

0,226887−1
0,168612
0,799480

0,968092

0,269001
0,969099

1,238100
1,335293

0,902807−1

0,985917−1
0,293389
0,933393−1

0,212699
0,163192
0,326384

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

+6,772336
371448
6400888

-0,346556
-0,023460
-0,001492

+0,023460

3,286168
3910
3,282258

2,391618
86639
2,304979
2,478257

2,390078

-6,772336
-0,346556
-0,001492
7120324
23460
7,096864

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

$$\frac{\partial^2 V}{\partial x^2} = -\frac{da\,db}{r^3} + 3\,\frac{(a-x)^2\,da\,db}{r^5}$$

$$\frac{\partial^2 U}{\partial x^2} = -\frac{db\,dc}{r^3} + 3\,\frac{(a-x)^2\,db\,dc}{r^5}$$

$$-\frac{b-y}{(a^2+c^2)r} + 3\,a^2\left(\frac{1}{3(a^2+c^2)r^3} + \frac{2}{3(a^2+c^2)^2 r}\right)b-y$$

$$2a^2+2b^2+2c^2$$

$$a^2 \qquad c^2$$

$$\frac{a^2\,b-y}{(a^2+c^2)^2 r^3}\left[(3a^2+3c^2)+2b^2\right]$$

$$-\frac{(a^2+c^2)(a^2+b^2+c^2)\,b}{(a^2+c^2)^2 r^3}$$

$$\frac{b}{(a^2+c^2)r^3}\left(3a^4+3a^2c^2+2a^2b^2\right)$$

$$-a^4-a^2b^2-a^2c^2$$

$$-a^2c^2-c^2b^2-c^4$$

$$2a^4+a^2b^2+a^2c-c^2b^2-c^4$$

$$-\frac{1}{1+\frac{b^2c^2}{a^2(a^2+b^2+c^2)}}\left(\frac{c}{a\sqrt{a^2+b^2+c^2}} - \frac{b^2c}{(a^2+b^2+c^2)^{\frac{3}{2}}}\right)$$

$$-\frac{c}{b^2c^2+a^2(a^2+b^2+c^2)}\left(a\sqrt{a^2+b^2+c^2} - \frac{ab^2}{\sqrt{a^2+b^2+c^2}}\right)$$

$$-\frac{ac}{b^2c^2+a^2(a^2+b^2+c^2)}\,\frac{a^2+c^2}{\sqrt{a^2+b^2+c^2}}$$

$$(a^2+b^2)(a^2+c^2)$$

$$-\frac{ac}{a^2+b^2}\,\frac{1}{r}$$

C magasságban $dc$ vastagságú rétegre:

$$\cancel{\frac{1}{f\sigma}\frac{\partial^2 U}{\partial x^2} = -dc\frac{(b-x)(a-x)}{((a-x)^2+c^2)\sqrt{(a-x)^2+(b-y)^2+c^2}}}$$

$a-x$ helyett $a$-t
$b-y$ helyett $b$-t írva
$\sqrt{(a-x)^2+(b-y)^2+c^2} = r$
$((a-x)^2+c^2) = \rho^2$

$$\cancel{\frac{1}{f\sigma}\frac{\partial^2 U}{\partial x^2} = -dc\,b\,\frac{1}{\rho^2}\frac{\partial}{\partial x}\frac{1}{r}}$$

$$\frac{1}{f\sigma}\frac{\partial^2 U}{\partial x^2} = -dc(a-x)\int\frac{db}{((a-x)^2+(b-y)^2+(c-z)^2)^{\frac{3}{2}}}$$

$$= -dc(a-x)\left|\frac{(b-y)}{((a-x)^2+(c-z)^2)\sqrt{(a-x)^2+(b-y)^2+(c-z)^2}}\right.$$

$$\frac{1}{f\sigma}\frac{\partial^2 U}{\partial x^2} = -dc\left|\int_{a_1}^{a_2}\right|_{b_1}^{b_2}\frac{ab}{(a^2+c^2)\sqrt{a^2+b^2+c^2}}$$

$\sqrt{a^2+b^2+c^2} = r$

$(a^2+c^2) = \rho^2.$

$$\frac{1}{f\sigma}\frac{\partial^2 U}{\partial x^2} = -dc\iint\frac{ab}{a\rho^2}\frac{1}{r}$$

$$\frac{\partial^n(\varphi f)}{\partial x^n} = \left(f\frac{\partial^n\varphi_n}{\partial x^n}+\varphi\frac{\partial^n f_n}{\partial x_n}\right)+n\left(\frac{\partial f}{\partial x}\cdot\frac{\partial^{n-1}\varphi}{\partial x^{n-1}}+\frac{\partial\varphi}{\partial x}\frac{\partial^{n-1}f}{\partial x^{n-1}}\right)+\frac{n.n-1}{1.2}\left(\frac{\partial^2 f}{\partial x^2}\frac{\partial^{n-2}\varphi}{\partial x^{n-2}}+\frac{\partial^2\varphi}{\partial x^2}\frac{\partial^{n}}{\partial}\right.$$

$$\frac{\partial^n\left(\frac{a}{\rho^2}\right)}{\partial x^n} = a\frac{\partial^n}{\partial x^n}\left(\frac{1}{\rho^2}\right)+n\frac{\partial^{n-1}}{\partial x^{n-1}}\left(\frac{1}{\rho^2}\right)\frac{\partial a}{\partial x}$$

$\frac{1}{\mathcal{B}} = 0.3$

$$\frac{\partial^n(\varphi f)}{\partial x^n} = \varphi^n f + n\varphi^{n-1}f' + \frac{n\ n-1}{1.2}\varphi^{n-2}f'' + \cdots$$

$\varphi = \frac{1}{r}$

$\psi = b.$

$$\frac{1}{2}\log\left(1+\frac{a^2}{b^2}\right) + \log\, c+\sqrt{c^2+b^2} - \log\, c+\sqrt{a^2+b^2+c^2}$$

$$-\left(-\frac{1}{2}\,\frac{b^2}{a^2+b^2}\,\frac{2a^2}{b^3} + \frac{1}{c+\sqrt{c^2+b^2}}\,\frac{b}{\sqrt{c^2+b^2}} - \frac{1}{c+\sqrt{a^2+b^2+c^2}}\,\frac{b}{\sqrt{a^2+b^2+c^2}}\right)$$

$$-b\left[-\frac{a^2}{(a^2+b^2)b^2} + \frac{1}{(c+\sqrt{c^2+b^2})\sqrt{c^2+b^2}} - \frac{1}{(c+\sqrt{a^2+b^2+c^2})\sqrt{a^2+b^2+c^2}}\right]$$

$120$

$3\dfrac{(a-x)(b-y)\,da\,di}{\left((a-x)^2+(b-y)^2+c^2\right)^{\frac{5}{2}}}$

$30$

$$-\frac{b\,da}{(a_2^2+b^2+c^2)^{\frac{3}{2}}} + \frac{b\,di}{(a_1^2+b^2+c^2)^{\frac{3}{2}}}.$$

$$-\frac{bc}{(a_2^2+b^2)\sqrt{a_2^2+b^2+c^2}} + \frac{bc}{(a_1^2+b^2)\sqrt{a_1^2+b^2+c^2}}$$

$$\boxed{-\frac{bc}{(a^2+b^2)\sqrt{a^2+b^2+c^2}} + \frac{bc}{b^2\sqrt{b_1^2+c^2}}}$$

$\left(\frac{\partial F}{\partial a}\right)_{\frac{\pi}{4}} = -2,262994$

$-3,062826$
$-0,477060$
$83768$
$-3,623654$
$239004$
$-3,384650$

$\left(\frac{\partial F}{\partial a}\right)_{\frac{\pi}{2}} = +3,384650$

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

$-1{,}860496\{-0{,}413035\cdot 45 - 0{,}422778\cdot 15 + 0{,}576015\cdot 3 - 0{,}594660\cdot 0{,}5 + 0{,}902826\cdot 0{,}5 + 0{,}437454\cdot 0{,}5$

$-\dfrac{\partial b_c}{\partial x^5 \partial q^2}$

$+15{,}720992\{-0{,}413035\cdot 15 - 0{,}352315\cdot 3 - 0{,}774260\cdot 0{,}5 - 0{,}198220\cdot 0{,}5 + 0{,}150471\cdot 0{,}5\}$

$-6{,}195525$
$-1{,}056945$
$-0{,}387130$
$0{,}099110$
$-7{,}738710$
$75236$
$7{,}663474$

$1{,}987938$
$2{,}417890$
$-0{,}429952$

$-0{,}000448 +$

$+238{,}492142$
$+963{,}819280$
$1202{,}311422$
$1196{,}315770$
$5{,}995652$

$0{,}000130$

$-\dfrac{\partial L}{\partial x^2 \partial q^2}$

$+15{,}720992\{\cancel{0{,}413035} + 0{,}211389\cdot 0{,}5 - 0{,}103203\cdot 0{,}5 - 0{,}077426\cdot 0{,}5$

$180629$
$\cancel{103203}$
$77426$
$0{,}292618$

$77426$

$0{,}030760$
$0{,}015380$

$665898$
$218$
$666216$

$\lg \dfrac{\sqrt{1{,}020217}}{\sqrt{4{,}451473}} \cdot \dfrac{4{,}299964}{3{,}386464}$

0,008694

0,004347
0,633461
0,637811
0,853998
$-$ 0,216187

0,648503

0,324252
0,152 9746

0,293389
0,365556 $-1$
0,987833
84° 7′ 42″

0,152857 $-1$
0,152699

1,4660766
20062
2036
1,4683164
5,8732656

0,293389
0,637258
0,656131 $-1$
24° 22′ 20″

0,269001
368257

0,4188790
63995
970
0,4253755
1,7015020

12

$-24 - 15{,}24$

$-24.16$

2840
144

24.16
144
24
384

$-\frac{12}{1920}\,16 = -\frac{6.16}{960} = -\frac{3.16}{480} = -\frac{3.8}{240} = -\frac{3.4}{120} = -\frac{3}{30} = -\frac{1}{10}$

1421340
1965124
2334840

$\left(\frac{5}{2^{2}} + 5\right)(7-9) - \dfrac{25^{2}_{2} + {}_{2}(7-9)^{2}_{2}}{\phantom{x}}$ 72

$-0{,}995578$

~~5~~ 345
5,451473
3 861712
9313185

2,020217
3,861712
5881929

0,078072
4 881929
4960001

$$\left\{ \frac{0{,}142186 \cdot 5{,}881929}{4{,}960001} \cdot \frac{1}{\sqrt{4{,}881929}} + \frac{1{,}857874 \cdot 9{,}313185}{21{,}641780} \cdot \frac{1}{\sqrt{8{,}313185}} \right.$$

0,919768

1475978
969496
0,688592
~~,06982~~
$-2{,}445474$

13,328595
8,313185
~~18,641780~~
21,641780

0,152857 −1
0,769520

0,044296
0,695482
1,039778
0,922377 −1
0,882599 −2

0,269001
0,969099

1475927
969412
2445306
~~0,763130~~
0,076313
0,277283
0,353596

1,876636

0,459884
1,335293
1,795177
1,238100
0,442923 −1

0,548507 −1
0,933393 −1
0,293389
0,775289 −1

0,596059
$-1{,}192118$
995578
$+2{,}187696$

---

0,152699
0,008694
0,161393
0,226782
0,065389 4

0,368257
0,648503
1,016760
0,226782
0,210022 −1

$-0{,}133489$
$-2{,}933928$
1,162490
4649960

~~0,102334~~
~~0,01~~
0,102724
$+0{,}409336$
0,162189
648754

$$\frac{(a-l\cos\alpha)^2(a^2+b^2+c^2+l^2-2l(a\cos\alpha\mp b\sin\alpha)}{c^2(b\mp l\sin\alpha)^2+(a-l\cos\alpha)^2(a^2+b^2+c^2+l^2-2l(a\cos\alpha\pm b\sin\alpha)}\left\{\frac{\pm cl\cos\alpha}{(a-l\cos\alpha)\sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha\pm b\sin\alpha}}-\frac{c(b\pm l\sin\alpha)\left[\ \right]}{\left((a-l\cos\alpha)\sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha\pm b\sin\alpha)}\right)^2}\left\{+l\sin\alpha\sqrt{\ }+\frac{(a-l\cos\alpha)(a\sin\alpha\mp b\cos\alpha)}{\sqrt{\ }}\right\}\right\}$$

$a=n$
$b=c$
$c=a$

$\{-cl\cos\alpha$

$-cl\cos\alpha(a-l\cos\alpha)\sqrt{\ }$ $-c(b\pm l\sin\alpha)(2lb\sin\alpha-2lb\cos\alpha)$

$$\frac{cl}{c^2(b\mp l\sin\alpha)^2+(a-l\cos\alpha)^2 S^2}\left\{\mp\cos\alpha(a-l\cos\alpha)S-(b\mp l\sin\alpha)\left\{b\sin\alpha\, S+\frac{(a-l\cos\alpha)(a\sin\alpha\mp b\cos\alpha)}{S}\right\}\right\}$$

$\{-(a\cos\alpha+b\sin\alpha)+l\}\,S$
$\{+a\cos\alpha-b\sin\alpha-l\}$

2,445474

$\sqrt{17}$

| | |
|---|---|
| | 3 |
| | 3861712 |
| | 16,861712 |
| | −0,188665 |
| | 294896 |
| | −0,483541 |
| | −0,967082 |

0,188665
296083
~~91882~~
0,484748

0,969496

1,230449

1,226902

0,084864
0,615225
0,469639 −1

0,084864
613451
0,471413

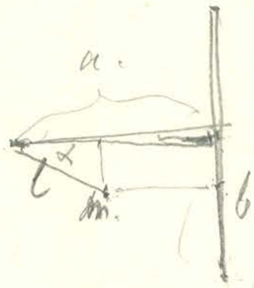
$\sqrt{a^2+b^2+c^2-2l}$
$a\sin\alpha-b\cos\alpha$

$a \sin\alpha - b\cos\alpha$

$r = a$

$b = c$

$a = b \pm$




$$Y_x - X_y$$

| $+$ | $-$ |
|---|---|
| $a = b - l\sin\alpha$ | $a = b + l\sin\alpha$ |
| $r = a - l\cos\alpha$ | $r = a - l\cos\alpha$ |
| $b = c$ | $b = c$ |

$$F = l\cos\alpha\, Y - l\sin\alpha\, X'.$$

$$Y = 2\Delta a/\sigma \Big\{ \log \frac{\sqrt{a^2+b^2+l^2-2l(a\cos\alpha+b\sin\alpha)}}{a-l\cos\alpha} - \log \frac{c+\sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha+b\sin\alpha)}}{c+\sqrt{c^2+(a-l\cos\alpha)^2}}$$

$$- \log \frac{\sqrt{a^2+b^2+l^2-2l(a\cos\alpha-b\sin\alpha)}}{a-l\cos\alpha} + \log \frac{c+\sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha-b\sin\alpha)}}{c+\sqrt{c^2+(a-l\cos\alpha)^2}} \Big\}$$

---

$$X' = 2\Delta a/\sigma \Big\{ \operatorname{arctg} \frac{c(b-l\sin\alpha)}{(a-l\cos\alpha)\sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha+b\sin\alpha)}}$$

$$+ \operatorname{arctg} \frac{c(b+l\sin\alpha)}{(a-l\cos\alpha)\sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha-b\sin\alpha)}} \Big\}$$

---

$$\left(\frac{\partial F}{\partial\alpha}\right)_{\alpha=0} = l\left(\frac{\partial Y}{\partial\alpha}\right)_{\alpha=0} - l X'_{\alpha=0} \qquad \left(\frac{\partial F}{\partial\alpha}\right)_{\alpha=\pi} = -l\left(\frac{\partial Y}{\partial\alpha}\right)_{\alpha=\pi} + l X'_{\alpha=\pi}$$

$$\left(\frac{\partial F}{\partial\alpha}\right)_{\alpha=\frac{\pi}{2}} = -l(Y)_{\alpha=\frac{\pi}{2}} - l\left(\frac{\partial X'}{\partial\alpha}\right)_{\alpha=\frac{\pi}{2}} \qquad \left(\frac{\partial F}{\partial\alpha}\right)_{\alpha=-\frac{\pi}{2}} = +l(Y)_{\alpha=-\frac{\pi}{2}} + l\left(\frac{\partial X'}{\partial\alpha}\right)_{\alpha=-\frac{\pi}{2}}$$

$$\frac{\partial F}{\partial\alpha} =$$

0471413

$$\frac{a\sin\alpha - b\cos\alpha}{\sqrt{a^2+b^2+l^2+c^2-2l(a\cos\alpha+b\sin\alpha)}}\left[\frac{\sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha+b\sin\alpha)}}{a^2+b^2+l^2-2l(a\cos\alpha+b\sin\alpha)} - \frac{1}{c+\sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha+b\sin\alpha)}}\right]$$

$$\left[\frac{c\sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha+b\sin\alpha)}+c^2}{\left[a^2+b^2+l^2-2l(a\cos\alpha+b\sin\alpha)\right]\left[c+\sqrt{\phantom{xxxxxx}}\right]}\right]$$

$$\frac{\partial y}{\partial \alpha} = 2f\sigma\Delta a\, l\left\{\frac{c(a\sin\alpha-b\cos\alpha)}{a^2+b^2+l^2-2l(a\cos\alpha+b\sin\alpha)}\,\frac{1}{\sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha+b\sin\alpha)}} - \frac{c(a\sin\alpha+b\cos\alpha)}{a^2+b^2+l^2-2l(a\cos\alpha-b\sin\alpha)}\,\frac{1}{\sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha-b\sin\alpha)}}\right\}$$

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

$\sqrt{(a-b)^2+b^2+c^2} = 2,976860$

$\underline{c}$ 1,965124 $\underline{a=7}$ $c^2 = 3,861712$ $\sqrt{(a+b)^2+b^2+c^2} = 4,567480$

$\log\sqrt{(a-b)^2+b^2+c^2} = \log\sqrt{8,861712} = \frac{1}{2} 0,947518 = \underline{0,473759}$

$\log\sqrt{(a+b)^2+b^2+c^2} = \log\sqrt{20,861712} = \frac{1}{2} 1,319350 = \underline{0,659675}$

0,293389
0,774789
0,518600
73° 8' 41"

0,293389
1,261705
0,031654 − 1
6° 8' 21"

301030
602060

1,965124
1,421340

1,2740904
23271
1988
1,2766163

5,1064552

0,1047198
23271
1018
0,1071487

2,334840
1,965124
4,299964

$\log \frac{3}{\sqrt{13}} \cdot \frac{c+\sqrt{16,861712}}{c+\sqrt{12,861712}}$

$\log \frac{\sqrt{13}}{3}$

0,477121
0,783290
1,260411
1,301379
−0,040968

0,556972
~~0,551454~~
0,744407

~~4,959032~~

$\sqrt{-4,116505}$
−0,184665

1,226901
0,613451
4,106300
1,965124
6,071424

1,109299
0,554650
3,586220
1,965124
1,113943

9.16,861712

$\log \frac{\sqrt{1,020217}}{\sqrt{4,451472}} \cdot \frac{4,299964}{3,386464}$

0,008694
0,648503
15,446848
151,755408

0,004347
0,633464
0,637811
0,853998
0,783813 − 1
−0,216187

0,324252
529746

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

$\frac{2 \cdot 25.861712}{167,202256}$

0,154673
= 1,215806 pr $\Delta a$

$\frac{1}{5} \cdot \frac{1}{\phantom{x}}$

4484

0,698970
0,473759
1,172729

1,230449
0,659675
1,890124

0,827271 − 2
0,602060
0,292389
0,722720 − 1
− 0,528105

0,109876 − 2
0,602060
0,293389
0,005325 − 1
+ 0,101234.

$$\frac{\partial Y}{\partial \alpha} = + \frac{l(a\sin\alpha - b\cos\alpha)}{\sqrt{a^2+b^2+l^2-2l(a\cos\alpha+b\sin\alpha)}} - \frac{l(a\sin\alpha+b\cos\alpha)}{\sqrt{a^2+b^2+l^2-2l(a\cos\alpha-b\sin\alpha)}}$$

$$- \frac{l(a\sin\alpha-b\cos\alpha)}{c+\sqrt{a^2+b^2+l^2+c^2-2l(a\cos\alpha+b\sin\alpha)}}\,\frac{1}{\sqrt{\phantom{x}}} + \frac{l(a\sin\alpha+b\cos\alpha)}{c+\sqrt{a^2+b^2+l^2+c^2-2l(a\cos\alpha-b\sin\alpha)}}\cdot\frac{1}{\sqrt{\phantom{x}}}$$

298,2

$\boxed{b = 149,1}$

127,9

$$\left(\frac{\partial Y}{\partial \alpha}\right)_{\alpha=0} = lb\left\{-\frac{2}{a^2+b^2+l^2-2la} + \frac{2}{c+\sqrt{a^2+b^2+l^2+c^2-2la}}\,\frac{1}{\sqrt{\phantom{x}}}\right\} = -\frac{2c}{a^2+b^2+l^2-2la}\,\frac{1}{\sqrt{a^2+b^2+l^2+c^2-2la}}$$

255,8

$$\left(\frac{\partial Y}{\partial \alpha}\right)_{\alpha=\pi} = lb\left\{+\frac{2}{a^2+b^2+l^2+2la} - \frac{2}{c+\sqrt{a^2+b^2+l^2+c^2+2la}}\,\frac{1}{\sqrt{\phantom{x}}}\right\} = +\frac{2c}{a^2+b^2+l^2+2la}\,\frac{1}{\sqrt{a^2+b^2+l^2+c^2+2la}}$$

$$\left(\frac{\partial Y}{\partial \alpha}\right)_{\alpha=\frac{\pi}{2}} = la\left\{\frac{1}{a^2+b^2+l^2-2lb} - \frac{1}{a^2+b^2+l^2+2lb} - \frac{1}{c+\sqrt{a^2+b^2+l^2+c^2-2lb}}\,\frac{1}{\sqrt{\phantom{x}}} + \frac{1}{c+\sqrt{a^2+b^2+l^2+c^2+2lb}}\,\frac{1}{\sqrt{\phantom{x}}}\right\}$$

$$\left(\frac{\partial Y}{\partial \alpha}\right)_{\alpha=-\frac{\pi}{2}} = la\left\{-\frac{1}{a^2+b^2+l^2+2lb} + \frac{1}{a^2+b^2+l^2-2lb} + \frac{1}{c+\sqrt{a^2+b^2+l^2+c^2+2lb}}\,\frac{1}{\sqrt{\phantom{x}}} - \frac{1}{c+\sqrt{a^2+b^2+l^2+c^2-2lb}}\,\frac{1}{\sqrt{\phantom{x}}}\right\}$$

833
796
37

$$\frac{\partial L_1 bc}{\partial x} = \operatorname{arctg} \frac{bc}{a\sqrt{a^2+b^2+c^2}} \qquad \frac{\partial L_1 bc}{\partial y} = \log \sqrt{1+\frac{a^2}{c^2}}\,\frac{1+\sqrt{1+\frac{b^2}{c^2}}}{1+\sqrt{1+\frac{a^2}{c^2}+\frac{b^2}{c^2}}}$$

$$\frac{\partial L_1 ac}{\partial x} = \log \sqrt{1+\frac{b^2}{a^2}}\,\frac{1+\sqrt{1+\frac{a^2}{c^2}}}{1+\sqrt{1+\frac{a^2}{c^2}+\frac{b^2}{c^2}}} \qquad \frac{\partial L_1 ac}{\partial y} = \operatorname{arctg} \frac{ac}{b\sqrt{a^2+b^2+c^2}}$$

$$\frac{\partial L_1 ab}{\partial x} = \log \sqrt{1+\frac{c^2}{a^2}}\,\frac{1+\sqrt{1+\frac{a^2}{b^2}}}{1+\sqrt{1+\frac{a^2}{b^2}+\frac{c^2}{b^2}}} \qquad \frac{\partial L_1 ab}{\partial y} = \operatorname{arctg} \frac{ab}{c\sqrt{a^2+b^2+c^2}}$$

$$\frac{\partial L_1 bc}{\partial x^2} = + \cancel{\frac{bc\,(2a^2+b^2+c^2)}{a\sqrt{M}}} + \frac{bc}{(a^2+c^2)\sqrt{M}} + \frac{bc}{(a^2+b^2)\sqrt{M}}$$

$$\frac{\partial L_1 bc}{\partial x\,\partial y} = - \frac{ac}{(a^2+b^2)\sqrt{M}}$$

$$\frac{\partial L_1 bc}{\partial y^2} = - \frac{bc}{(a^2+b^2)\sqrt{M}}$$

$$\frac{\partial L_1 ac}{\partial x^2} = - \frac{ac}{(a^2+b^2)\sqrt{M}}$$

$$\frac{\partial L_1 ac}{\partial x\,\partial y} = - \frac{bc}{(a^2+b^2)\sqrt{M}}$$

$$\frac{\partial L_1 ac}{\partial y^2} = + \frac{ac}{(b^2+c^2)\sqrt{M}} + \frac{ac}{(a^2+b^2)\sqrt{M}}.$$

$$\frac{\partial L_1 ab}{\partial x^2} = - \frac{ab}{(a^2+c^2)\sqrt{M}}$$

$$\frac{\partial L_1 ab}{\partial x\,\partial y} = - \frac{bc}{(a^2+c^2)\sqrt{\ }} \qquad - \frac{bc}{(a^2+c^2)\sqrt{M}}$$

$$\frac{\partial L_1 ab}{\partial y^2} = + \frac{ab}{(b^2+c^2)\sqrt{M}} + \frac{ab}{(a^2+c^2)\sqrt{\ }} \qquad - \frac{ac}{(b^2+c^2)\sqrt{M}}$$

$(b - l \sin\alpha)(a - l\cos\alpha)(a\sin\alpha - b\cos\alpha)$

$ab - al\sin\alpha - bl\cos\alpha + l^2 \sin\alpha\cos\alpha$

$\sin\alpha\,[a^2+b^2+c^2+l^2 - 2l(a\cos\alpha + b\sin\alpha)] + (a - l\cos\alpha)\{(a\sin\alpha - b\cos\alpha)$

$- a\cos\alpha + l\sin^2\alpha - b\sin\alpha + l\sin^2\alpha$

$- (a\cos\alpha + b\sin\alpha) + l\,]\,\}$ $+ a^2\sin\alpha$ ... $\sin\alpha\cos\alpha$

0,933393 − 1

2

0,866786 − 1
0,291642

0,158428

+1,44022
0,006156
0,000066

+1,446442
0 000774

+1,445668

4

0,733572 − 1
0,055760 − 2

0,789332 − 3
0

0,733572 − 1
0,524015 − 1

0,257587 − 1

0,180962
160 18

0,180980
140

+0,180840

6

0,600358 − 1
0,288249 − 3

0,888607 − 4

0,600358 − 1
0,546543 − 4

0,146901 − 4

2,891336
723360

3,614696
0 322772

3,291924

8

0,467144 − 1
0,352183 − 4

0,819327 − 5

0,467144 − 1
0,792392 − 5

0,259536 − 5

~~0,22~~

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

− 2,891336
83768

− 2,975104
962364

− 2,012740
3 291924

− 5,304664

10,609328

+0,723360
0,239004

− 10,391908

$+\frac{1}{2}\cdot 0{,}586402 + \frac{1}{24}\,0{,}123252 + \frac{1}{768}\,0{,}023413 - \frac{1}{46080}\,0{,}223227.$

$+\frac{1}{48}\,1{,}489734 + \frac{1}{960}\,0{,}225627 - \frac{1}{46080}\cdot \cancel{0{,}276703}\ 0{,}236703$

$+\frac{1}{3840}\,9{,}288101 + \frac{1}{107520}\,1{,}723413$

$+\frac{1}{645120}\cdot 89{,}028473.$

1,187511
5,565676
6,747187
− 6,983890
− 0,236703

+0,602456
−0,579043
+0,023413

+0,020103
−0,243330
−0,223227

1489734
+0,415428
−1,074006

−~~0,121665~~.

+0,602456 0,579043
8,685645
+9,288101

+ ~~0,124862~~
~~+0,210735~~
+0,335627

− 0,043147

+1,181511
+5,525676
+6,707187
−6,983890
−0,276703

1,381419

+ 0,020103
1,703310
1,723413

0,121665

~~11~~
+1,181511
38,959702
48,887260
89,028473

1,291419

$$\frac{1}{2}\left(\frac{\partial^2 V}{\partial x^2}-\frac{\partial^2 V}{\partial y^2}\right)=+1{,}957230$$

$$\frac{1}{24}\left(\frac{\partial^4 V}{\partial x^4}-\frac{\partial^4 V}{\partial y^4}\right)=+0{,}169726$$

$$\left[\frac{1}{768}(\quad)+\quad\right]=-0{,}001942$$

$$\left[\frac{1}{46080}(\quad)\quad\right]=+0{,}000225$$

---

$$\frac{1}{48}\left[\quad\right]=-0{,}220037$$

$$\left[\frac{1}{960}(\quad)\;''\right]=-0{,}000352$$

$$\left[\frac{1}{46080}\quad\right]=+0{,}000062$$

---

$$\left[\frac{1}{3840}\quad\right]=-0{,}099952$$

$$\left[\frac{1}{107520}\quad\right]=-0{,}000035$$

---

$$-0{,}035715$$

$$\frac{\partial X}{\partial x}=x \qquad \frac{\partial Y}{\partial y}=y \qquad \frac{\partial X}{\partial y}=z \qquad \frac{\partial Z}{\partial x}=u \qquad \frac{\partial Z}{\partial y}=v \qquad Z=w$$

0° $\frac{\cancel{3}k_1+\cancel{7}k_2}{2}\,l\,X.x - \frac{k_1-k_2}{2}\,lXy + (k_1+3k_2)\,l\,Y.z + (k_1+k_2)\,l\,u.w \cancel{=A} + (k_1-k_2)Xw = 2A$

90° $\frac{3k_1+5k_2}{2}\,l\,X\,x + \frac{k_1-k_2}{2}\,lXy + (k_1+3k_2)\,l\,Y\,z + (k_1+k_2)\,l\,u\,w + (k_1-k_2)\,Yw = 2A'$

0° $-\frac{k_1-k_2}{2}\,lYx + \frac{k_1+7k_2}{2}\,l\,Y\,y + (k_1+3k_2)\,l\,X\,z + (k_1-k_2)\,l\,v\,w + (k_1-k_2)\,Yw = 2B$

90° $+\frac{k_1-k_2}{2}\,l\,Y\,x + \frac{3k_1+5k_2}{2}\,l\,Y\,y + (k_1+3k_2)\,l\,X\,z + (k_1-k_2)\,l\,v\,w - (k_1-k_2)\,Xw = 2B'$

$$\Big\}\ (k_1-k_2)\,l\,Xx - \cancel{lYy}$$

$$l\,Xx + lYy + (Y-X)\,w = G$$

$$l\,Yx + l\,Yy - (Y+X)\,w = H$$

$$l\,(x+y)\,X + (Y-X)\,w = G$$

$$l\,(x+y)\,Y - (Y+X)\,w = H.$$

$\underline{(x+y)}$ $\underline{w}$,

$u, v$

$(x-y)$ $z$

$$+\cancel{w}x - wy + Xu - Yv = a$$

$$+2\cancel{w}z + X\cancel{v} + Yu = b$$

---

$$Xu + Yv + \cancel{w}x + \cancel{w}y = C$$

$$Xu + Yv + w(x+y) = C$$

$$Xv - Yu = D$$

$$Xx - Xy - 2Yz = E$$

$$Yx - Yy + 2XZ = F$$

$\lambda = 0$ | $\lambda = 90$

$$\sin\alpha \left| \frac{k_1-k_2}{2} XZ + \frac{k_1+3k_2}{2} l\left[X\frac{\partial X}{\partial x} + Y\frac{\partial Y}{\partial x}\right] + \frac{k_1-k_2}{2}\frac{l}{2} X\frac{\partial Z}{\partial z} + \frac{k_1+k_2}{2} l Z \frac{\partial Z}{\partial x} = A \right.$$

$$+\frac{k_1-k_2}{2} YZ + \frac{k_1+3k_2}{2} l\left[X\frac{\partial X}{\partial x} + Y\frac{\partial Y}{\partial x}\right] - \frac{k_1-k_2}{2}\frac{l}{2} X\frac{\partial Z}{\partial z} + \frac{k_1+k_2}{2} l Z \frac{\partial Z}{\partial x} = A'$$

$$\cos\alpha \left| \frac{k_1-k_2}{2} YZ + \frac{k_1+3k_2}{2} l\left[X\frac{\partial X}{\partial y} + Y\frac{\partial Y}{\partial y}\right] + \frac{k_1-k_2}{2}\frac{l}{2} Y\frac{\partial Z}{\partial z} + \frac{k_1+k_2}{2} l Z \frac{\partial Z}{\partial y} = B \right.$$

$$-\frac{k_1-k_2}{2} XZ + \frac{k_1+3k_2}{2} l\left[X\frac{\partial X}{\partial y} + Y\frac{\partial Y}{\partial y}\right] - \frac{k_1-k_2}{2}\frac{l}{2} Y\frac{\partial Z}{\partial z} + \frac{k_1+k_2}{2} l Z \frac{\partial Z}{\partial y} = B'$$

$a$

$$\sin 2\alpha \left| -\frac{k_1-k_2}{2}(Y^2-X^2) + \frac{k_1-k_2}{2}\cdot\frac{l}{2}\left[Z\left(\frac{\partial X}{\partial x} - \frac{\partial Y}{\partial y}\right) + X\frac{\partial Z}{\partial x} - Y\frac{\partial Z}{\partial y}\right] = C \right.$$

$b$

$$+(Y^2-X^2)\frac{k_1-k_2}{2} + \frac{k_1-k_2}{2}\frac{l}{2}\left(X\frac{\partial Z}{\partial y} + Y\frac{\partial Z}{\partial x} + 2Z\frac{\partial X}{\partial y}\right) = C'$$

$$\cos 2\alpha \left| +\frac{k_1-k_2}{2} 2XY + \frac{k_1-k_2}{2}\frac{l}{2}\left(X\frac{\partial Z}{\partial y} + Y\frac{\partial Z}{\partial x} + 2Z\frac{\partial X}{\partial y}\right) = D \right.$$

$$-\frac{k_1-k_2}{2} 2XY - \frac{k_1-k_2}{2}\frac{l}{2}\left[Z\left(\frac{\partial X}{\partial x} - \frac{\partial Y}{\partial y}\right) + X\frac{\partial Z}{\partial x} - Y\frac{\partial Z}{\partial y}\right] = D'$$

$$C + D = -\frac{(k_1-k_2)}{2}(Y^2-X^2) - \frac{k_1-k_2}{2} 2XY$$

$$D + C' = +\frac{k_1-k_2}{2}(Y^2-X^2) - \frac{k_1-k_2}{2} 2XY$$

$$Y^2 - X^2 = a$$

$$XY = b.$$

$$y = \frac{b}{x}$$

$$C + C' = a + b$$

$$D + D' = b - a$$

az utóbbi párokat ismerem. $(y^2-x^2)$ és $xy$ tehát $X, Y$

és

$$\begin{cases} Z\left(\frac{\partial X}{\partial x} - \frac{\partial Y}{\partial y}\right) + X\frac{\partial Z}{\partial x} - Y\frac{\partial Z}{\partial y} = a \\ X\frac{\partial Z}{\partial y} + Y\frac{\partial Z}{\partial x} + 2Z\frac{\partial X}{\partial y} = b. \end{cases}$$

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

$$a_{23}w\Big[-yZ\sin\alpha - XZ\cos\alpha + \left(-Z\frac{\partial y}{\partial x} - y\frac{\partial Z}{\partial x} + Z\frac{\partial X}{\partial y} - X\frac{\partial Z}{\partial y}\right)\frac{l}{2}\sin 2\alpha$$

$$+ \left(Z\frac{\partial X}{\partial x} + X\frac{\partial Z}{\partial x}\right) l\sin^2\alpha + \left(Z\frac{\partial y}{\partial y} + y\frac{\partial Z}{\partial y}\right) l\cos^2\alpha$$

---

$$a_{31}w\Big[-XZ\sin\alpha + yZ\cos\alpha + \left(Z\frac{\partial y}{\partial y} + y\frac{\partial Z}{\partial y} - Z\frac{\partial X}{\partial x} - X\frac{\partial Z}{\partial x}\right)\frac{l}{2}\sin 2\alpha$$

$$-\left(Z\frac{\partial y}{\partial x} + y\frac{\partial Z}{\partial x}\right)\sin^2\alpha + \left(Z\frac{\partial y}{\partial x} + X\frac{\partial Z}{\partial y}\right)\cos^2\alpha\Big]$$

$\cos^2\alpha = \frac{1}{2} + \frac{1}{2}\cos 2\alpha$

$$F = A_1\sin\alpha + B_1\cos\alpha + A_2\sin 2\alpha + B_2\cos 2\alpha + A_3\sin 3\alpha + B_3\cos 3\alpha$$
$$+ C\sin^2\alpha + D\cos^2\alpha.$$

$\alpha$

1 0 $+B_1 + B_2 + B_3 + D$

2 45 $+\frac{1}{\sqrt2}A_1 + \frac{1}{\sqrt2}B_1 + A_2 + \frac{1}{\sqrt2}A_3 - \frac{1}{\sqrt2}B_3 + \frac{1}{2}C + \frac{1}{2}D$

3 90 $+A_1 - B_2 - A_3 + C$

4 135 $+\frac{1}{\sqrt2}A_1 - \frac{1}{\sqrt2}B_1 - A_2 + \frac{1}{\sqrt2}A_3 + \frac{1}{\sqrt2}B_3 + \frac{1}{2}C + \frac{1}{2}D$

5 180 $-B_1 + B_2 - B_3 + D$

6 225 $-\frac{1}{\sqrt2}A_1 - \frac{1}{\sqrt2}B_1 + A_2 - \frac{1}{\sqrt2}A_3 + \frac{1}{\sqrt2}B_3 + \frac{1}{2}C + \frac{1}{2}D$

7 270 $-A_1 - B_2 + A_3 + C$

8 315 $-\frac{1}{\sqrt2}A_1 + \frac{1}{\sqrt2}B_1 - A_2 + \frac{1}{\sqrt2}A_3 - \frac{1}{\sqrt2}B_3 + \frac{1}{2}C + \frac{1}{2}D$

$$\Sigma = 4C + 4D$$

$n_1 + n_5 = 2B_2 + 2D$

$n_1 - n_5 = 2B_1 + 2B_3$

$n_3 + n_7 = -2B_2 + 2C$

$n_3 - n_7 = 2A_1 - 2A_3$

$(n_2 + n_6) + (n_4 + n_8) = 2C + 2D$

$(n_2 + n_6) - (n_4 + n_8) = +4A_2$

$(n_2 - n_6) + (n_4 - n_8) = 2\sqrt2 A_1 + 2\sqrt2 A_3$

$(n_2 - n_6) - (n_4 - n_8) = 2\sqrt2 B_1 - 2\sqrt2 B_3$

$n_2 + n_6 = +2A_2 + C + D$

$n_2 - n_6 = +\sqrt2 A_1 + \sqrt2 B_1 + \sqrt2 A_3 - \sqrt2 B_3$

$n_4 + n_8 = -2A_2 + C + D$

$n_4 - n_8 = +\sqrt2 A_1 - \sqrt2 B_1 + \sqrt2 A_3 + \sqrt2 B_3$

Ha a vörös $\Sigma^*_j$-ben $y=0$ $\frac{\partial Z}{\partial y}=0$ $\frac{\partial X}{\partial y}=0$ akkor.

$$\frac{F_1}{w} = a_{23}\frac{l}{2}\left(X\frac{\partial Z}{\partial x} - Z\frac{\partial Z}{\partial z}\right)$$

$$-\sin\alpha\left\{a_{31}XZ + \frac{a_{11}+a_{22}}{2}lX\frac{\partial X}{\partial x} + \frac{a_{11}-a_{22}}{2}\frac{l}{2}X\frac{\partial Z}{\partial z} + a_{33}lZ\frac{\partial Z}{\partial x}\right\}$$

$$+\cos\alpha\left\{-a_{23}XZ + \ldots - a_{12}\frac{l}{2}X\frac{\partial Z}{\partial z}\right\}$$

$$-\sin 2\alpha\left\{\frac{a_{11}-a_{22}}{2}X^2 + a_{31}\frac{l}{2}\left[Z\left(\frac{\partial X}{\partial x}-\frac{\partial y}{\partial y}\right) + X\frac{\partial Z}{\partial x}\right]\right\}$$

$$+\cos 2\alpha\left\{-a_{12}X^2 - a_{23}\frac{l}{2}\left(Z\frac{\partial X}{\partial x} + X\frac{\partial Z}{\partial x}\right)\right.$$

$$-\sin 3\alpha\left\{\frac{a_{11}-a_{22}}{2}\frac{l}{2}X\left(\frac{\partial X}{\partial x}-\frac{\partial y}{\partial y}\right)\right\}$$

$$+\cos 3\alpha\left\{-a_{12}\frac{l}{2}X\left(\frac{\partial X}{\partial x}-\frac{\partial y}{\partial y}\right)\right.$$

---

$K_1$ X felé

$a_{11} = K_1$
$a_{22} = K_2$
$a_{33} = K_2$
$a_{12} = 0$
$a_{23} = 0$
$a_{31} = 0$

~~$X, y$~~

~~$\frac{\partial X}{\partial y}$ $\frac{\partial X}{\partial x}$ $\frac{\partial y}{\partial y}$~~ $\varphi = 0$

$\frac{a_{11}-a_{22}}{2} = \frac{K_1-K_2}{2}$

$\frac{a_{11}+a_{22}}{2} = \frac{K_1+K_2}{2}$

$a_{33} = K_2$

$K_1$ Z felé

$a_{11} = K_2$
$a_{22} = K_2$
$a_{33} = K_1$
$a_{12} = 0$
$a_{23} = 0$
$a_{31} = 0$

$\frac{a_{11}-a_{22}}{2} = 0$

$\frac{a_{11}+a_{22}}{2} = K_2$

$a_{33} = K_1$

$K_1$ horizontalis

$a_{11} = K_1\cos^2\varphi + K_2\sin^2\varphi$
$a_{22} = K_1\sin^2\varphi + K_2\cos^2\varphi$
$a_{33} = K_2$
$a_{12} = (K_1-K_2)\frac{\sin 2\varphi}{2}$
$a_{23} = 0$
$a_{31} = 0$

$\lambda = 90^\circ$

$\frac{a_{11}-a_{22}}{2} = -\frac{K_1-K_2}{2}$

$\frac{a_{11}+a_{22}}{2} = \frac{K_1+K_2}{2}$

$a_{33} = K_2$

$\lambda = 0$ $\varphi = 45$

$a_{11} = \frac{K_1+K_2}{2}$
$a_{22} = K_2$
$a_{33} = \frac{K_1+K_2}{2}$
$a_{12} = 0$
$a_{23} = 0$
$a_{31} = \frac{K_1-K_2}{2}$

$\frac{a_{11}+a_{22}}{2} = +\frac{K_1+3K_2}{2}$

$\frac{a_{11}-a_{22}}{2} = +\frac{K_1-K_2}{2}$

$a_{33} = +\frac{K_1+K_2}{2}$

$a_{12} = 0$

$a_{23} = 0$

$a_{31} = \frac{K_1-K_2}{2}$

$\lambda = \frac{\pi}{2}$ $\varphi = 45$

$a_{11} = K_2$
$a_{22} = \frac{K_1+K_2}{2}$
$a_{33} = \frac{K_1+K_2}{2}$
$a_{12} = 0$
$a_{23} = \frac{K_1-K_2}{2}$
$a_{31} = 0$

$+\frac{K_1+3K_2}{2}$

$-\frac{K_1-K_2}{2}$

$+\frac{K_1+K_2}{2}$

$0$

$a_{23} = \frac{K_1-K_2}{2}$

$a_{31} = 0$

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

$X$ $y$ $Z$ $\frac{\partial X}{\partial x}$ $\frac{\partial X}{\partial y}$ $\frac{\partial X}{\partial z}$ $\frac{\partial y}{\partial y}$ $\frac{\partial y}{\partial z}$

$\varphi = 0 \qquad \lambda = 0$

$$\frac{a_{11}+a_{22}}{2} l \left[ X \frac{\partial X}{\partial x} + Y \frac{\partial Y}{\partial x} \right] + \frac{a_{11}-a_{22}}{2} \frac{l}{2} X \frac{\partial Z}{\partial z} + a_{33} l Z \frac{\partial Z}{\partial x} = A$$

$$\frac{a_{11}+a_{22}}{2} l \left[ X \frac{\partial X}{\partial y} + Y \frac{\partial Y}{\partial y} \right] + \frac{a_{11}-a_{22}}{2} \frac{l}{2} Y \frac{\partial Z}{\partial z} + a_{33} l Z \frac{\partial Z}{\partial y} = B,$$

$$\left. \begin{aligned} \frac{a_{11}-a_{22}}{2}(Y^2-X^2) &= C \\ \frac{a_{11}-a_{22}}{2}(XY) &= D \end{aligned} \right\} \quad X, Y$$

$$\left. \begin{aligned} \frac{a_{11}-a_{22}}{2} \frac{l}{2} \left[ X \left( \frac{\partial X}{\partial x} - \frac{\partial Y}{\partial y} \right) - 2Y \frac{\partial X}{\partial y} \right] &= E \\ \frac{a_{11}-a_{22}}{2} \frac{l}{2} \left[ Y \left( \frac{\partial X}{\partial x} - \frac{\partial Y}{\partial y} \right) + 2X \frac{\partial X}{\partial y} \right] & F. \end{aligned} \right\} \quad \frac{\partial X}{\partial x} - \frac{\partial Y}{\partial y} \qquad \frac{\partial X}{\partial y}$$

$\varphi = 0 \qquad \lambda = 90^\circ$

$$X \frac{\partial X}{\partial x} + X \frac{\partial Y}{\partial y} + Y \frac{\partial X}{\partial x} + Y \frac{\partial Y}{\partial y} = A - B$$

$$\left. \begin{aligned} X \frac{\partial X}{\partial x} - X \frac{\partial Y}{\partial y} - 2Y \frac{\partial X}{\partial y} &= E \\ Y \frac{\partial X}{\partial x} - Y \frac{\partial Y}{\partial y} + 2 \cancel{X} \frac{\partial X}{\partial y} &= F \end{aligned} \right\}$$

$$2(X^2 + Y^2) \frac{\partial X}{\partial y} = FX - EY.$$

$$2XY \frac{\partial X}{\partial x} - 2XY \frac{\partial Y}{\partial y} = FX + EY$$

$$(X+Y) \frac{\partial X}{\partial x} + (X+Y) \frac{\partial Y}{\partial y} = A - B$$

$$X, \; Y, \; \frac{\partial X}{\partial x} \; \frac{\partial X}{\partial y} \; \frac{\partial Y}{\partial y}$$

$\cancel{\sin 60^\circ \tfrac{1}{2} \quad \cos 60^\circ = \tfrac{\sqrt{3}}{2}}$

$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \cos 60 = \frac{1}{2}$

$$\frac{F_1}{w} = A_1 \sin\alpha + B_1 \cos\alpha + A_2 \sin 2\alpha + B_2 \cos 2\alpha + A_3 \sin 3\alpha + B_3 \cos 3\alpha$$

| | $\alpha$ | |
|---|---|---|
| 1 | $0^\circ$ | $B_1 + B_2 + B_3$ |
| 2 | $60^\circ$ | $\frac{\sqrt{3}}{2} A_1 + \frac{1}{2} B_1 + \frac{\sqrt{3}}{2} A_2 - \frac{1}{2} B_2 - B_3$ |
| 3 | $120^\circ$ | $\frac{\sqrt{3}}{2} A_1 - \frac{1}{2} B_1 - \frac{\sqrt{3}}{2} A_2 - \frac{1}{2} B_2 + B_3$ |
| 4 | $180^\circ$ | $-B_1 + B_2 - B_3$ |
| 5 | $240^\circ$ | $-\frac{\sqrt{3}}{2} A_1 - \frac{B_1}{2} + \cancel{A_2 \sin 2\alpha} + \frac{\sqrt{3}}{2} A_2 - \frac{1}{2} B_2 + B_3$ |
| 6 | $300^\circ$ | $-\frac{\sqrt{3}}{2} A + \frac{1}{2} B_1 - \frac{\sqrt{3}}{2} A_2 - \frac{1}{2} B_2 - B_3$ |

$$C \frac{n_1 + n_4}{2} = B_2 \qquad C \frac{n_1 - n_4}{2} = B_1 + B_3$$

$C(n_2 + n_5$

$$c.(n_1 + n_4) = 2 B_2$$
$$c(n_2 + n_5) = \cancel{\phantom{xx}} + \sqrt{3} A_2 - B_2$$
$$C(n_3 + n_6) = -\sqrt{3} A_2 - B_2$$

$$C(n_1 - n_4) = 2(B_1 + B_3)$$
$$C(n_2 - n_5) = \sqrt{3} A_1 + B_1 - 2 B_3$$
$$C(n_3 - n_6) = \sqrt{3} A_1 - B_1 + 2 B_3$$

$$A_1 = C \frac{(n_2 - n_5) + (n_3 - n_6)}{2\sqrt{3}}$$

$$A_2 = C \frac{(n_2 + n_5) - (n_3 + n_6)}{2\sqrt{3}}$$

$$B_1 = C \frac{2(n_1 - n_4) + (n_2 - n_5) - (n_3 - n_6)}{6}$$

$$B_2 = C \frac{n_1 + n_4}{2}$$

$$B_3 = C \frac{(n_1 - n_4) + (n_3 - n_6) - (n_2 - n_5)}{6}$$

$$C(n_2 - n_5) - C(n_3 - n_6) = +2B_1 - 4B_3$$
$$2C(n_1 - n_4) = 4B_1 + 4B_3$$
$$C(n_1 - n_4) = 2B_1 + 2B_3$$

$n_2 +$

$\alpha = 90^\circ \quad n - n_0 = A_1 - A_3 \qquad 30^\circ$

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$a_1 = 5 \quad a_2 = 20 \quad R_1 = 0 \quad R_2 = 4$

$$\log \frac{4+\sqrt{416}}{20} = \log \frac{24{,}3961}{20} \qquad \log \frac{4+\sqrt{41}}{5} = \frac{10{,}4032}{5}$$

---

$a_1 = 5{,}5 \quad a_2 = 20{,}5 \quad R_1 = 0 \quad R_2 = 4.$

$$\log \frac{4+\sqrt{436{,}25}}{20{,}5} = \frac{24{,}8866}{20{,}5} \qquad \log \frac{4+\sqrt{46{,}25}}{5{,}5} = \frac{10{,}8007}{5{,}5}$$

---

$a_1 = 4{,}5 \quad a_2 = 19{,}5 \quad R_1 = 0 \quad R_2 = 4$

$$\log \frac{4+\sqrt{396{,}25}}{19{,}5} = \frac{23{,}9060}{19{,}5} \qquad \log \frac{4+\sqrt{36{,}25}}{4{,}5} = \frac{10{,}02080}{4{,}5}$$

---

$$+a_{23}\,\omega\Big\{\cancel{-Xz} - z\frac{\partial y}{\partial x} l \sin\alpha \overset{\mathrm{I}}{+} z\frac{\partial y}{\partial y} l\cos\alpha - y\frac{\partial z}{\partial x} l \sin\alpha + y\frac{\partial z}{\partial y} l\cos\alpha\Big)\cos\alpha$$

$$\overset{a_{23}\omega}{+}\Big\{-yz + z\frac{\partial x}{\partial x} l\sin\alpha \overset{\mathrm{II}}{-} z\frac{\partial x}{\partial y} l\cos\alpha + x\frac{\partial z}{\partial x} l\sin\alpha - x\frac{\partial z}{\partial y} l\cos\alpha\Big)\sin\alpha.$$

---

$$+a_{31}\,\omega\{-\mathrm{II}\}\cos\alpha + \{\mathrm{I}\}\sin\alpha.$$

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

---

| | | | |
|---|---|---|---|
| 1,219851 | | 2,08064 | |
| 0,086293 | 2,639735 | 0,318195 | 1,665112 |
| ~~0,362216~~ | 1,319868 | 0,362216 | 0,832556 |
| 0,935981 −2 | 20,8866 | 0,502694 −1 | 6,80071 |
| 0,298197 −1 | | 0,864910 −1 | 1,559208 |
| 0,198700 | 2,597969 | 0,702671 | 0,779654 |
| 8 397400 | 1,298985 | | 6,02080 |
| 3,66336 | 19,9060 | 63227 | |
| 0,31064 | | 31613 | |

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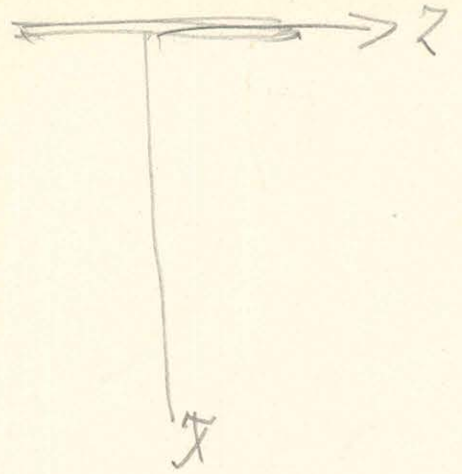
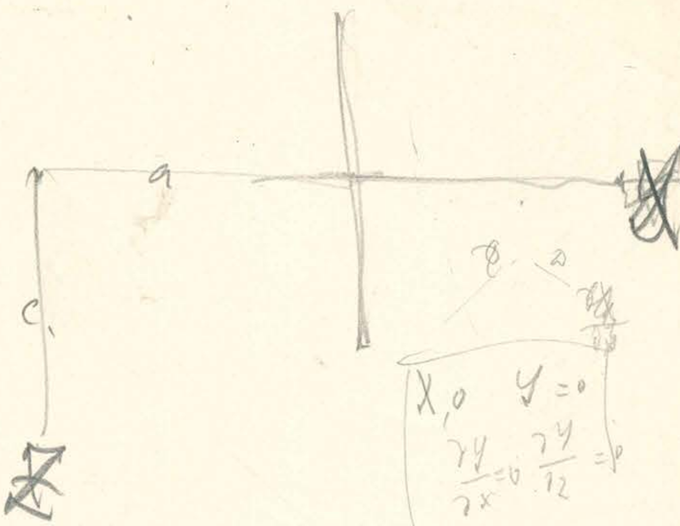
| | | | |
|---|---|---|---|
| 1,395965 | 1,033452 | 1,378489 | 1,000903 |
| 1,311754 | 0,740263 | 1,290035 | 0,653213 |
| 0,084211 | 0,293089 | 0,088454 | 0,347690 |
| 0,925369 −2 | 0,466997 −1 | 946717 −2 | 541192 −1 |
| ~~0,121397~~ | ~~0,196376~~ | ~~0,122590~~ | ~~0,222634~~ |
| 362216 | 362216 | 362216 | 362216 |
| ~~0,483615 −2~~ | ~~0,558592 −1~~ | ~~0,484806 −2~~ | 0,903408 −1 |
| 0,287585 −1 | 87 | ~~43 1 528~~ | |
| | 0,829213 −1 | 0,308933 −1 | |
| 0,192903 | 0,674859 | 0,202673 | 0,800585 |
| 0,287585 −1 | 0,829213 −1 | 0,308933 −1 | 0,903408 −1 |
| 1,311754 | 0,740263 | 1,290035 | 0,653213 |
| 0,599339 | 0,569576 | 0,598968 | 0,556621 |
| 3,97502 | 3,71173 | 3,97162 | 3,60264 |
| 71173 0,26329 | | 60264 | |

36898




$X = 0 \quad Y = 0$

$\frac{\partial Y}{\partial x} = 0 \quad \frac{\partial Y}{\partial z} = 0$

$\chi(\varrho, z) = \varphi.$

$M_x = df\,i \qquad M_x = \varrho\, d\varrho\, d\varphi$

$\dim \frac{\partial X}{\partial z} = -\frac{3c}{r^5} + \frac{15a^2c}{r^7} \qquad a = c. \qquad c = \varrho\cos\varphi \qquad b = \varrho\sin\varphi$

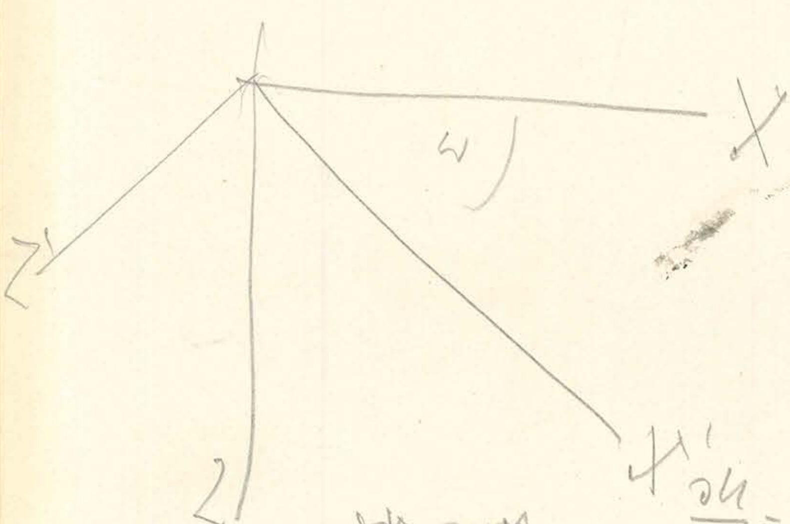
$r^2 = a^2 + b^2 + c^2$

$a_{23} \quad a_{33} \quad a_{31}$

$a_{23} \quad (a_{11} + a_{22}) \quad a_{33}$

$a_{11} - a_{22} \quad a_{12}$

$a_{31}$




$x' = x\cos\omega + z\sin\omega$

$z' = -x\sin\omega + z\cos\omega$

$\cancel{X = X'\cos\omega - Z'\sin\omega}$

$Z = \cancel{X'\sin\omega} + Z'\cos\omega$

$\cancel{\frac{\partial X}{\partial x} = \frac{\partial X}{\partial x'}}$

$X' \quad \frac{\partial u}{\partial x} = \frac{\partial u}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial u}{\partial z'}\frac{\partial z'}{\partial x} \qquad X = X'\cos\omega - Z'\sin\omega$

$\frac{\partial x'}{\partial x} = \cos\omega \qquad \frac{\partial z'}{\partial x} = -\sin\omega$

$\frac{\partial u}{\partial z} = \frac{\partial u}{\partial x'}\frac{\partial x'}{\partial z} + \frac{\partial u}{\partial y'}\frac{\partial y'}{\partial z} \qquad Z = X'\sin\omega + Z'\cos\omega$

$\frac{\partial x'}{\partial z} = \sin\omega \qquad \frac{\partial z'}{\partial z} = \cos\omega$

$\frac{\partial^2 u}{\partial x^2} \qquad \frac{\partial^2 u}{\partial z^2}$

$\frac{\partial^2 u}{\partial x \partial z}.$

---

$\cancel{\frac{\partial X}{\partial x} = \frac{\partial X}{\partial z}} \qquad \cancel{\frac{\partial X}{\partial}} = \frac{\partial X'}{\partial z}$

$\cancel{\frac{\partial^2 u}{\partial x^2}} =$

$\frac{\partial X}{\partial x} = \frac{\partial X'}{\partial x'}\frac{\partial x'}{\partial x}\cos\omega + \frac{\partial X'}{\partial z'}\frac{\partial z'}{\partial x}\cos\omega - \frac{\partial Z'}{\partial x'}\frac{\partial x'}{\partial x}\sin\omega - \frac{\partial Z'}{\partial z'}\frac{\partial z'}{\partial x}\sin\omega$

$\frac{\partial X}{\partial x} = \frac{\partial X'}{\partial x'}\cos^2\omega - \frac{1}{2}\frac{\partial X'}{\partial z'}\sin 2\alpha - \frac{1}{2}\frac{\partial Z'}{\partial x'}\sin 2\alpha + \frac{\partial Z'}{\partial z'}\sin^2\omega$

$\frac{\partial Z}{\partial z} = \frac{\partial X'}{\partial x'}\sin^2\omega + \frac{1}{2}\frac{\partial X'}{\partial z'}\sin 2\omega + \frac{\partial Z'}{\partial z'}\cos^2\omega + \frac{1}{2}\frac{\partial Z'}{\partial x'}\sin 2\alpha$

$\frac{\partial X}{\partial z} = \frac{1}{2}\frac{\partial X'}{\partial x'}\sin 2\omega + \frac{\partial X'}{\partial z'}\cos^2\omega - \frac{\partial Z'}{\partial x'}\sin^2\omega - \frac{1}{2}\frac{\partial Z'}{\partial z'}\sin 2\omega$

$\frac{\partial Z}{\partial x} = \frac{1}{2}\frac{\partial X'}{\partial x'}\sin 2\omega - \frac{\partial X'}{\partial z'}\sin^2\omega + \frac{\partial Z'}{\partial x'}\cos^2\omega - \frac{1}{2}\frac{\partial Z'}{\partial z'}\sin 2\omega$

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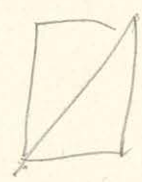
$$X = 2\pi i \int_{R_1}^{R_2}\int_{a_1}^{a_2} \frac{R^2}{(R^2+a^2)^{\frac{3}{2}}}\, da\, dR \qquad \frac{\partial X}{\partial x} = 6\pi i \int_{R_1}^{R_2}\int_{a_1}^{a_2} \frac{R^2 a}{(R^2+a^2)^{\frac{5}{2}}}\, da\, dR.$$

a konst.

$$X = 2\pi i \int_{R_1}^{R_2} R^2 dR \left| \frac{a}{R^2\sqrt{R^2+a^2}} \right|_{a_1}^{a_2}$$

a konst.

$$\frac{\partial X}{\partial x} = 6\pi i \int_{R_1}^{R_2} R^2 dR \left| \left(-\frac{1}{3(R^2+a^2)^{\frac{3}{2}}}\right) \right|_{a_1}^{a_2}$$



$\cancel{X = 2\pi i \int_{R_1}^{R_1}}$

$$X = 2\pi i\, a_2 \int \frac{dR}{\sqrt{R^2+a_2^2}} - 2\pi i\, a_1 \int \frac{dR}{\sqrt{R^2+a_1^2}}$$

$$X = 2\pi i\, a_2 \log \frac{R_2+\sqrt{R_2^2+a_2^2}}{R_1+\sqrt{R_1^2+a_2^2}} - 2\pi i\, a_1 \log \frac{R_2+\sqrt{R_2^2+a_1^2}}{R_1+\sqrt{R_1^2+a_1^2}}$$

$$\frac{\partial X}{\partial x} = 2\pi i \int \frac{R^2 dr}{(R^2+a_1^2)^{\frac{3}{2}}} - 2\pi i \int \frac{R^2 dr}{(R^2+a_2^2)^{\frac{3}{2}}}$$

$$\frac{\partial X}{\partial x} = 2\pi i \left\{ \frac{R_1}{\sqrt{R_1^2+a_1^2}} - \frac{R_2}{\sqrt{R_2^2+a_1^2}} + \frac{R_2}{\sqrt{R_2^2+a_2^2}} - \frac{R_1}{\sqrt{R_1^2+a_2^2}} \right\}$$

$$+ 2\pi i \log \frac{R_2+\sqrt{R_2^2+a_1^2}}{R_1+\sqrt{R_1^2+a_1^2}} - 2\pi i \log \frac{R_2+\sqrt{R_2^2+a_2^2}}{R_1+\sqrt{R_1^2+a_2^2}}$$

$a_1 = 2 \quad R_1 = 5$

$a_2 = 4 \quad R_2 = 10$

10 °C.

$\cancel{a = \frac{R}{7} = \frac{5}{7} \quad R_1 = 5 \quad R_1^2 + a_1^2 = 25 + \frac{25}{7} = \frac{200}{7}}$

$$X = 2\pi i \left\{ 4 \log \frac{10+\sqrt{116}}{5+\sqrt{41}} - 2 \log \frac{10+\sqrt{104}}{5+\sqrt{29}} \right\}$$

$$\frac{\partial X}{\partial x} = 2\pi i \left\{ \frac{5}{\sqrt{29}} - \frac{10}{\sqrt{104}} + \frac{10}{\sqrt{116}} - \frac{5}{\sqrt{41}} + \log \frac{10+\sqrt{104}}{5+\sqrt{29}} - \log \frac{10+\sqrt{116}}{5+\sqrt{29}} \right\}$$

---

$a_1 = 5 \quad a_2 = 20 \quad R_1 = 1 \quad R_2 = 10$

4

$i = \frac{1}{10} = 1$ Amp.

$$X = 2\pi i \left\{ 20 \cdot \log \frac{\overset{32{,}2607}{10+\sqrt{500}}}{\underset{21{,}0250}{1+\sqrt{401}}} - 5 \log \frac{\overset{21{,}1803}{10+\sqrt{125}}}{\underset{6{,}099}{1+\sqrt{126}}} \right\} = 2\pi i \cdot \overset{2{,}348}{\cancel{1{,}520}}$$

$$\frac{\partial X}{\partial x} = 2\pi i \left\{ \underbrace{\frac{1}{\sqrt{26}} - \frac{10}{\sqrt{101}}}_{+0{,}04066} + \frac{10}{\sqrt{500}} - \frac{1}{\sqrt{401}} + \log \frac{10+\sqrt{125}}{1+\sqrt{26}} - \log \frac{10+\sqrt{500}}{1+\sqrt{401}} \right\} = 2\pi i\, 1{,}043$$

1791

0,8640

$\cancel{1{,}4334}$

1,2543

1791

0,4310

$i = 12$

$X = 144{,}27 \qquad \frac{\partial X}{\partial x} = 78{,}65$

$X \frac{\partial X}{\partial x} = 11346{,}6.$

2,681467 | 1,340734 | 21,9046
2,581210 | 1,290605 | 19,5256

2,080085 | 1,040043 | 10,9659
1,327259 | 0,663680 | 4,6098

2,716212 | 1,358106 | 22,8090
2,624540 | 1,312270 | 20,5244

2,114778 | 1,057389 | 11,4123
1,494850 | 0,747475 | 5,5908

1,503853
1,312296
0,191557
0,282298-1
362216
0,644514-1
0,441076

1,321514
0,748947
0,572567
0,757827-1
362216
0,120043
1,04723
1,31839

1,515993
1,332931
0,183062
0,262598-1
362216
0,624814-1
0,421515

1,330662
0,818938
0,511725
0,709037-1
362216
0,071253
1,17829

0,644517-1
~~1219~~
1,290035
0,934552

0,120044
0,653213
~~66831~~
0,773257
860115
5,93277
2,66838

41219
82438
0,65551 1,94883
670716

0,624814-1
1,311754
0,936568

0,071253
0,740363
0,811616

8,64108
6,48061
2,16047
66808
482885
2,41442

492466
246483

MAGYAR
TUDOMÁNYOS AKADÉMIA
KÖNYVTÁRA

1,330682
818899
0,511783
0709086
362216
1,071302

1,335699
850217
0,485482
~~362~~
0,686173
362216
1,048389
1,11786

0,21076
843040

1,17842
5,8921
58921
648131

864108
648131
2,15977

$$X = 2\pi i\left\{a_2 \log \frac{R_2+\sqrt{R_2^2+a_2^2}}{R_1+\sqrt{R_1^2+a_2^2}} - a_1 \log \frac{R_2+\sqrt{R_2^2+a_1^2}}{R_1+\sqrt{R_1^2+a_1^2}}\right\}$$

$$\frac{\partial X}{\partial x} = 2\pi i\left\{\frac{R_1}{\sqrt{R_1^2+a_1^2}} - \frac{R_2}{\sqrt{R_2^2+a_1^2}} + \frac{R_2}{\sqrt{R_2^2+a_2^2}} - \frac{R_1}{\sqrt{R_1^2+a_2^2}} + \right.$$

$$\left. + \log \frac{R_2+\sqrt{R_2^2+a_1^2}}{R_1+\sqrt{R_1^2+a_1^2}} - \log \frac{R_2+\sqrt{R_2^2+a_2^2}}{R_1+\sqrt{R_1^2+a_2^2}}\right\}$$

---

$a_1 = 5 \quad a_2 = 20 \quad R_1 = 1 \quad R_2 = 10$

$X = 2\pi i\,\{20 . 0{,}43076 - 5 . 1{,}24495\} = 2\pi i . 2{,}3904$

$\frac{\partial X}{\partial x} = 2\pi i . 0{,}8640$

---

$a_1 = 4 \quad a_2 = 19 \quad R_1 = 1 \quad R_2 = 10$

$X = 2\pi i\,\{19 . 0{,}45205 - 4 . 1{,}40203\} = 2\pi i\; 2{,}98083$

$\frac{\partial X}{\partial x}$

kruep $2{,}4648$

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$a_1 = 6 \quad a_2 = 21 \quad R_1 = 1 \quad R_2 = 10$

$X = 2\pi i\,\{21 . 0{,}41219 - 6 . 1{,}11786\} = 2\pi i\; 1{,}94883.$

$\frac{\partial X}{\partial x}$

---

$a_1 = 4{,}5 \quad a_2 = 19{,}5 \quad R_1 = 1 \quad R_2 = 10.$

$X = 2\pi i\,\{19{,}5 . 0{,}44108 - 4{,}5 . \cancel{1{,}04723}\ 1{,}31839\} = 2\pi i . 2{,}66838$

$\frac{\partial X}{\partial x}$

kriz. $2{,}4141$

---

$a_1 = 5{,}5 \quad a_2 = 20{,}5$

$X = 2\pi i\,\{20{,}5 . 0{,}42152 - 5{,}5 . 1{,}17842 = 2\pi i\; 2{,}15977$

$\frac{\partial X}{\partial x}$

---

$a_1 = 5 \quad a_2 = 20 \quad R_2 = \frac{4}{\cancel{5}} \quad R_1 = 0$

$X^2 = 2\pi i \{20 \cdot 0{,}198700 - 5 \cdot 0{,}732657\} = 2\pi i \cdot 0{,}31064$

$\frac{\partial X}{\partial x}$

$a_1 = 5{,}5 \quad a_2 = 20{,}5 \quad R_2 = \frac{4}{\cancel{4}} \quad R_1 = 0$

$X^2 = 2\pi i \{20{,}5 \cdot 0{,}193903 - 5{,}5 \cdot 0{,}674859\} = 2\pi i \cdot 0{,}26329$

$\frac{\partial X}{\partial x}$

Közép $= 0{,}31613$

$a_1 = 4{,}5 \quad a_2 = 19{,}5 \quad R_2 = \frac{4}{\cancel{5}} \quad R_1 = 0$

$X^2 = 2\pi i \{19{,}5 \cdot 0{,}203673 - 4{,}5 \cdot 0{,}800585\} = 2\pi i \, 0{,}36898$

$X \frac{\partial X}{\partial x}$

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$$a_{23}\left[\left(\cancel{+}\,\tfrac{1}{2}Z\frac{\partial Y}{\partial y} + 2Y\frac{\partial Z}{\partial y}\right)\cos^2\alpha + \left(2Z\frac{\partial X}{\partial x} + X\frac{\partial Z}{\partial x}\right)\sin^2\alpha.\right]$$

$$a_{31}\left[+2Z\frac{\partial X}{\partial y} + X\frac{\partial Z}{\partial y}\right]\cos^2\alpha - \left(Z\frac{\partial Y}{\partial x} + Y\frac{\partial Z}{\partial x}\right)\sin^2\alpha$$

$$\cos^2\alpha = \frac{1}{2} + \frac{1}{2}\cos 2\alpha$$

$$\sin^2\alpha = \frac{1}{2} - \frac{1}{2}\cos 2\alpha$$

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$a_1 = 4 \quad a_2 = 19 \quad R_1 = 1 \quad R_2 = 10$

$$X = 2\pi i \left\{ 19 \log \frac{10 + \overset{21,471}{\sqrt{461}}}{1 + \underset{19,026}{\sqrt{362}}} - 4 \log \frac{10 + \overset{10,817}{\sqrt{116}}}{1 + \underset{4,123}{\sqrt{17}}} \right\} = 2\pi i \{ 19 \cdot 0,45205 - 4 \cdot 1,40203 \}$$
$$= 2\pi i \cdot 2,98083$$

$a_1 = 6 \quad a_2 = 21 \quad R_1 = 1 \quad R_2 = 10$

$$X = 2\pi i \left\{ 21 \log \frac{10 + \overset{23,259}{\sqrt{541}}}{1 + \underset{21,024}{\sqrt{442}}} - 6 \log \frac{10 + \overset{11,662}{\sqrt{136}}}{1 + \underset{6,083}{\sqrt{37}}} \right\} = 2\pi i = \{ 21 \cdot 0,41219 - 6 \cdot 1,11786 \}$$
$$= 2\pi i \cdot 1,94883$$

| | | |
|---|---|---|
| 1,497911 | 1,318418 | 4,06845 |
| 1,301594 | 0,709524 | 45205 |
| 0,196317 | 0,608894 | 8,58895 |
| | | 566812 |
| 0,292965 − 1 | 0,784539 − 1 | 2,98083 |
| 262216 | 262216 | |
| 0,655181 − 1 | 0,146755 | |

493766
246883
2748

| | | |
|---|---|---|
| 1,521909 | 1,335699 | 41219 |
| 1,342896 | 0,850217 | 82438 |
| 0,179013 | 0,485482 | 8,65599 |
| | | 6,70716 |
| 0,252877 − 1 | 0,686171 − 1 | 1,84883 |
| 262216 | 262216 | |
| 0,615093 | 0,048387 | |

6,70716

95
82
70

86200
62715
23485

$a_1 = 4,5 \quad a_2 = 19,5$

$$\log \frac{10 + \sqrt{480,25}}{1 + \sqrt{381,25}} = \frac{31,9046}{20,5256} \qquad \log = \frac{10 + \sqrt{120,25}}{1 + \sqrt{21,25}} = \frac{20,9659}{5,6098}$$

$a_1 = 5,5 \quad a_2 = 20,5$

$$\log \frac{10 + \sqrt{520,25}}{1 + \sqrt{421,25}} = \frac{32,8090}{21,5244} \qquad \log \frac{10 + \sqrt{130,25}}{1 + \sqrt{31,25}} = \frac{21,4132}{6,5902}$$

2,114811 0,557406
1,057406
36
1494850
11432
5,55017 0,747425

510009
322839
0,187170

1,330008
~~0,540229~~
~~0789679~~
707483
0,622525

~~2868~~
1330008
785259
0,544749

8,620
~~7~~1062

8,620
7,100
1,520

0,272236 -1
0,362216
0,634452 -1
0,42109

0,794160 -1
0,362216
0,156382

0,736197 -1
362216
0,098413

1,2543
62715
~~8620~~
2348

1,4334

1,414973
0,000000
0,707487
0,292513 -1
0,19612
~~98058~~
~~1,17670~~
92848
1,12460
108394
0,04066

2,017000
1,000000
1,008517
0,991483 -1
0,98058
15546
1,4334
407
~~144~~ 14741
4310
1,0431

2,064458
1,000000
1,032229
0,967771 -1
0,92848

612784
0,000000
0,808392
0,191608 -1
0,15546

31415
6,283
12566
75,396

~~10862~~

1,877348
281844
2,159192
1895674
4,054866
144,27
78,65
1134,66

1,877348
018326
1,895674

298083
194883
492966
24648

---

1,510018
1,322943
0,187075

0,272015 -1
362216
0,634231 -1
0,430756

1,325932
0,785259
0,540673

0,732935 -1
362216
0,095151
124495

8,6152
6,2248
2,3904

2,66838
2,15977
482815

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Ms 5102/6

$\frac{35}{2000}$ $\frac{70}{2000}$ $\frac{7}{200}$

| | | | | | |
|---|---|---|---|---|---|
| +0,108870 | +0,065250 | +0,040557 | +0,014802 | −0,028542 | −0,114414 |
| −0,077750 | −0,072540 | −0,065250 | −0,054560 | −0,024670 | +0,057084 |
| +0,031060 | −0,007290 | −0,024693 | −0,039758 | −0,053212 | −0,057330 |

| | | | | | |
|---|---|---|---|---|---|
| +0,084616 | −0,014998 | −0,122965 | −0,165275 | +0,165781 | +1,429050 |
| −0,205689 | −0,169232 | +0,044994 | +0,491860 | +0,826375 | −0,994686 |
| −0,121073 | −0,184230 | −0,077971 | +0,326585 | +0,992156 | +0,434364 |

$-4\{0,264591 \cdot 0,057330 - 0,248768 \cdot 0,053212 - 0,275475 \cdot 0,039758 - 0,061980 \cdot 0,024690$
$\pm 0,108435 \cdot 0,007290 - 0,105942 \cdot 0,031060 - 0,028899 \cdot 0,233150\}$

Péntek este

0,0153983
0,0185586
0,0109523
0,0015305
0,0007905 −
0,0032906
0,0067378

0,0572586
15810

0,0556776

$-\frac{1}{b}$ 0,233150 ~~10~~

$\cancel{\frac{\partial^8 L_{ac}}{\partial x^8} = \frac{1}{6^6} 0,229034.}$

5964

$\boxed{\frac{\partial^8 L_{ab}}{\partial x^8} = \frac{1}{6^6} 0,222710,}$ jó

0,024876
0,069752

940287
0,015976

0,924411

940287
925544

0,014843

$2.\Delta c. \frac{\partial^8 L_{ab}}{\partial x^8} = +\frac{1}{6^6} 0,015534$

115200
48000
67200

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$\frac{a}{a^2+c^2}$ $\frac{b}{b^2+c^2}$

$$\left(\frac{bc}{a^2+c^2}+\frac{2bc}{a^2+b^2}\right)\frac{1}{2}.$$

$$8\left\{\left(\frac{3}{5}\cdot\frac{1}{\sqrt{6}}-\underline{0{,}2777}:0{,}2686\right)\right.$$

0,389076
69 89.70
001030
1.088046
0,212984 −1

0,1633

477121
1088046
0,389075 −1

429106
443576
0,872682 −2

+0,0791029
+0,0128608
+0,0053040
0,0003508
0,037817 5

15,720992

+0,0981842
+0,0015693
0,0047648
0,0033827
0,0479017

45378
44635
90013
0,0213877
0,0123864

9013

0,25065
0,07419
0,17606
1,40848
423344
150 | 0,1831824
30
31,8
0,0012212

$\frac{1}{200}$

~~011068~~
122965
~~134000~~
19200
142165

940
925
015

13

142905

| 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|
| 0,787402 −3 | 0,672103 −4 | 0,562804 −5 | 0,453505 −6 | 0,344206 −7 | 0,234907 −8 |
| 0,778151 | 1,665361 | 2,568151 | 3,239703 | 4,634147 | 6,346449 |
| 0,559553 −2 | 0,337464 −2 | 0,130955 −2 | 0,693208 −3 | 0,978353 −3 | 0,581356 −2 |

4,86171 0,686789 0,313211 −1

| | | | | | |
|---|---|---|---|---|---|
| 0,626422 −2 | 0,939633 −3 | 0,252844 −3 | 0,566055 −4 | 0,879266 −5 | 0,192477 −5 |
| 0,301030 | 0,236391 | 1,836827 | 2,652153 | 3,340268 | 9,962570 |
| 0,927452 −2 | 0,176024 −2 | 0,089781 −1 | 0,218208 −1 | 0,219534 −1 | 0,155047 −4 |

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$\frac{1}{r} = 0{,}413035 \frac{1}{6} = +\underline{0{,}413035}$

$\frac{\partial}{\partial x} = + 0{,}070463 = +\underline{0{,}070463}$

$\frac{\partial}{\partial x^2} = -0{,}070463 + 0{,}036062 = -\underline{0{,}034401}$

$\frac{\partial}{\partial x^3} = -0{,}108188 + 0{,}030761) = -\underline{0{,}077427}$

$\frac{\partial}{\partial x^4} = +0{,}108188 - 0{,}184566 + 0{,}036734 = -\underline{0{,}039644}$

$\frac{\partial}{\partial x^5} = +0{,}461414 - 0{,}367344 + 0{,}056401 = +\underline{0{,}150471}$

$\frac{\partial}{\partial x^6} = -0{,}461414 + 1{,}653046 - 0{,}846019 + 0{,}105841 = +\underline{0{,}451454}$

$\frac{\partial}{\partial y} = +0{,}070463 = +\underline{0{,}070463}$

$\frac{\partial}{\partial y^2} = -0{,}070463 + 0{,}036062 = -\underline{0{,}034401}$ megegyez

$\frac{\partial}{\partial y^3} = -0{,}108188 + 0{,}030761) = -\underline{0{,}077427}$ megegyez

$\frac{\partial}{\partial y^4} = +0{,}108188 - 0{,}184566 + 0{,}036734 = -\underline{0{,}039644}$

$\frac{\partial}{\partial y^5} = +0{,}461414 - 0{,}367344 + 0{,}056401 = +\underline{0{,}150471}$

$\frac{\partial}{\partial y^6} = -0{,}461414 + 1{,}653046 - 0{,}846019 + 0{,}105841 = +\underline{0{,}451454}$

$\frac{\partial}{\partial x \partial y} = +0{,}036061 = +\underline{0{,}036062}$

$\frac{\partial}{\partial x \partial y^2} = -0{,}036062 + 0{,}030761 = -\underline{0{,}005301}$

$\frac{\partial}{\partial x^2 \partial y} = -0{,}036062 + 0{,}030761 = -\underline{0{,}005301}$

$\frac{\partial}{\partial x \partial y^3} = -0{,}092283 + 0{,}036734 = -\underline{0{,}055549}$

$\frac{\partial}{\partial x^2 \partial y^2} = +0{,}036062 - 0{,}061522 + 0{,}036734 = +\underline{0{,}011274}$

$\frac{\partial}{\partial x^3 \partial y} = -0{,}092283 + 0{,}036734 = -0{,}055549$

$\frac{\partial}{\partial x^2 \partial y^3} = +0{,}092283 - 0{,}146937 + 0{,}056401 = +\underline{0{,}001747}$

$\frac{\partial}{\partial x^3 \partial y^2} = +0{,}092283 - 0{,}146937 + 0{,}056401 = +\underline{0{,}001747}$

$\frac{\partial}{\partial x^3 \partial y^3} = +0{,}330609 - 0{,}338408 + 0{,}105841 = +\underline{\underline{0{,}098042}}$

$\frac{1704}{12}$ 10959

641,142

860,996

+ 0,926469

+ 0,491774

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$\frac{2}{\sqrt{\ }}\left\{\frac{\sqrt{\ }}{a^2+b^2+c^2} - \frac{1}{c+}\right)$

$\frac{2}{\sqrt{\ }}\left\{\frac{c\sqrt{\ } + \cancel{a^2}+\cancel{b^2}+\cancel{c^2}+c^2+2ca - \cancel{a^2}-\cancel{b^2}-\cancel{c^2}-\cancel{2ca}}{(a^2+b^2+c^2+2ca)(c+\sqrt{\ })}\right)$

$\frac{2c(c+\sqrt{\ })}{\sqrt{\ }(a\quad)(\quad)}$

+0,070414 = +0,070414

−0,070414 + 0,036062 = −0,034352
36062

−0,108187 + 0,030761 = −0,077426
30761

+0,108187 − 0,184566 + 0,036734 = −0,039645
36734 144921

+0,461415 − 0,367340 + 0,056401 = +0,150476
056401 +517816

−0,461415 + 1,653044 − 0,846018 + 0,105841 = +0,451452
105841 461415
1,758885
1,307403

+0,070414

−0,070414 + 0,036062

−0,108187

+0,036062 = +0,036062

−0,036062 + 0,030761 = −0,005301
30761

−0,036062 + 0,030761 = −0,005301

−0,092283 + 0,036734 = −0,055549
36734

+0,036062 − 0,061522 + 0,036734 = +0,011274
72796 36062

−0,092283 + 0,036734 = −0,055549
36734

+0,092283 − 0,146937 + 0,056401 = +0,001747
56401 +0,148684

+0,092283 − 0,146937 + 0,056401 = + 0,001747
56401 148684

+0,330609 − 0,169204 + 0,105841 = +0,267246
105841 436450

| $w_0$ | $w_1$ | $\varphi_1$ | $r_1$ | $r_2$ | $\delta$ | |
|---|---|---|---|---|---|---|
| 1°38'54" | 177°57'17" | 0 | 27000 | 139058 | + 1,70 | + ~~0,000278~~ 0,000646 |
| 88°21'6" | 1°38'54" | 1°38'54" | 139058 | 4000 | + 1,70 | +0,139042 |
| 46°21'52" | 14°43'3" | 1°38'54" | 139058 | 40361 | −0,261535 | −0,062203 |
| 46°21'52" | 1°38'54" | 1°38'54" | 139058 | 5382 | +0,261535 | +0,011226 |
| 41°59'14" | 48°0'46" | 48°0'46" | 40361 | 30000 | −0,261535 | −0,076241 |
| 41°59'14" | 48°0'46" | 48°0'46" | 5382 | 4000 | +0,261535 | +0,010403 |
| | | | | | | −0,138547 |
| | | | | | | 21628 |
| | | | | | | 0,116916 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1°38'54" | 177°57'17' | 0 | 27000 | 139058 | +1,70 | + 0,000646 |
| 88°21'6" | 1°38'54' | 1°38'54" | 139058 | 4000 | +1,70 | + 0,139042 |
| 64°7'26" | 28°12'48" | 1°38'54" | 139058 | 65795 | −0,121429 | − 0,070534 |
| 64°7'26" | 1°38'54" | 1°38'54" | 139058 | 4386 | +0,121429 | + 0,007208 |
| 24°13'40" | 65°46'20" | 65°46'20" | 65795 | 60000 | −0,121429 | − 0,040851 |
| 24°13'40" | 65°46'20" | 65°46'20" | 4386 | 4000 | +0,121429 | + 0,002723 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 52°1'38" | 26°48'51" | 13°44'42" | 141460 | 65795 | −0,261535 | −0,157439 |
| 52°1'38" | 13°44'42" | 13°44'42" | 141460 | 37284 | +0,261535 | +0,105843 |
| 24°13'40" | 65°46'20" | 65°46'20" | 65795 | 60000 | −0,261535 | −0,087986 |
| 24°13'40" | 65°46'20" | 65°46'20" | 37284 | 34000 | +0,261535 | +0,049859 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 6°49'6" | 179°4'33" | 0 | 32000 | 280029 | + 1,70 | +0,000241 |
| 89°10'54" | 0°49'6" | 0°49'6" | 280029 | 4000 | +1,70 | +0,140327 |
| 55°0'14" | 12°54'31" | 6°55'25" | 282057 | 68000 | −0,261535 | −0,176220 |
| 55°0'14" | 6°55'25" | 6°55'25" | 282057 | 38533 | +0,261535 | +0,113197 |
| 28°4'21" | 61°55'39" | 61°55'39" | 68000 | 6000 | −0,261535 | −0,101949 |
| 28°4'21" | 61°55'39" | 61°55'39" | 38533 | 34000 | +0,261535 | +0,057771 |

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KÖNYVTÁRA

| $w_0$ | $w_1$ | $\varphi_1$ | $r_1$ | $r_2$ | $\vartheta$ | |
|---|---|---|---|---|---|---|
| 0°42'34" | 178°53'54" | 0° | 115000 | 323025 | +1,70 | +0,000373 |
| 89°17'26" | 0°42'34" | 0°42'34" | 323025 | 4000 | +1,70 | +0,140506 |
| 21°32'38" | 13°8'2" | 6°0'32" | 324785 | 129712 | −0,261535 | −0,066352 |
| 21°32'38" | 6°0'32" | 6°0'32" | 324785 | 73503 | +0,261535 | +0,044335 |
| 62°26'50" | 27°33'10" | 27°33'10" | 129712 | 60000 | −0,261535 | −0,226787 |
| 62°26'50" | 27°33'10" | 27°33'10" | 73503 | 34000 | +0,261535 | +0,128512 |
| | | Alpesek. | | | | |
| 66°30'57" | 35°40'15" | 23°29'3" | 158095 | 94315 | −0,261535 | −0,327933 |
| 66°30'57" | 23°29'3" | 23°29'3" | 158095 | 63000 | +0,261535 | +0,259675 |
| 55°55'11" | 71°24'47" | 90° | 94315 | 112428 | −0,261535 | −0,269426 |
| 55°55'11" | 90° | 90° | 63000 | 112428 | +0,261535 | +0,213234 |
| +) [illegible] ~~55°55'11"~~ | ~~34°4'49"~~ | ~~34°4'49"~~ | ~~112428~~ | ~~63000~~ | ~~+0,261535~~ | |
| ~~55°55'11"~~ | ~~52°40'2"~~ | ~~34°4'49"~~ | ~~112428~~ | ~~94315~~ | ~~−0,261535~~ | |
| | | | Milanó (alsó rész) | | | |
| 19°23'34" | 33°20'52" | 14°45'39" | 235492 | 162652 | −0,261535 | −0,205907 |
| 19°23'34" | 14°45'39" | 14°45'39" | 235492 | 106873 | +0,261535 | +0,164388 |
| 65°40'48" | 21°58'1" | 34°9'13" | 162652 | 60895 | −0,261535 | −0,151570 |
| 65°40'48" | 34°9'13" | 34°9'13" | 106873 | 60895 | +0,261535 | +0,117041 |
| | | Alpesek (Milano) felső rész. | | | | |
| 1°14'22" | 176°58'10" | −1°11'7" | 134631 | 227720 | +2,73 | +0,135590 |
| 178°45'38" | +3'15" | +3'15" | 227720 | 10402 | +2,73 | +0,024051 |

$$\frac{\partial}{\partial x}\left(\frac{1}{(a-x)^2+(c-z)^2}\right) = + \frac{2(a-x)}{[(a-x)^2+(c-z)^2]^2}$$

$$\frac{\partial}{\partial x^2}\left(\frac{1}{(a-x)^2+(c-z)^2}\right) = - \frac{2}{[(a-x)^2+(c-z)^2]^2} + \frac{8(a-x)^2}{[(a-x)^2+(c-z)^2]^3}$$

$$\frac{\partial}{\partial x^3}\left(\frac{1}{(a-x)^2+(c-z)^2}\right) = - \frac{24(a-x)}{[(a-x)^2+(c-z)^2]^3} + \frac{48(a-x)^3}{[(a-x)^2+(c-z)^2]^4}$$

$$\frac{\partial}{\partial x^4}\left(\frac{1}{(a-x)^2+(c-z)^2}\right) = + \frac{24}{[(a-x)^2+(c-z)^2]^3} - \frac{288(a-x)^2}{[(a-x)^2+(c-z)^2]^4} + \frac{384(a-x)^4}{[(a-x)^2+(c-z)^2]^5}$$

$$\frac{\partial}{\partial x^5}\left(\frac{1}{(a-x)^2+(c-z)^2}\right) = + \frac{720(a-x)}{[(a-x)^2+(c-z)^2]^4} - \frac{3840(a-x)^3}{[(a-x)^2+(c-z)^2]^5} + \frac{3840(a-x)^5}{[(a-x)^2+(c-z)^2]^6}$$

$$\frac{\partial}{\partial x^6}\left(\frac{1}{(a-x)^2+(c-z)^2}\right) = - \frac{720}{[(a-x)^2+(c-z)^2]^4} + \frac{17280(a-x)^2}{[(a-x)^2+(c-z)^2]^5} - \frac{57600(a-x)^4}{[(a-x)^2+(c-z)^2]^6} + \frac{46080(a-x)^6}{[(a-x)^2+(c-z)^2]^7}$$

$$\frac{\partial^n}{\partial x^n}\frac{a}{a^2+c^2} = + a\frac{\partial^n}{\partial x^n}\frac{1}{a^2+c^2} - n\frac{\partial^{n-1}}{\partial x^{n-1}}\frac{1}{a^2+c^2}$$

$a = 3$ $\qquad b = 1$ $\qquad$ M1 $\qquad c = 1{,}979752$

$$a = 3 \qquad (a-x)^2 + (c-2)^2 = 9 + 3{,}919418 = 12{,}919418$$

$$a^2 = 9 \qquad \log[a^2+c^2] = 1{,}111243$$

$$a^3 = 27 \qquad \log[a^2+c^2]^2 = 2{,}222486$$

$$a^4 = 81 \qquad \log[a^2+c^2]^3 = 3{,}333729$$

$$a^5 = 243 \qquad \log[a^2+c^2]^4 = 4{,}444972$$

$$a^6 = 729 \qquad \log[a^2+c^2]^5 = 5{,}556215$$

$$a^7 = 2187 \qquad \log[a^2+c^2]^6 = 6{,}667458$$

$$a^8 = 6561 \qquad \log[a^2+c^2]^7 = 7{,}778701$$

$$\frac{1}{a^2+c^2} = +0{,}0774028 \qquad \frac{a}{a^2+c^2} = +0{,}232208$$

$$\frac{\partial}{\partial x}\left(\frac{1}{a^2+c^2}\right) = +0{,}0359472 \qquad \frac{\partial}{\partial x}\left(\frac{a}{a^2+c^2}\right) = +0{,}030439$$

$$\frac{\partial^2}{\partial x^2}\left(\frac{1}{a^2+c^2}\right) = +0{,}0214066 \qquad \frac{\partial^2}{\partial x^2}\left(\frac{a}{a^2+c^2}\right) = -0{,}007675$$

$$\frac{\partial^3}{\partial x^3}\left(\frac{1}{a^2+c^2}\right) = +0{,}0131303 \qquad \frac{\partial^3}{\partial x^3}\left(\frac{a}{a^2+c^2}\right) = -0{,}024831$$

$$\frac{\partial^4}{\partial x^4}\left(\frac{1}{a^2+c^2}\right) = +0{,}0045085 \qquad \frac{\partial^4}{\partial x^4}\left(\frac{a}{a^2+c^2}\right) = -0{,}038993$$

$$\frac{\partial^5}{\partial x^5}\left(\frac{1}{a^2+c^2}\right) = -0{,}009857 \qquad \frac{\partial^5}{\partial x^5}\left(\frac{a}{a^2+c^2}\right) = -0{,}052125$$

$$\frac{\partial^6}{\partial x^6}\left(\frac{1}{a^2+c^2}\right) = -0{,}03793 \qquad \frac{\partial^6}{\partial x^6}\left(\frac{a}{a^2+c^2}\right) = -0{,}054630$$

TUDOMÁNYOS AKADÉMIA
KÖNYVTÁRA

folytatás a túloldalon!

$$\frac{\partial^n(\varphi\psi)}{\partial x^n} = \varphi^{(n)}\psi + n\varphi^{(n-1)}\psi^{(1)} + \frac{n.n-1}{1.2}\varphi^{(n-2)}\psi^{(2)} + \frac{n.(n-1)(n-2)}{1.2.3}\varphi^{(n-3)}\psi^{(3)} + \ldots$$

$$\varphi = \frac{a}{a^2+c^2} \qquad \psi = \frac{1}{r}.$$

$\underline{a=1}$ $\underline{b=3}$ $\underline{c=1{,}979752}$

$(a-x)^2+(c-2)^2 = 4{,}919418$

$\log[a^2+c^2] = 0{,}691914$

$\log[a^2+c^2]^2 = 1{,}383828$

$\log[a^2+c^2]^3 = 2{,}075742$

$\log[a^2+c^2]^4 = 2{,}767656$

$\log[a^2+c^2]^5 = 3{,}459570$

$\log[a^2+c^2]^6 = 4{,}157484$

$\log[a^2+c^2]^7 = 4{,}843398$

$\frac{1}{a^2+c^2} = +0{,}20326$

$\frac{a}{a^2+c^2} = +0{,}203260$

✓ $\frac{\partial}{\partial x}\left(\frac{1}{a^2+c^2}\right) = +0{,}082642$

$\frac{\partial}{\partial x}\left(\frac{a}{a^2+c^2}\right) = -0{,}120618$

$\frac{\partial^2}{\partial x^2}\left(\frac{1}{a^2+c^2}\right) = -0{,}0154455$

$\frac{\partial^2}{\partial x^2}\left(\frac{a}{a^2+c^2}\right) = -0{,}180730$

$\frac{\partial^3}{\partial x^3}\left(\frac{1}{a^2+c^2}\right) = -0{,}119633$

$\frac{\partial^3}{\partial x^3}\left(\frac{a}{a^2+c^2}\right) = -0{,}073295$

$\frac{\partial^4}{\partial x^4}\left(\frac{1}{a^2+c^2}\right) = -0{,}156871$

$\frac{\partial^4}{\partial x^4}\left(\frac{a}{a^2+c^2}\right) = +0{,}321661$

$\frac{\partial^5}{\partial x^5}\left(\frac{1}{a^2+c^2}\right) = +0{,}13950$

$\frac{\partial^5}{\partial x^5}\left(\frac{a}{a^2+c^2}\right) = +0{,}923855$

$\frac{\partial^6}{\partial x^6}\left(\frac{1}{a^2+c^2}\right) = +1{,}39321$

$\frac{\partial^6}{\partial x^6}\left(\frac{a}{a^2+c^2}\right) = +0{,}556210$

$a=3$ $b=1$ $c=1,979752$

$r^2 = 13,919418$

$\log r = 0,571810$ $\log \frac{1}{r} = 0,428190-1$

$\log r^3 = 1,715430$

$\log r^5 = 2,859050$

$\log r^7 = 4,002670$

$\log r^9 = 5,146290$

$\log r^{11} = 6,289910$

$\log r^{13} = 7,433530$

$\log r^{15} = 8,577150$

$\log r^{17} = 9,720770$

$\frac{1}{r} = +0,268034$

$\frac{\partial(\frac{1}{r})}{\partial x} = +0,0577685$

$\frac{\partial^2 \frac{1}{r}}{\partial x^2} = +0,0180958$

$\frac{\partial^3 \frac{1}{r}}{\partial x^3} = +0,0028997$

$\frac{\partial^4 \frac{1}{r}}{\partial x^4} = -0,0073258$

$\frac{\partial^5 \frac{1}{r}}{\partial x^5} = -0,017544$

$\frac{\partial^6 \frac{1}{r}}{\partial x^6} = -0,028432$

$\frac{\partial^7 \frac{1}{r}}{\partial x^7} = -0,03430$

$\frac{\partial^8 \frac{1}{r}}{\partial x^8} = -0,01078$

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

$a=1$ $b=3$ $c=1{,}979752$

$$\frac{1}{r} = +0{,}268034$$

$$\frac{\partial \frac{1}{r}}{\partial x} = +0{,}0192562$$

$$\frac{\partial^2 \frac{1}{r}}{\partial x^2} = -0{,}0151060$$

$$\frac{\partial^3 \frac{1}{r}}{\partial x^3} = -0{,}0109599$$

$$\frac{\partial^4 \frac{1}{r}}{\partial x^4} = +0{,}0042555$$

$$\frac{\partial^5 \frac{1}{r}}{\partial x^5} = +0{,}0152983$$

$$\frac{\partial^6 \frac{1}{r}}{\partial x^6} = +0{,}0045386$$

MAGYAR
TUDOMÁNYOS AKADÉMIA
KÖNYVTÁRA

$$\frac{1}{r} = \underline{0{,}268591}$$

$$\frac{\partial}{\partial x} = \underline{+0{,}058128}$$

$$\frac{\partial^2}{\partial x^2} \quad -0{,}019276 + 0{,}037741_4 = +\underline{0{,}018365}$$

$$\frac{\partial^3}{\partial x^3} \quad -0{,}037741_4 + 0{,}040840_6 = +\underline{0{,}003099}$$

$$\frac{\partial^4}{\partial x^4} \quad +0{,}012580_5 - 0{,}081681_2 + 0{,}061871_6 = -\underline{0{,}007229}$$

$$\frac{\partial^5}{\partial x^5} \quad +0{,}068067_7 - 0{,}206238_8 + 0{,}120514 = -\underline{0{,}017657}$$

$$\frac{\partial^6}{\partial x^6} \quad -0{,}022689_2 + 0{,}309358_3 + 0{,}286903 - 0{,}602471 = -0{,}028899$$

$$\frac{\partial}{\partial y} \quad +\underline{0{,}019376}$$

$$\frac{\partial^2}{\partial y^2} \quad -0{,}019376 + 0{,}004193_5 = -\underline{0{,}015182}$$

$$\frac{\partial^3}{\partial y^3} \quad -0{,}012580_4 + 0{,}001512_6 = -0{,}\underline{011068}$$

$$\frac{\partial^4}{\partial y^4} \quad +0{,}012580_4 - 0{,}009075_7 + 0{,}000763_8 = +0{,}\underline{004269}$$

$$\frac{\partial^5}{\partial y^5} \quad +0{,}022689_2 - 0{,}007638_4 + 0{,}000495_9 = +\underline{0{,}015547}$$

$$\frac{\partial^6}{\partial y^6} \quad -0{,}022689_2 + 0{,}034373_1 - 0{,}007439_2 + 0{,}000393_6 = +\underline{0{,}004639} \times$$

$$\frac{\partial^2}{\partial x \partial y} \quad \underline{+0{,}012580_5}$$

$$\frac{\partial^3}{\partial x \partial y^2} \quad -0{,}012580_5 + 0{,}004537_8 = -\underline{0{,}008043}$$

$$\frac{\partial^3}{\partial x^2 \partial y} \quad -0{,}004193_5 + 0{,}013613_5 = +\underline{0{,}009420}$$

$$\frac{\partial^4}{\partial x \partial y^3} \quad -0{,}013613_5 + 0{,}002291_5 = -\underline{0{,}011322}$$

$$\frac{\partial^4}{\partial x^2 \partial y^2} \quad +0{,}004193_5 - 0{,}015126_1 + 0{,}006874_6 = -\underline{0{,}004058}$$

$$\frac{\partial^4}{\partial x^3 \partial y} \quad -0{,}013613_5 + 0{,}020623_9 = +\underline{0{,}007010} \times$$

$$\frac{\partial^5}{\partial x^2 \partial y^3} \quad +0{,}004537_8 - 0{,}021387_7 + 0{,}004463_5 = -\underline{0{,}012386_4}$$

$$-\frac{\partial^5}{\partial x^3 \partial y^2} \quad +0{,}013613_5 - 0{,}027498_5 + 0{,}013390_5 = -\underline{0{,}000495}$$

$$\frac{\partial^6}{\partial x^3 \partial y^3} \quad +0{,}020623_9 - 0{,}044634_9 + 0{,}010626_0 = -\underline{0{,}013385}$$

MAGYAR
TUDOMÁNYOS AKADÉMIA
KÖNYVTÁRA

$\psi = \frac{1}{\sqrt{3}} = 0{,}077750$

$\frac{\partial}{\partial\psi} = \left(\frac{1}{\sqrt{3}}\right)^2 6 \qquad = +\,0{,}036270$

$\frac{\partial}{\partial x^2} = \left(\frac{1}{\sqrt{3}}\right)^3 46{,}2766 \quad = +\,0{,}021750$

$\frac{\partial}{\partial x^3} = \frac{1}{\sqrt{3}^4} 369{,}957 \quad = +\,0{,}013579$

$\frac{\partial}{\partial x^4} = \left(\frac{1}{\sqrt{3}}\right)^5 \cdot 1736{,}61 = +\,0{,}004934$

$\frac{\partial}{\partial x^5} = \left(\frac{1}{\sqrt{3}}\right)^6 - 43067{,}2 = -\,0{,}009514$

$\frac{\partial}{\partial x^6} \left(\frac{1}{\sqrt{3}}\right)^7 - 222\,0495 = -\,0{,}038138$

$\psi \quad = 0{,}205689$

$\frac{\partial}{\partial y}\left(\frac{1}{\sqrt{3}}\right)^2 \cdot +2 \qquad = +\,0{,}084616$

$\frac{\partial}{\partial y^2}\left(\frac{1}{\sqrt{3}}\right)^3 - 1{,}72342 = -\,0{,}014998$

$\frac{\partial}{\partial y^3}\left(\frac{1}{\sqrt{3}}\right)^4 - 68{,}6810 = -\,0{,}122965$

$\frac{\partial}{\partial y^4}\left(\frac{1}{\sqrt{3}}\right)^5 - 448{,}903 = -\,0{,}165275$

$\frac{\partial}{\partial y^5}\left(\frac{1}{\sqrt{3}}\right)^6 + 2189{,}11 = +\,0{,}165781$

$\frac{\partial}{\partial y^6}\left(\frac{1}{\sqrt{3}}\right)^7 + 91742{,}37 = +\,1{,}429030$

0,070534
40857

−0,111385
9931

−0,101454

+0,007208
2723

9931

$$\frac{\partial^6 L_{bc}}{\partial x^6} = +7{,}860496 \{ 0{,}268591 \cdot 0{,}080774 + 0{,}232572 \cdot 0{,}071119 + 0{,}110190 \cdot 0{,}073750 + 0{,}012396 \cdot 0{,}096270 - 0{,}007229 \cdot 0{,}177750 \}$$

+0,021 6952 −0,0012850
0,016 5360
0,008 1265
0,001 1934
0,0475511
12850
+0,0462661

$$\frac{\partial L}{\partial x^6} = +0{,}363674\ \Delta a$$

$$\frac{\partial^6 L_{bc}}{\partial x^4 \partial y^2} = -7{,}860496 \{ 0{,}268591 \cdot 0{,}075840 + 0{,}232572 \cdot 0{,}057600 + 0{,}110190 \cdot 0{,}052000 + 0{,}012396 \cdot 0{,}060000 - 0{,}007229 \cdot 0{,}100000 \}$$

+0,020 3699 − 0,0007229
+0,013 3927
0,005 7299
0,0007438
0,0372363 402362
7229 7229
0,0365134 0,395134

$$\frac{\partial L}{\partial x^4 \partial y^2} = -0{,}310595\ \Delta a$$

$$\frac{\partial^6 L_{bc}}{\partial x^2 \partial y^4}\ +7{,}860496 \{ 0{,}268591 \cdot \cancel{0{,}077504}\ 0{,}075840 + 0{,}077504 \cdot 0{,}016800 + 0{,}091092 \cdot 0{,}052 + 0{,}044272 \cdot 0{,}157750 + 0{,}004269 \cdot \cancel{0{,}177}\ 0{,}177750 \}$$

+0,0203699
0,0013021
0,0047368
0,0069839
0,0007588
0,0341515

$$\frac{\partial L}{\partial x^2 \partial y^4} = +0{,}268448$$

$$\frac{\partial^6 L}{\partial y^6} = -7{,}860496 \{ 0{,}268591 \cdot 0{,}075840 + 0{,}077504 \cdot 0{,}016800 + 0{,}091092 \cdot 0{,}052 + 0{,}044272 \cdot 0{,}08 + 0{,}004269 \cdot 0{,}1 \}$$

+0,0203699
0,0013021
0,0047368
0,0035418
4269
0,0303775

$$+7{,}860496 \{ +0{,}044272 \cdot 0{,}077750 + 0{,}077750 \cdot 0{,}004269 \}$$

0,0037741 44292 0,048541

$$\frac{\partial^6 L}{\partial y^6} = -0{,}238782$$

$$\frac{\partial^6 L}{\partial x^2 \partial y^4} + \frac{\partial^6 L}{\partial y^6} = +0{,}029666$$

$$\frac{\partial}{\partial x} \operatorname{arctg} \frac{bc}{a\sqrt{a^2+b^2+c^2}} = \left(\frac{bc}{a^2+c^2} + \frac{bc}{(a^2+b^2)}\right)\frac{1}{r}$$

$$\frac{\partial}{\partial y} \operatorname{arctg} \frac{bc}{a\sqrt{a^2+b^2+c^2}} = -\frac{ac}{a^2+b^2}\frac{1}{r}.$$

$$\frac{\partial}{\partial x} \log \sqrt{1+\frac{a^2}{b^2}} \cdot \frac{1+\sqrt{1+\frac{b^2}{c^2}}}{1+\sqrt{1+\frac{a^2}{c^2}+\frac{b^2}{c^2}}} = -\frac{ac}{(a^2+b^2)}\frac{1}{r}$$

$$\frac{\partial}{\partial y} \log \sqrt{1+\frac{a^2}{b^2}} \frac{1+\sqrt{1+\frac{b^2}{c^2}}}{1+\sqrt{1+\frac{a^2}{c^2}+\frac{b^2}{c^2}}} =$$

$n = b \quad b = c \quad a = a$

$n = a \quad b = c \quad a = b.$

$$\Bigg|_{b_1}^{b_2} \; - \log \frac{1+\sqrt{\frac{a_1^2}{c_1^2}+\frac{b_2^2}{c_1^2}+1}}{1+\sqrt{\frac{a_2^2}{c_1^2}+\frac{b_2^2}{c_1^2}+1}} \cdot \frac{1+\sqrt{\frac{a_2^2}{c_2^2}+\frac{b_2^2}{c_2^2}+1}}{1+\sqrt{\frac{a_1^2}{c_2^2}+\frac{b_2^2}{c_2^2}+1}}$$

$$+ \log \frac{1+\sqrt{\frac{a_1^2}{c_1^2}+\frac{b_1^2}{c_1^2}+1}}{1+\sqrt{\frac{a_2^2}{c_1^2}+\frac{b_1^2}{c_1^2}+1}} \cdot \frac{1+\sqrt{\frac{a_2^2}{c_2^2}+\frac{b_1^2}{c_2^2}+1}}{1+\sqrt{\frac{a_1^2}{c_2^2}+\frac{b_1^2}{c_2^2}+1}}$$

$$\Bigg|_{a_1}^{a_2} \; - \log \frac{1+\sqrt{\frac{b_1^2}{c_1^2}+\frac{a_2^2}{c_1^2}+1}}{1+\sqrt{\frac{b_2^2}{c_1^2}+\frac{a_2^2}{c_1^2}+1}} \cdot \frac{1+\sqrt{\frac{b_2^2}{c_2^2}+\frac{a_2^2}{c_2^2}+1}}{1+\sqrt{\frac{b_1^2}{c_2^2}+\frac{a_2^2}{c_2^2}+1}}$$

$$+ \log \frac{1+\sqrt{\frac{b_1^2}{c_1^2}+\frac{a_1^2}{c_1^2}+1}}{1+\sqrt{\frac{b_2^2}{c_1^2}+\frac{a_1^2}{c_1^2}+1}} \cdot \frac{1+\sqrt{\frac{b_2^2}{c_2^2}+\frac{a_1^2}{c_2^2}+1}}{1+\sqrt{\frac{b_1^2}{c_2^2}+\frac{a_1^2}{c_2^2}+1}}$$

$b = c$

$a = a.$

$n = b$

$$\frac{\partial^2 L_{bc}}{\partial x^2} = \| bc \left\{ \frac{1}{a^2+c^2} + \frac{1}{a^2+b^2} \right\} \frac{1}{r} = +7{,}860496 \left\{ 0{,}268591 \cdot 0{,}177750 \right\} \qquad \frac{1}{f_6} \frac{\partial^2 L_{bc}}{\partial x^2} = +0{,}375276\,\Delta a$$

$$\frac{\partial^2 L_{tc}}{\partial y^2} = -\| bc \frac{1}{r} \frac{1}{a^2+b^2} = -7{,}860496 \cdot 0{,}268591 \cdot 0{,}1 \qquad \frac{1}{f_6} \frac{\partial^2 L_{bc}}{\partial y^2} = -0{,}211126\,\Delta a$$

$$\frac{\partial^4 L_{bc}}{\partial x^4} = +7.860496 \left\{ 0{,}268591 \cdot 0{,}073750 + 0{,}116256 \cdot 0{,}096270 + 0{,}018365 \cdot 0{,}177750 \right\} \qquad \frac{1}{f_6} \frac{\partial^4 L_{bc}}{\partial x^4} = +0{,}269340\,\Delta a$$

0,0198086
0,0111920
0,0032644
0,0342650

$$\frac{\partial^4 L_{bc}}{\partial x^2 \partial y^2} = -7.860496 \left\{ 0{,}268591 \cdot 0{,}052000 + 0{,}116256 \cdot 0{,}060000 + 0{,}018365 \cdot 0{,}1 \right\} \qquad \frac{1}{f_6} \frac{\partial^4 L_{bc}}{\partial x^2 \partial y^2} = -0{,}179051\,\Delta a$$

0,0139667
0,0069754
0,0018365
0,0227786

$$\frac{\partial^4 L_{bc}}{\partial y^4} = -7.860496 \left\{ -0{,}268591 \cdot 0{,}052000 + 0{,}038752 \cdot 0{,}080000 + 0{,}015182 \cdot 0{,}1 \right\} \qquad \frac{1}{f_6} \frac{\partial^4 L_{bc}}{\partial y^4} = +0{,}146088\,\Delta a$$

0,0139667
0,0031002
0,0015182
0,0185851

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

Als ey XZ [illegible] $y = 0$ [illegible]

$$F = -\sin\alpha \Big\{ ml\left(\frac{\partial^2 V}{\partial x \partial z}\right)_{000}(z'-z) + \frac{1}{8}\left(ml^3 - \int l^3 dm\right)\left[\frac{\partial^3 V}{\partial x^3} + \frac{\partial^3 V}{\partial x \partial y^2}\right]_{000}$$

$$+ \frac{ml}{2}\left(\frac{\partial^3 V}{\partial x \partial z^2}\right)_{000}(z'^2 - z^2) + \frac{1}{8}\left(ml^3 z' - \int dm\, l^3 . z\right)\left[\frac{\partial^4 V}{\partial x^3 \partial z} + \frac{\partial^4 V}{\partial x \partial y^2 \partial z}\right]_{000}$$

$$+ \frac{ml}{6}\left(\frac{\partial^4 V}{\partial x \partial z^3}\right)_{000}(z'^3 - z^3) \Big\}$$

$$- \sin 2\alpha \Big\{ \frac{1}{2}\left(ml^2 + \int l^2 dm\right)\left(\frac{\partial^2 V}{\partial x^2} - \frac{\partial^2 V}{\partial y^2}\right)_{000} + \frac{1}{24}\left(ml^4 + \int l^4 dm\right)\left(\frac{\partial^4 V}{\partial x^4} - \frac{\partial^4 V}{\partial y^4}\right)_{000}$$

$$+ \frac{1}{2}\left(ml^2 z' + \int l^2 dm . z\right)\left(\frac{\partial^3 V}{\partial x^2 \partial z} - \frac{\partial^3 V}{\partial y^2 \partial z}\right)_{000}$$

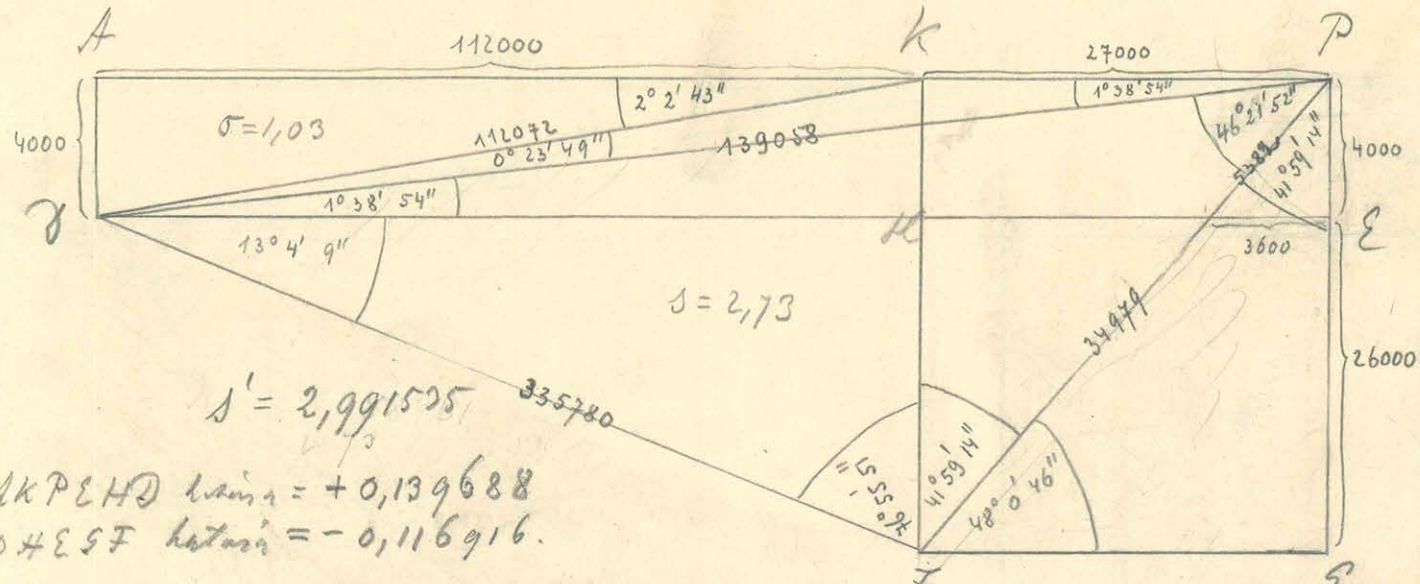
$$+ \frac{1}{4}\left(ml^2 z'^2 + \int l^2 dm\, z^2\right)\left(\frac{\partial^4 V}{\partial x^2 \partial z^2} - \frac{\partial^4 V}{\partial y^2 \partial z^2}\right)_{000}$$

$$- \sin 3\alpha \Big\{ \frac{1}{8}\left(ml^3 - \int dm\, l^3\right)\left(\frac{\partial^3 V}{\partial x^3} - 3\frac{\partial^3 V}{\partial x \partial y^2}\right)_{000}$$

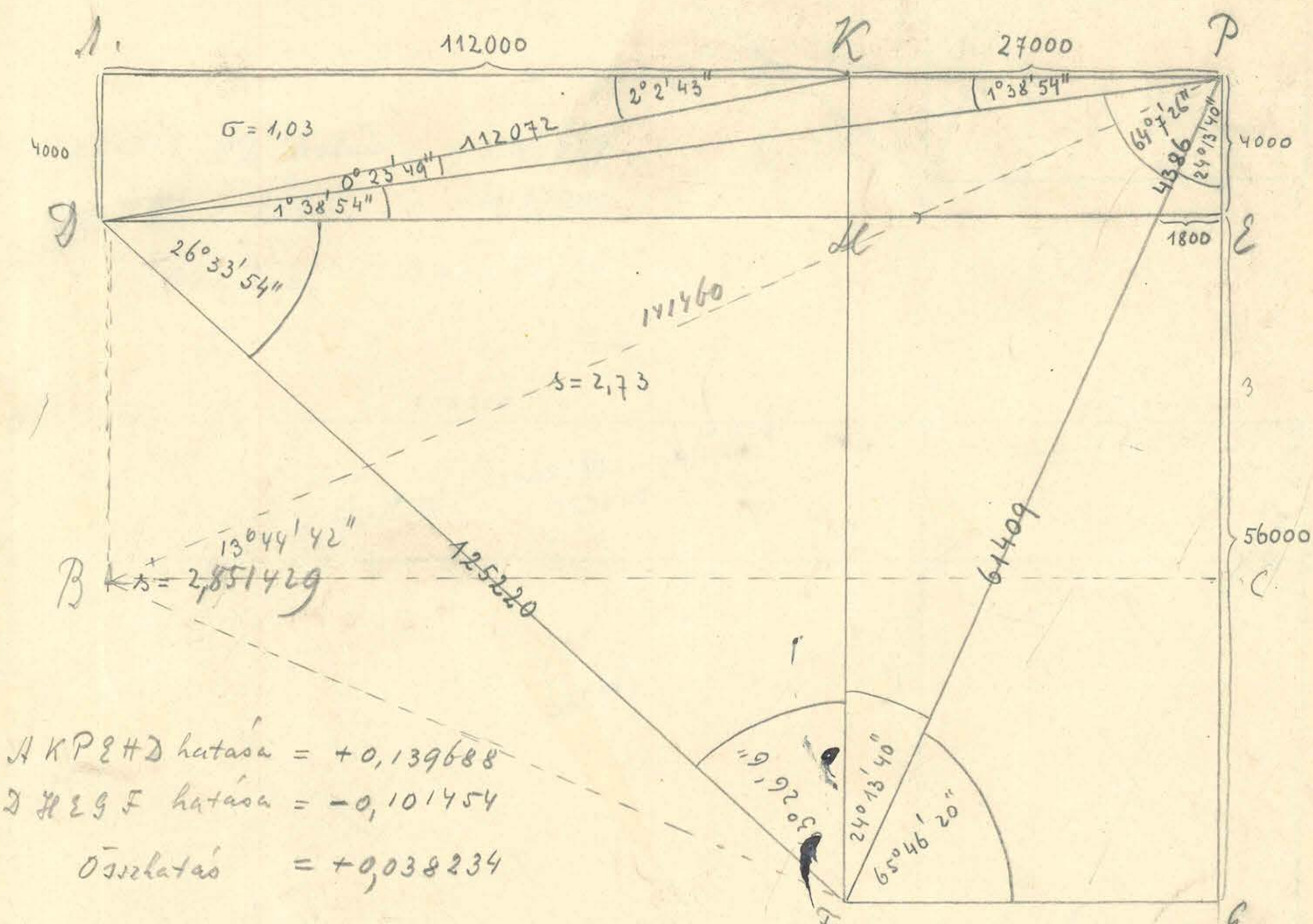
$$+ \frac{1}{8}\left(ml^3 z' - \int dm\, l^3 . z\right)\left(\frac{\partial^4 V}{\partial x^3 \partial z} - 3\frac{\partial^4 V}{\partial x \partial y^2 \partial z}\right)_{000}$$

$$- \sin 4\alpha \Big\{ \frac{1}{48}\left(ml^4 + \int dm\, l^4\right)\left[\left(\frac{\partial^4 V}{\partial x^4} + \frac{\partial^4 V}{\partial y^4}\right)_{000} - 6\left(\frac{\partial^4 V}{\partial x^2 \partial y^2}\right)_{000}\right] \Big\}$$

$s' - s = 0{,}261535 \quad \frac{s}{s'-s} = 10{,}43837$



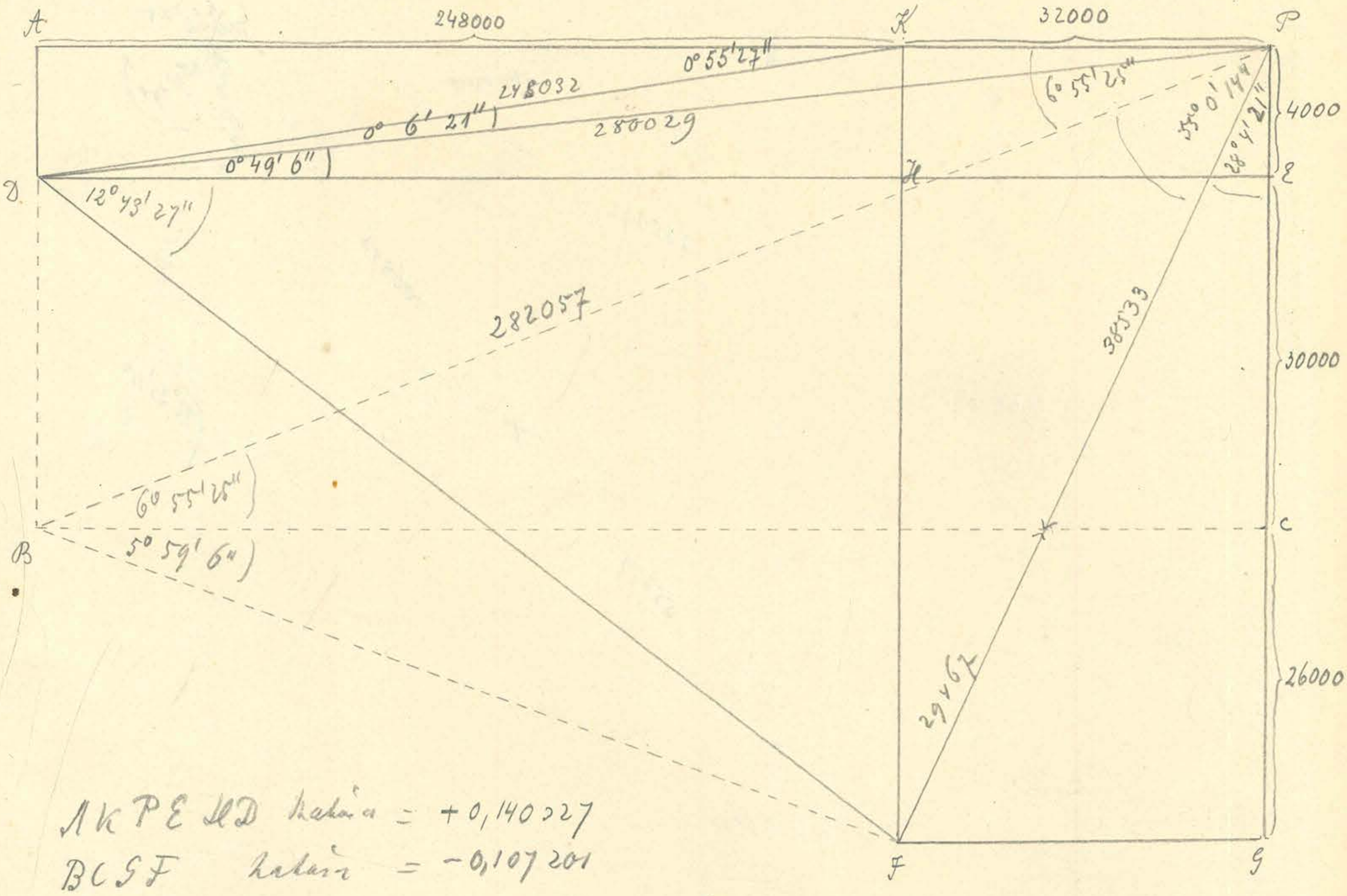

AKPEHD hatása = +0,139688
DHESF hatása = −0,116916.
Összhatás = +0,022772




AKPEHD hatása = +0,139688
DHESF hatása = −0,101454
Összhatás = +0,038234

Az AKPEHD hatása = +0,139688
a BCSF hatása = −0,089723
Összhatás = +0,049965

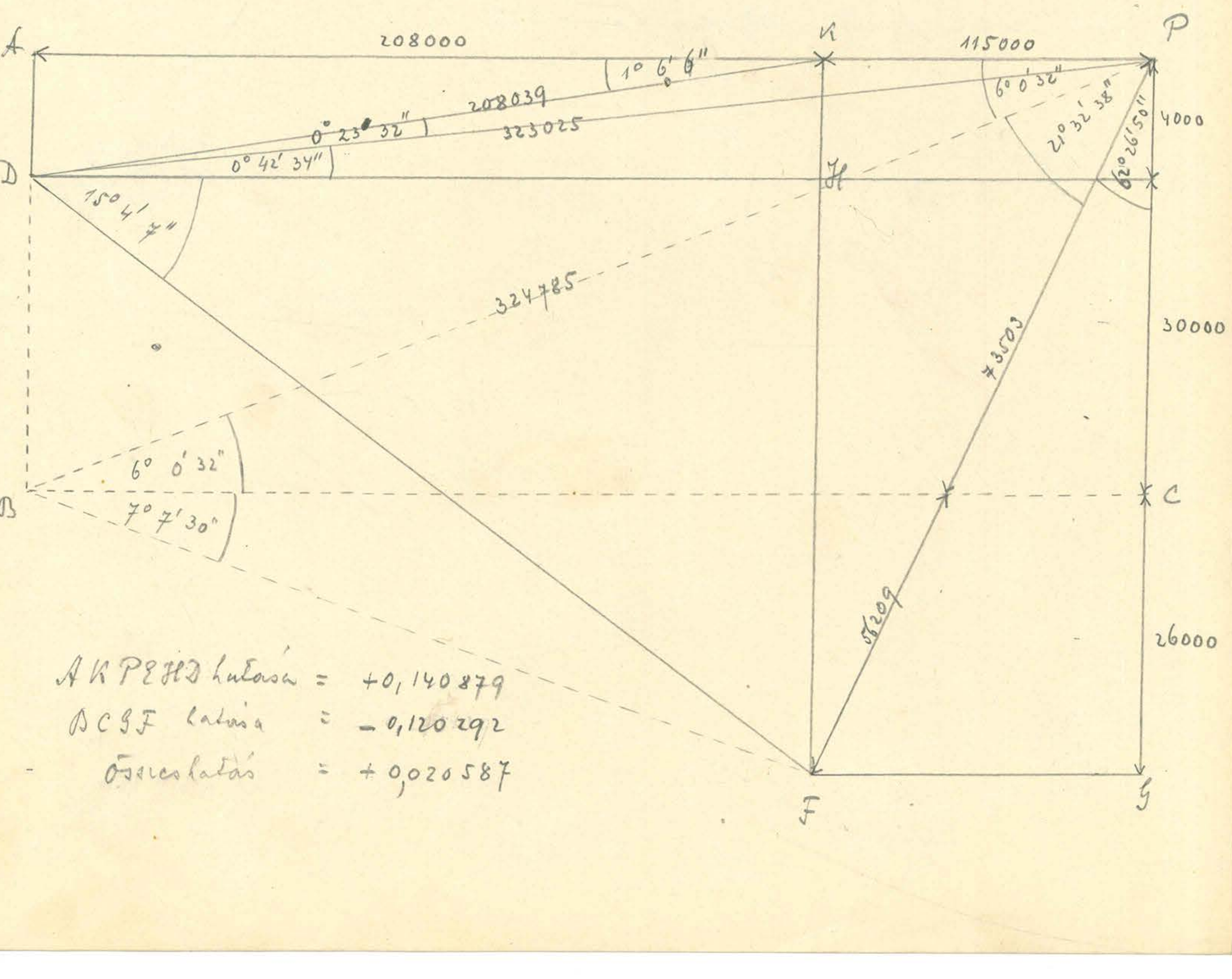
MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA




AKPEHD hatása = +0,140327

BCGF hatása = −0,107201

Összhatás = +0,033126

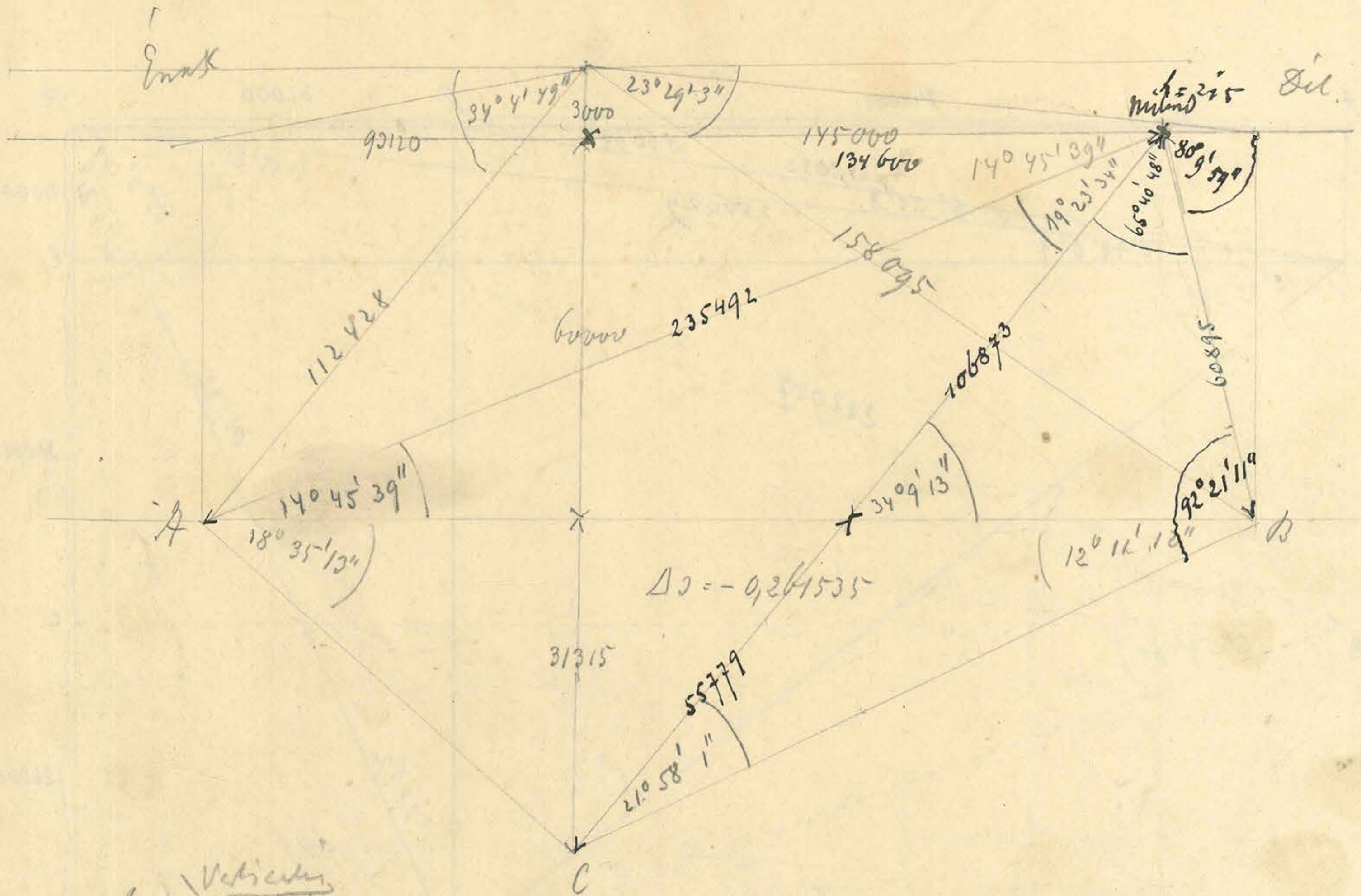



AKPEHD hatása = +0,140879

BCGF hatása = −0,120292

Összhatás = +0,020587

Alpesi



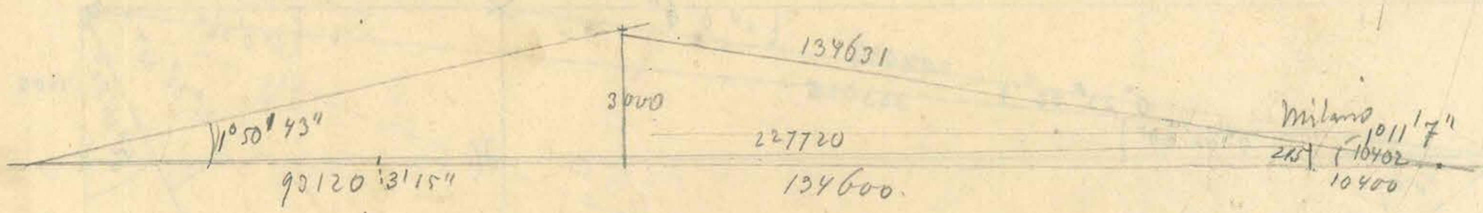

Vertikális

ABC hatása = −0,124460

horizontális

ABC hatása (Milanóban) horizontális komponens észak felé = −0,07604815

Tenger színe feletti alpesi tömeg . km




MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

$$\frac{\partial^6 V}{\partial x^4 \partial y^2} = -\frac{\partial^5 U_{be}}{\partial x^3 \partial y^2} = +15{,}720992\{\cancel{0{,}412035} + 0{,}211389 \cdot 0{,}5 - 0{,}103203 \cdot 0{,}5 - 0{,}077426 \cdot 0{,}5\}$$

0,1056945
0,0516015
0,0387130
−0,0903145

+0,0153800

$$\frac{\partial^6 V}{\partial x^4 \partial y^2} = 0{,}241789 = \frac{\partial^6 V}{\partial x^2 \partial y^4}$$

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

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$$\frac{\partial^8 V}{\partial x^8} = -\frac{\partial^7 U_{be}}{\partial x^7} = 15{,}720992\{-0{,}412035 \cdot \overset{14{,}834220}{15{,}165780} - 0{,}352315 \cdot 3{,}165275$$
$$+ 0{,}344010 \cdot 0{,}122936 - 0{,}774260 \cdot 0{,}485002 - 0{,}198220 \cdot 0{,}584616$$
$$+ 0{,}150471 \cdot 0{,}705689\}$$

−6,1270521
1,1151739
0,3755176
0,1158826
7,7336262
1484769
7,5851493

+0,0422912
0,1061857

$$\frac{\partial^8 V}{\partial x^8} = +119{,}246071$$

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$$\frac{\partial^8 V}{\partial x^6 \partial y^2} = -\frac{\partial^7 U_{be}}{\partial x^5 \partial y^2} = +15{,}720992\{-0{,}412035 \cdot 15 - 0{,}352315 \cdot 3 \; \cancel{0{,}344010}$$
$$- 0{,}774260 \cdot 0{,}5 - 0{,}198220 \cdot 0{,}5 + 0{,}150471 \cdot 0{,}5$$

−6,1955250
~~5,69...~~
1,0569450
0,3871300
0,0991100
7,738710
0752355
7,663474

$$\frac{\partial^8}{\partial x^6 \partial y^2} = -120{,}477413$$

---

$$\frac{\partial^8 V}{\partial x^4 \partial y^4} = -\frac{\partial^7 U}{\partial x^3 \partial y^4} = +15{,}720992\{0{,}412035 \cdot 15 + 0{,}070463 \cdot 6 + 0{,}070463 \cdot 9$$
$$+ 0{,}036062 \cdot 9 - 0{,}005301 \cdot 1{,}5 - 0{,}005301 \cdot 3 + 0{,}077426 \cdot 0{,}5$$
$$+ 0{,}011274 \cdot 1{,}5 + 0{,}001747 \cdot 0{,}5$$

+6,1955250
0,4227780
0,6341676
0,3245580
~~0,38...~~
7,5770280
584975

−0,0238545
+7,6335255
+7,6096710

+0,0387130
0,0169110
0,0008735

$$\frac{\partial^8}{\partial x^4 \partial y^4} = +119{,}631577.$$

$\psi = \frac{1}{a^2+c^2} = \frac{1}{4{,}861712} = \underline{0{,}2056889}\ \frac{1}{b^2}$ $\quad c^2 = 3{,}861712$

$c^4 = 14{,}912820$

$\psi^2 = 0{,}04230792$ $\quad c^6 = 57{,}589016$

$\psi^3 = 0{,}00870227$

$\psi^4 = 0{,}00178996$

$\psi^5 = 0{,}000368175$

$\psi^6 = 0{,}0000757295$

$\psi^7 = 0{,}0000155767$

$+ 2 \cdot 0{,}04230792 \quad = + \underline{0{,}084616}\ \frac{1}{b^3}$

$- 1{,}723424 \cdot 0{,}00870227 \quad = - \underline{0{,}014998}\ \frac{1}{b^4}$

$- 68{,}681088 \cdot 0{,}00178996 \quad = - \underline{0{,}129805}\ \frac{1}{b^5}$

$- 448{,}903200 \quad 0{,}000368175 \quad = - \underline{0{,}165275}$

$+ \overset{240}{9{,}121260} \cdot 0{,}0000757295 \quad = + 2189{,}102400 \quad = \underline{+0{,}165780}$

$\overset{720}{+} 127{,}420284 \cdot 0{,}0000155767 \quad = + 91742{,}60160 = + \cancel{0}1{,}429047$

$\psi = 0{,}2056889$ $\quad 2{,}861712$

$+ 2 \cdot \psi^2$

$- \psi^3\, 1{,}723424$

$- 68{,}681088\ \psi^4$

$+$

$2{,}861712 \cdot 24 \cdot 0{,}00178996$

$a=1 \quad b=1 \quad c=1{,}965124$

$c = 1{,}965124$

$r^2 = 5{,}861712_3$ $\qquad \log r^2 = 0{,}768026$

$c^2 = 3{,}861712_3$ $\qquad \log r = 0{,}384013$

$\log \frac{1}{r} = 0{,}615987-1$

$\log \frac{1}{r} = 0{,}615987-1 \qquad \frac{1}{r} = 0{,}413035$

$l\left(\frac{1}{r}\right)^3 = 0{,}847961-2 \qquad \left(\frac{1}{r}\right)^3 = 0{,}0704143$

$l\left(\frac{1}{r}\right)^5 = 0{,}079935-2 \qquad \left(\frac{1}{r}\right)^5 = 0{,}0120208 \qquad 0{,}036062$

$l\left(\frac{1}{r}\right)^7 = 0{,}311909-3 \qquad \left(\frac{1}{r}\right)^7 =$

$l\left(\frac{1}{r}\right)^9 = 0{,}543883-4 \qquad \left(\frac{1}{r}\right)^9 =$

$\quad 201974-1$

$l\left(\frac{1}{r}\right)^{11} = 0{,}775857-5 \qquad \left(\frac{1}{r}\right)^{11} =$

$l\left(\frac{1}{r}\right)^{13} = 0{,}007831-5 \qquad \left(\frac{1}{r}\right)^{13} =$

$0{,}311909-3$
$1{,}176091$
$0{,}488000-2$

$0{,}311909-3$
$2{,}352183$
$0{,}664092-1$

$153805$
$3076110$
$461415$

$15\frac{1}{r^7} = 0{,}030761$

~~$90\frac{1}{r^7} = 0{,}184566$~~

~~$225\frac{1}{r^7} = 0{,}461415$~~

~~$45\frac{1}{r^7} = 0{,}092283$~~

$30\frac{1}{r^7} = 0{,}061522$

$2{,}021189$
$0{,}543883-4$
~~$2{,}002166$~~
~~$0{,}522694$~~
$0{,}565072-2$

$0{,}543883-4$
$3{,}674402$
$0{,}218285$

$0{,}543883-4$
$2{,}623249$
$0{,}167132-1$

$0{,}543883-4$
$2{,}975432$
$0{,}519315-1$

$105\left(\frac{1}{r}\right)^9 = 0{,}036734$

$4725\left(\frac{1}{r}\right)^9 = 1{,}653044$

$420\left(\frac{1}{r}\right)^9 = 0{,}146927$

$945\left(\frac{1}{r}\right)^9 = 0{,}330609$

$0{,}775857-5$
$2{,}975432$
$0{,}751289-2$

$0{,}775857-5$
$4{,}151523$
$0{,}927380-1$

$0{,}775857-5$
$3{,}452553$
$0{,}228410-1$

$945\left(\frac{1}{r}\right)^{11} = 0{,}056401$

$14175\left(\frac{1}{r}\right)^{11} = 0{,}846018$

$2835\left(\frac{1}{r}\right)^{11} = 0{,}169204$ ~~$846006020$~~

$0{,}007831-5$
$4{,}016824$
$0{,}024655-1$

$0{,}105841$

15,720992

40 7,860496 . 0,705689 . 0,413035.

0,895450
0,848613 − 1
0,615987 − 1

0,360050 2,291130

0,895450
0,615987 − 1
0,698970

0,210407 − 1 1,623330

1,965124 0,413035

0,292289
615987 − 1

0,909376 − 1 39° 0' 54"

0,6806784
8727
2618

0,6818129

5,4545032

$$N: -\frac{\partial^2 L_{bc}}{\partial x^2} = -15,720992\left\{0,413035 . 0,584616 + 0,070463 . 0,705689\right\}$$

0,241466g
0,049725[illegible]
0,291192

$$\frac{\partial^4 V}{\partial x^2} = -4,577827$$

$$\frac{\partial^4 V}{\partial x^2 \partial y^2} = -\frac{\partial^2 L_{bc}}{\partial x \partial y^2} = +15,720992\left\{0,413035 . 0,5 + 0,5 . 0,070463\right\}$$ 0,485002

0,2065175
0352315
0,2417490

$$\frac{\partial^4 V}{\partial x^2 \partial y^2} = +3,800534$$

$$\frac{\partial^6 V}{\partial x^6} = -\frac{\partial^5 L_{bc}}{\partial x^5} = -15,720992\left\{0,413035 . \overset{0,122926}{\cancel{[illegible]}} + 0,211289 . \overset{0,485002}{\cancel{0,514998}}\right.$$

$$\left. - 0,103203 . 0,584616 - 0,077426 . 0,705689\right\}$$

− 0,0507769
− 0,0603347
− 0,0546387

− 0,1657497
+ 0,1025241

− 0,0632256

$$\frac{\partial^6 V}{\partial x^6} = +0,993969$$

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

1916 Júliusban a [illegible]
[illegible] [illegible]

Ms 5102/7

Ms 5102/7

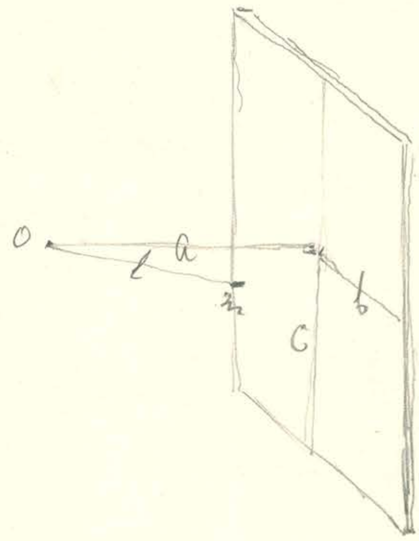
Nagy ontogo

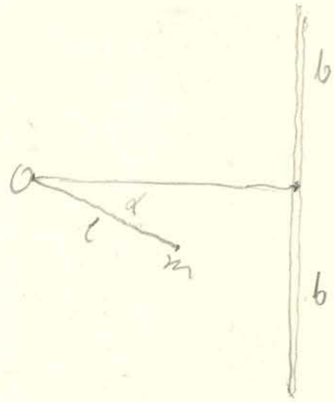
vonzása

Lap. forg. momentuma.

1915 Nov. 3.

$\frac{X}{1}$




Egy tömegpontra $F_1 = ml\,Y\cos\alpha - ml\,X\sin\alpha$

$$\frac{\partial F_1}{\partial\alpha} = ml\frac{\partial Y}{\partial\alpha}\cos\alpha - ml\,Y\sin\alpha - ml\frac{\partial X}{\partial\alpha}\sin\alpha - ml\,X\cos\alpha.$$

$$Y = 2f\sigma\Delta a\left\{\frac{1}{2}\log\frac{a^2+b^2+l^2-2l(a\cos\alpha+b\sin\alpha)}{a^2+b^2+l^2-2l(a\cos\alpha-b\sin\alpha)} - \log\frac{c+\sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha+b\sin\alpha)}}{c+\sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha-b\sin\alpha)}}\right\}$$

$$X = 2f\sigma\Delta a\left\{\operatorname{arctg}\frac{c(b-l\sin\alpha)}{(a-l\cos\alpha)\sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha+b\sin\alpha)}} + \operatorname{arctg}\frac{c(b+l\sin\alpha)}{(a-l\cos\alpha)\sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha-b\sin\alpha)}}\right\}$$

$$\frac{\partial Y}{\partial\alpha} = 2f\sigma\Delta a\,lc\left\{\frac{a\sin\alpha-b\cos\alpha}{a^2+b^2+l^2-2l(a\cos\alpha+b\sin\alpha)}\,\frac{1}{\sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha+b\sin\alpha)}}\right.$$

$$\left. - \frac{a\sin\alpha+b\cos\alpha}{a^2+b^2+l^2-2l(a\cos\alpha-b\sin\alpha)}\,\frac{1}{\sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha-b\sin\alpha)}}\right\}$$

$$\frac{\partial X}{\partial\alpha} = 2f\sigma\Delta a\,cl\left\{\frac{1}{c^2(b-l\sin\alpha)^2+(a-l\cos\alpha)^2 G^2}\left[(l-(a\cos\alpha+b\sin\alpha))G - \frac{(a-l\cos\alpha)(b-l\sin\alpha)(a\sin\alpha-b\cos\alpha)}{G}\right]\right.$$

$$\left. + \frac{1}{c^2(b+l\sin\alpha)^2+(a-l\cos\alpha)^2 G'^2}\left[(l+a\cos\alpha-b\sin\alpha)G' - \frac{(a-l\cos\alpha)(b+l\sin\alpha)(a\sin\alpha+b\cos\alpha)}{G'}\right]\right\}$$

$$G = \sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha+b\sin\alpha)}$$

$$G' = \sqrt{a^2+b^2+c^2+l^2-2l(a\cos\alpha-b\sin\alpha)}$$

$$\left(\frac{\partial F_1}{\partial \alpha}\right)_{\alpha=0} = ml\left(\frac{\partial Y}{\partial \alpha}\right)_{\alpha=0} - ml X_{\alpha=0} \qquad \left(\frac{\partial F_1}{\partial \alpha}\right)_{\alpha=\pi} = -ml\left(\frac{\partial Y}{\partial \alpha}\right)_{\alpha=\pi} + ml X_{\alpha=\pi}$$

$$\left(\frac{\partial F_1}{\partial \alpha}\right)_{\alpha=+\frac{\pi}{2}} = -ml Y_{\alpha=+\frac{\pi}{2}} - ml\left(\frac{\partial X}{\partial \alpha}\right)_{\alpha=+\frac{\pi}{2}} \qquad \left(\frac{\partial F_1}{\partial \alpha}\right)_{\alpha=-\frac{\pi}{2}} = +ml Y_{\alpha=-\frac{\pi}{2}} + ml\left(\frac{\partial X}{\partial \alpha}\right)_{\alpha=-\frac{\pi}{2}}$$

$$X_{\alpha=0} = 4 f\sigma \Delta a \operatorname{arctg} \frac{cb}{(a-l)\sqrt{(a-l)^2+b^2+c^2}}$$

$$X_{\alpha=\pi} = 4 f\sigma \Delta a \operatorname{arctg} \frac{cb}{(a+l)\sqrt{(a+l)^2+b^2+c^2}}$$

$$Y_{\alpha=\frac{\pi}{2}} = 2 f\sigma \Delta a \left\{\frac{1}{2}\log\frac{a^2+(b-l)^2}{a^2+(b+l)^2} - \log\frac{c+\sqrt{a^2+(b-l)^2+c^2}}{c+\sqrt{a^2+(b+l)^2+c^2}}\right\}$$

$$Y_{\alpha=-\frac{\pi}{2}} = 2 f\sigma \Delta a \left\{\frac{1}{2}\log\frac{a^2+(b+l)^2}{a^2+(b-l)^2} - \log\frac{c+\sqrt{a^2+(b+l)^2+c^2}}{c+\sqrt{a^2+(b-l)^2+c^2}}\right\} = -Y_{\alpha=\frac{\pi}{2}}$$

$$\left(\frac{\partial X}{\partial \alpha}\right)_{\alpha=+\frac{\pi}{2}} = -2 f\sigma \Delta a . cl \left\{\frac{(b-l)(2a^2+(b-l)^2+c^2)}{c^2(b-l)^2+a^2(a^2+(b-l)^2+c^2)}\frac{1}{g} + \frac{(b+l)(2a^2+(b+l)^2+c^2)}{c^2(b+l)^2+a^2(a^2+(b+l)^2+c^2)}\frac{1}{g'}\right\}$$

$$g = \sqrt{a^2+(b-l)^2+c^2}$$
$$g' = \sqrt{a^2+(b+l)^2+c^2}$$

$$\left(\frac{\partial X}{\partial \alpha}\right)_{\alpha=-\frac{\pi}{2}} = -\left(\frac{\partial X}{\partial \alpha}\right)_{\alpha=+\frac{\pi}{2}}$$

$$\left(\frac{\partial Y}{\partial \alpha}\right)_{\alpha=0} = -4 f\sigma \Delta a . bcl \frac{1}{(a-l)^2+b^2} \cdot \frac{1}{\sqrt{(a-l)^2+b^2+c^2}}$$

$$\left(\frac{\partial Y}{\partial \alpha}\right)_{\alpha=\pi} = +4 f\sigma \Delta a . bcl \frac{1}{(a+l)^2+b^2} \cdot \frac{1}{\sqrt{(a+l)^2+b^2+c^2}}$$

MAGYAR TUDOMÁNYOS AKADÉMIA KONYVTÁRA

$$F = f\sigma \frac{\Delta a}{b}\Big[ -\sin 2\alpha \left\{1{,}957230 \int l^2 dm \cos 2\varepsilon + \ldots \right.$$
$$-\sin 4\alpha \left\{+0{,}334207 \frac{1}{b^2}\int l^4 dm \cos 4\varepsilon - \ldots \right.$$
$$-\sin 6\alpha \left\{-0{,}099952 \frac{1}{b^4}\int l^6 dm \cos 6 \ldots \right.$$
$$-\sin 8\alpha \left\{-0{,}035715 \frac{1}{b^6}\int l^8 dm \ldots \right.$$

$\sqrt{a^2+(b-l)^2+c^2} = \sqrt{(a-l)^2+b^2+c^2} = 2{,}20951$ $\quad \log V = 0{,}344296$

$\sqrt{a^2+(b+l)^2+c^2} = \sqrt{(a+l)^2+b^2+c^2} = 2{,}88326$ $\quad \log V = 0{,}459884$

$a^2+(b-l)^2+c^2 = 4{,}881929 \mid a^2+(b+l)^2+c^2 = 8{,}313185$

$a-l = 0{,}142186$ $\quad (a-l)^2 = 0{,}020217$

$a+l = 1{,}857814$ $\quad (a+l)^2 = 3{,}451473$

$b-l = a-l$ $\quad c^2 = 3{,}861712$

$\frac{a}{b} = \underline{a} \quad \frac{b}{b} = \underline{c} \qquad b = 149{,}1$

$\frac{l}{b} = \underline{l} \qquad l = 127{,}9$

ha $\underline{a} = 1 \quad \underline{b} = 1 \quad \underline{c} = 1{,}965124 \quad \underline{l} = 0{,}857814.$

$X_{\alpha=0} = +5{,}649068\, f\sigma\Delta a \qquad X_{\alpha=\pi} = +1{,}406464\, f\sigma\Delta a$

$y_{\alpha=\frac{\pi}{2}} = -1{,}173978\, f\sigma\Delta a \qquad y_{\alpha=-\frac{\pi}{2}} = +1{,}173978\, f\sigma\Delta a$

$\left(\frac{\partial X}{\partial\alpha}\right)_{\alpha=+\frac{\pi}{2}} = \cancel{-0{,}326384}\ -1{,}192118\, f\sigma\Delta a \qquad \left(\frac{\partial X}{\partial\alpha}\right)_{\alpha=-\frac{\pi}{2}} = +\cancel{0{,}326384}\ +1{,}192118\, f\sigma\Delta a$

$\left(\frac{\partial y}{\partial\alpha}\right)_{\alpha=0} = -2{,}991248\, f\sigma\Delta a \qquad \left(\frac{\partial y}{\partial\alpha}\right)_{\alpha=\pi} = +0{,}537596\, f\sigma\Delta a$

$\left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=0} = -7{,}411784\, f\sigma\Delta a\, m b \qquad \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=\pi} = +0{,}745327\, f\sigma\Delta a\, m b$

$\left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=+\frac{\pi}{2}} = +2{,}029670\, f\sigma\Delta a m b \qquad \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=-\frac{\pi}{2}} = +2{,}029670\, f\sigma\Delta a m b$

Vid m lömnyonten.

$\left(\frac{\partial F}{\partial\alpha}\right)_0 = \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=0} + \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=\pi} = -6{,}666457\, f\sigma\Delta a\, m\, b$

$\left(\frac{\partial F}{\partial\alpha}\right)_{\frac{\pi}{2}} = \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=\frac{\pi}{2}} + \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=-\frac{\pi}{2}} = +4{,}059340\, f\sigma\Delta a\, m.b$

Kisspollen.

$\ldots dm \cos 2\varepsilon + 0{,}011370 \frac{1}{b^4}\int l^4 dm \cos 2\varepsilon - 0{,}001942 \frac{1}{b^6}\int l^6 dm \cos 2\varepsilon + 0{,}000225 \frac{1}{b^8}\int l^8 dm \cos 2\varepsilon + \cdots \}$

$\ldots l^4 dm \cos 4\varepsilon - 0{,}000352 \frac{1}{b^6}\int l^6 dm \cos 4\varepsilon + 0{,}000062 \frac{1}{b^8}\int l^8 dm \cos 4\varepsilon + \cdots \}$

$\ldots \int l^6 dm \cos 6\varepsilon - 0{,}000035 \frac{1}{b^8}\int l^8 dm \cos 6\varepsilon + \cdots \}$

$\ldots \frac{1}{b^8}\int l^8 dm \cos 8\varepsilon + \cdots \}$

1 m. körmagyarra

$$F_1 = f\sigma\Delta a m b \cdot \underline{l}^2 \Big\{ -\sin 2\alpha\left(1{,}957230 + 0{,}011370\,\underline{l}^2 - 0{,}001942\,\underline{l}^4 + 0{,}000225\,\underline{l}^6\right)$$
$$-\sin 4\alpha\left(+0{,}334207\,\underline{l}^2 - 0{,}000352\,\underline{l}^4 + 0{,}000062\,\underline{l}^6\right)$$
$$-\sin 6\alpha\left(-0{,}099952\,\underline{l}^4 - 0{,}000035\,\underline{l}^6\right)$$
$$-\sin 8\alpha\left(-0{,}035715\,\underline{l}^6\right)$$

$\underline{a}=1$ $\underline{l}=0{,}857814$ re.

$$F_1 = -f\sigma\Delta a m b \Big\{ \sin 2\alpha \cdot 1{,}445668 + \sin 4\alpha\, 0{,}180840 - \sin 6\alpha \cdot 0{,}039834 - \sin 8\alpha \cdot 0{,}010471 \Big\}$$

$$\frac{\partial F_1}{\partial\alpha} = -f\sigma\Delta a m b \Big\{ \cos 2\alpha \cdot 2{,}891336 + \cos 4\alpha \cdot 0{,}723360 - \cos 6\alpha \cdot 0{,}239004 - \cos 8\alpha \cdot 0{,}083768$$

$$\left(\frac{\partial F_1}{\partial\alpha}\right)_0 = \left(\frac{\partial F_1}{\partial\alpha}\right)_\pi = -3{,}291924\, f\sigma\Delta a m b \quad \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=+\frac{\pi}{2}} = \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=-\frac{\pi}{2}} = +2{,}012740\, f\sigma\Delta a m b$$

Két m. körmagyarra:

$$\left(\frac{\partial F}{\partial\alpha}\right)_0 = \left(\frac{\partial F_1}{\partial\alpha}\right)_0 + \left(\frac{\partial F_1}{\partial\alpha}\right)_\pi = -6{,}583848\, f\sigma\Delta a m b \quad \text{és} \quad \left(\frac{\partial F}{\partial\alpha}\right)_{\frac{\pi}{2}, -\frac{\pi}{2}} = +4{,}025480\, f\sigma\Delta a m b.$$

$$\cancel{\left\{\left(\frac{\partial F}{\partial\alpha}\right)_0 + \left(\frac{\partial F}{\partial\alpha}\right)_\pi\right\} - \left\{\left(\frac{\partial F}{\partial\alpha}\right)_{\frac{\pi}{2}} + \left(\frac{\partial F}{\partial\alpha}\right)_{-\frac{\pi}{2}}\right\} = -10{,}609328\, f\sigma\Delta a m b.}$$ ~~erősítésből~~

~~A tűl alvalan levő végső értek.~~

$$\cancel{\left\{\left(\frac{\partial F_1}{\partial\alpha}\right)_0 + \left(\frac{\partial F_1}{\partial\alpha}\right)_\pi\right\} - \left\{\left(\frac{\partial F}{\partial\alpha}\right)_{\frac{\pi}{2}} + \left(\frac{\partial F}{\partial\alpha}\right)_{-\frac{\pi}{2}}\right\}}$$

---

$\underline{a}=3$ $\underline{b}=1$ $\underline{l}=1$ $c=1{,}965124$ értékekre

$$(X)_{\alpha=0} = +1{,}275234\, f\sigma\Delta a \qquad (X)_{\alpha=\pi} = +0{,}428595\, f\sigma\Delta a$$

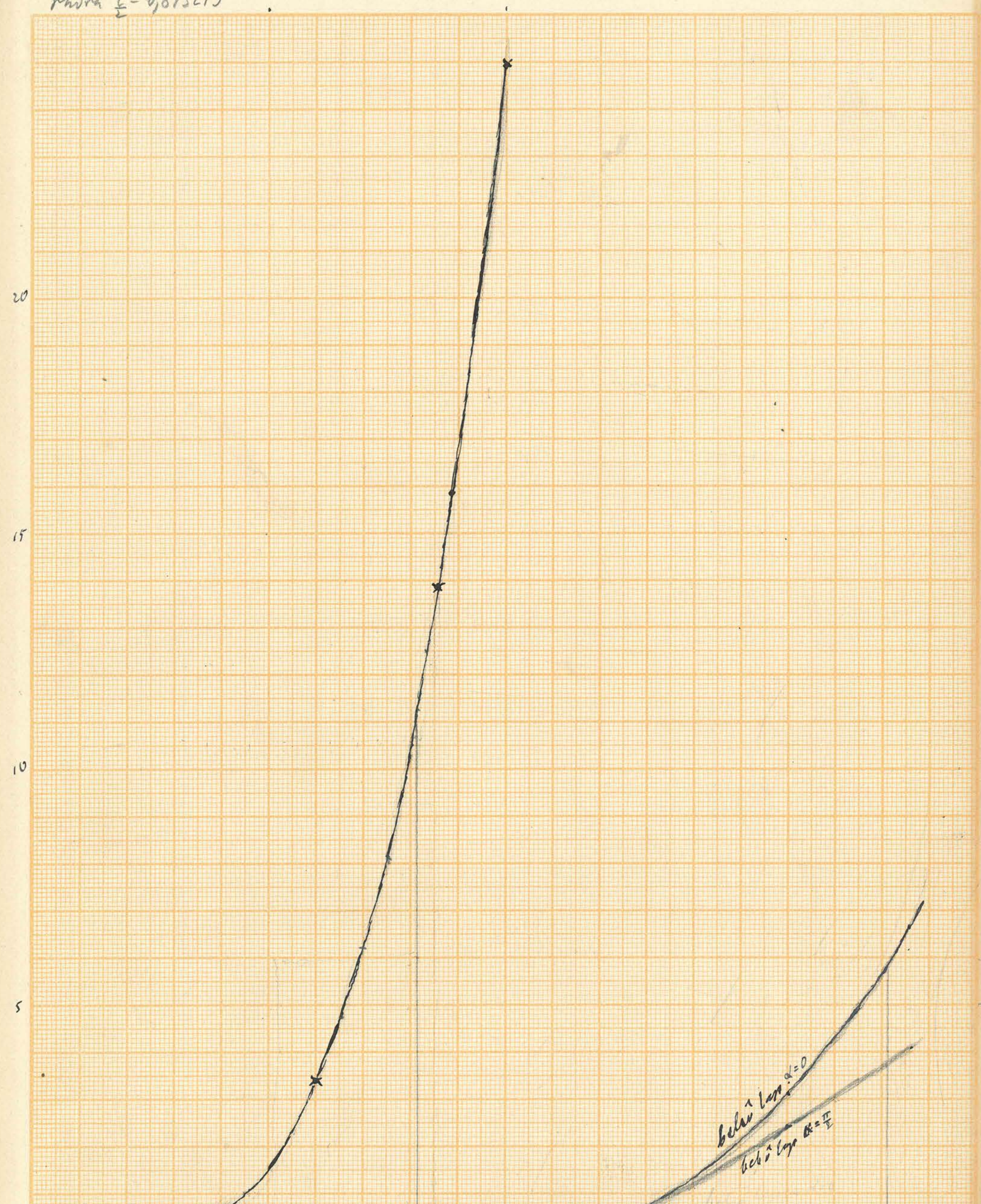
$$(Y)_{\alpha=\frac{\pi}{2}} = -0{,}188665\, f\sigma\Delta a \qquad (Y)_{\alpha=-\frac{\pi}{2}} = +0{,}188665\, f\sigma\Delta a$$

$$\left(\frac{\partial X}{\partial\alpha}\right)_{\alpha=\frac{\pi}{2}} = -0{,}296083\, f\sigma\Delta a \; \cancel{-1{,}215806\, f\sigma\Delta a} \qquad \left(\frac{\partial X}{\partial\alpha}\right)_{\alpha=-\frac{\pi}{2}} = +1{,}215806\, f\sigma\Delta a$$

$$\left(\frac{\partial Y}{\partial\alpha}\right)_{\alpha=0} = -0{,}528105\, f\sigma\Delta a \qquad \left(\frac{\partial Y}{\partial\alpha}\right)_{\alpha=-\frac{\pi}{2}} = +0{,}101234\, f\sigma\Delta a$$

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

radna $\frac{l}{2} = 0,813213$



2

$$\left(\frac{\partial F_1}{\partial \alpha}\right)_{\alpha=0} = -1,803339 \cdot f\sigma\Delta a m b \qquad \left(\frac{\partial F_1}{\partial \alpha}\right)_{\alpha=\pi} = +0,327361\, f\sigma\Delta a m b.$$

$$\left(\frac{\partial F_1}{\partial \alpha}\right)_{+\frac{\pi}{2}} = \overset{+0,484748}{\cancel{+1,404471}}\, f\sigma\Delta a m b \qquad \left(\frac{\partial F_1}{\partial \alpha}\right)_{-\frac{\pi}{2}} = \overset{+0,484748}{+\cancel{1,404471}}$$

$$\left(\frac{\partial F}{\partial \alpha}\right)_0 = \left(\frac{\partial F_1}{\partial \alpha}\right)_{\alpha=0} + \left(\frac{\partial F_1}{\partial \alpha}\right)_{\alpha=\pi} = -1,475978\, f\sigma\Delta a m b$$

$$\left(\frac{\partial F}{\partial \alpha}\right)_{\frac{\pi}{2}} = \left(\frac{\partial F_1}{\partial \alpha}\right)_{\alpha=+\frac{\pi}{2}} + \left(\frac{\partial F_1}{\partial \alpha}\right)_{\alpha=-\frac{\pi}{2}} = \overset{+0,969496}{\cancel{+2,808942}}\, f\sigma\Delta a m b.$$

$$F_1 = -f\sigma\Delta a m b \left\{ \sin 2\alpha \cdot 0,298362 + \sin 4\alpha\, 0,031381 + \sin 6\alpha \cdot 0,002435 + \sin 8\alpha \cdot 0,000138 \right\}$$

$$\frac{\partial F_1}{\partial \alpha} = -f\sigma\Delta a m b \left\{ \cos 2\alpha \cdot 0,596724 + \cos 4\alpha \cdot 0,125524 + \cos 6\alpha \cdot 0,014610 + \cos 8\alpha \cdot 0,001104 \right\}$$

$$\left(\frac{\partial F_1}{\partial \alpha}\right)_0 = \left(\frac{\partial F_1}{\partial \alpha}\right)_\pi = -0,737962\, f\sigma\Delta a m.b \qquad \left(\frac{\partial F_1}{\partial \alpha}\right)_{\frac{\pi}{2}} = \left(\frac{\partial F_1}{\partial \alpha}\right)_{-\frac{\pi}{2}} = +0,484706\, f\sigma\Delta a m.b$$

$$\left(\frac{\partial F}{\partial \alpha}\right)_0 = \left(\frac{\partial F_1}{\partial \alpha}\right)_0 + \left(\frac{\partial F_1}{\partial \alpha}\right)_\pi = -1,475924\, f\sigma\Delta a m.b \qquad \left(\frac{\partial F}{\partial \alpha}\right)_{-\frac{\pi}{2}} = +0,969412\, f\sigma\Delta a m b$$

$$\cancel{\left\{\left(\frac{\partial F}{\partial \alpha}\right)_0 + \left(\frac{\partial F}{\partial \alpha}\right)_\pi\right\} - \left\{\left(\frac{\partial F}{\partial \alpha}\right)_{\frac{\pi}{2}} + \left(\frac{\partial F}{\partial \alpha}\right)_{-\frac{\pi}{2}}\right\} = -2,445336\, f\sigma\Delta a m b.}$$

~~A fenti szigorú érték:~~

$$\cancel{\left\{\left(\frac{\partial F}{\partial \alpha}\right)_0 + \left(\frac{\partial F}{\partial \alpha}\right)_\pi\right\} - \left\{\left(\frac{\partial F}{\partial \alpha}\right)_{\frac{\pi}{2}} + \left(\frac{\partial F}{\partial \alpha}\right)_{-\frac{\pi}{2}}\right\} = -2,445474\, f\sigma\Delta a m b.}$$

$\underline{a} = 1 \quad \underline{b} = 1 \quad \underline{l} = 1 \quad \underline{c} = \infty$ ilitebre.

$X_0 = +6{,}283184\ f\sigma\Delta a \qquad X_\pi = +1{,}754587\ f\sigma\Delta a$

$Y_{\frac{\pi}{2}} = -1{,}609428\ f\sigma\Delta a \qquad Y_{-\frac{\pi}{2}} = +1{,}609428\ f\sigma\Delta a$

$\left(\frac{\partial X}{\partial\alpha}\right)_{+\frac{\pi}{2}} = -0{,}8\ f\sigma\Delta a \qquad \left(\frac{\partial X}{\partial\alpha}\right)_{-\frac{\pi}{2}} = +0{,}8\ f\sigma\Delta a$

$\left(\frac{\partial Y}{\partial\alpha}\right)_0 = -4\ f\sigma\Delta a \qquad \left(\frac{\partial Y}{\partial\alpha}\right)_\pi = +0{,}8\ f\sigma\Delta a.$

$\left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=0} = -10{,}283184\ f\sigma\Delta a m b \qquad \left(\frac{\partial F_1}{\partial\alpha}\right)_\pi = +0{,}954587$

$\left(\frac{\partial F_1}{\partial\alpha}\right)_{+\frac{\pi}{2}} = +2{,}409428\ f\sigma\Delta a m b \qquad \left(\frac{\partial F_1}{\partial\alpha}\right)_{-\frac{\pi}{2}} = +2{,}409428$

$\left(\frac{\partial F}{\partial\alpha}\right)_0 = \left(\frac{\partial F_1}{\partial\alpha}\right)_0 + \left(\frac{\partial F_1}{\partial\alpha}\right)_\pi = -9{,}328597\ f\sigma\Delta a m b$

$\left(\frac{\partial F}{\partial\alpha}\right)_{\frac{\pi}{2}} = \left(\frac{\partial F_1}{\partial\alpha}\right)_{\frac{\pi}{2}} + \left(\frac{\partial F_1}{\partial\alpha}\right)_{-\frac{\pi}{2}} = +4{,}818876\ f\sigma\Delta a m b.$

$F_1 = -f\sigma\Delta a m b. \left\{ \sin 2\alpha . 2. + \sin 4\alpha \cdot \frac{1}{3} - \sin 6\alpha \cdot \frac{1}{10} - \sin 8\alpha\ \frac{1}{28} \right\}$

$F_1 = -f\sigma\Delta a m b \left\{ \sin 2\alpha . 2 + \sin 4\alpha \cdot 0{,}333333 - \sin 6\alpha\ 0.1 - \sin 8\alpha\ 0{,}035714 \right\}$

$\frac{\partial F_1}{\partial\alpha} = -f\sigma\Delta a\ m.b \left\{ \cos 2\alpha .\ 4 + \cos 4\alpha .\ 1{,}333333 - \cos 6\alpha . 0{,}6 - \cos 8\alpha . 0{,}285712 \right\}$

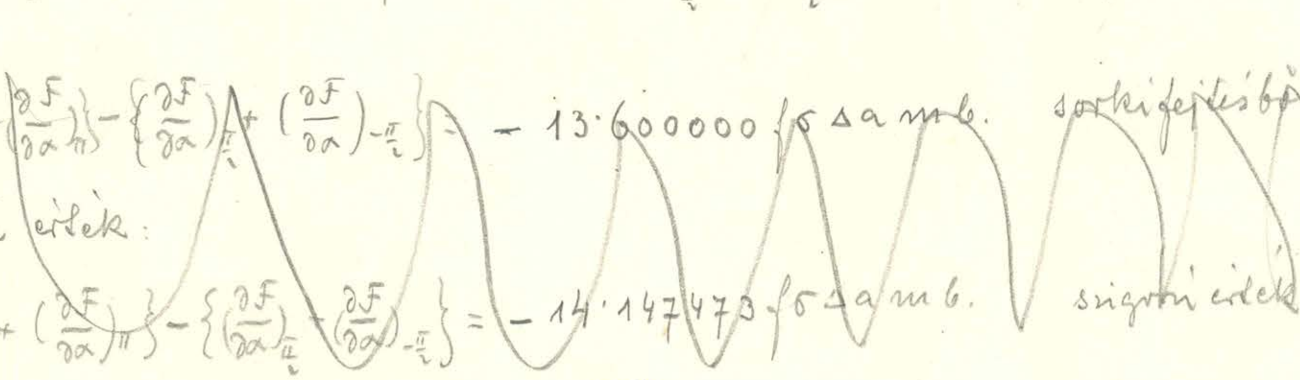
$\left(\frac{\partial F_1}{\partial\alpha}\right)_0 = \left(\frac{\partial F_1}{\partial\alpha}\right)_\pi = -4{,}447621\ f\sigma\Delta a m.b. \qquad \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=\frac{\pi}{2}} = \left(\frac{\partial F_1}{\partial\alpha}\right)_{-\frac{\pi}{2}} = +2{,}352379\ f\sigma\Delta a\ m.b$

$\overset{\left(\frac{\partial F}{\partial\alpha}\right)_0 \parallel}{\left(\frac{\partial F_1}{\partial\alpha}\right)_0 + \left(\frac{\partial F_1}{\partial\alpha}\right)_\pi} = -8{,}895242\ f\sigma\Delta a m b. \qquad \overset{\left(\frac{\partial F}{\partial\alpha}\right)_{\frac{\pi}{2}} \parallel}{\left(\frac{\partial F}{\partial\alpha}\right)_{\frac{\pi}{2}} + \left(\frac{\partial F}{\partial\alpha}\right)_{-\frac{\pi}{2}}} = +4.704758\ f\sigma\Delta a\ m.b$

$\left\{\left(\frac{\partial F}{\partial\alpha}\right)_0 + \left(\frac{\partial F}{\partial\alpha}\right)_\pi\right\} - \left\{\left(\frac{\partial F}{\partial\alpha}\right)_{\frac{\pi}{2}} + \left(\frac{\partial F}{\partial\alpha}\right)_{-\frac{\pi}{2}}\right\} = -13{\cdot}600000\ f\sigma\Delta a m b.$ sorkifejtésből

Szigorú érték:

$\left\{\left(\frac{\partial F}{\partial\alpha}\right)_0 + \left(\frac{\partial F}{\partial\alpha}\right)_\pi\right\} - \left\{\left(\frac{\partial F}{\partial\alpha}\right)_{\frac{\pi}{2}} + \left(\frac{\partial F}{\partial\alpha}\right)_{-\frac{\pi}{2}}\right\} = -14{\cdot}147473\ f\sigma\Delta a m b.$ szigorú érték



MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

Quadratikus oszlop.

$\underline{a}=1 \quad \underline{b}=1 \quad c=\infty \quad l=1$

~~$X_0 = -6{,}928050\, p_0$~~ ~~$X_\pi = +6{,}928050\, p_0$~~

~~$Y_{+\frac{\pi}{2}} = -6{,}928050\, p_0$~~ ~~$Y_{-\frac{\pi}{2}} = +6{,}928050\, p_0$~~

~~$\left(\frac{\partial X}{\partial \alpha}\right)_{\frac{\pi}{2}} = +4{,}428598\, p_0$~~ ~~$\left(\frac{\partial X}{\partial \alpha}\right)_{-\frac{\pi}{2}} = -4{,}428598\, p_0$~~

~~$\left(\frac{\partial Y}{\partial \alpha}\right)_0 = -4{,}428598\, p_0$~~ ~~$\left(\frac{\partial Y}{\partial \alpha}\right)_\pi = +4{,}428598\, p_0$~~

~~$\left(\frac{\partial F}{\partial \alpha}\right)_0 = +2{,}499452\, p_0 m$~~ ~~$\left(\frac{\partial F}{\partial \alpha}\right)_\pi = +2{,}499452\, p_0 m$~~

~~$\left(\frac{\partial F}{\partial \alpha}\right)_\pi = +2{,}499452\, p_0 m$~~ ~~$\left(\frac{\partial F}{\partial \alpha}\right)_{-\frac{\pi}{2}} = 2{,}499452\, p_0 m$~~

Közös számításból (lásd tulajdonképp lévő formulákat:

$\left(\frac{\partial F}{\partial \alpha}\right)_0 = \left(\frac{\partial F_1}{\partial \alpha}\right)_0 + \left(\frac{\partial F_1}{\partial \alpha}\right)_\pi = +4{,}998904\, p_0 m$

$\left(\frac{\partial F}{\partial \alpha}\right)_{\frac{\pi}{2}} = \left(\frac{\partial F_1}{\partial \alpha}\right)_{\frac{\pi}{2}} + \left(\frac{\partial F_1}{\partial \alpha}\right)_{-\frac{\pi}{2}} = +4{,}998904\, p_0 m$

Lantidejiből

$F_1 = (+0{,}666667 \sin 4\alpha - 0{,}023810 \sin 8\alpha)\, p_0 m b^2$

$\frac{\partial F_1}{\partial \alpha} = (+2{,}666667 \cos 4\alpha - 0{,}190476 \cos 8\alpha)\, p_0 m b^2$

és ebből

$\left(\frac{\partial F_1}{\partial \alpha}\right)_0 = \left(\frac{\partial F_1}{\partial \alpha}\right)_\pi = \left(\frac{\partial F_1}{\partial \alpha}\right)_{\frac{\pi}{2}} = \left(\frac{\partial F_1}{\partial \alpha}\right)_{-\frac{\pi}{2}} = +2{,}476191\, p_0 m b^2$

$\left(\frac{\partial F}{\partial \alpha}\right)_0 = \left(\frac{\partial F}{\partial \alpha}\right)_{\frac{\pi}{2}} = +4{,}952382$

Quadratikus oszlop.

$\underline{c}=1{,}965124 \quad \underline{a}=1 \quad \underline{b}=1 \quad \underline{l}=0{,}857874$

Általános formula

$$\left(\frac{\partial F_1}{\partial \alpha}\right)_0 = +4 f\sigma m l \Big\{ (a+l)\,\mathrm{arc\,tg}\,\frac{bc}{(a+l)\sqrt{(a+l)^2+b^2+c^2}} - (a-l)\,\mathrm{arc\,tg}\,\frac{bc}{(a-l)\sqrt{(a-l)^2+b^2+c^2}}$$

$$b \log \frac{\sqrt{(a+l)^2+b^2}}{\sqrt{(a-l)^2+b^2}} \cdot \frac{c+\sqrt{(a-l)^2+b^2+c^2}}{c+\sqrt{(a+l)^2+b^2+c^2}}$$

$$c \log \frac{\sqrt{(a+l)^2+c^2}}{\sqrt{(a-l)^2+c^2}} \cdot \frac{b+\sqrt{(a-l)^2+b^2+c^2}}{b+\sqrt{(a+l)^2+b^2+c^2}} \Big\}$$

$$-4 f\sigma m l^2 \Big\{ \mathrm{arc\,tg}\,\frac{(a-l)c}{b\sqrt{(a-l)^2+b^2+c^2}} + \mathrm{arc\,tg}\,\frac{(a+l)c}{b\sqrt{(a+l)^2+b^2+c^2}} \Big\}$$

$\left(\frac{\partial F_1}{\partial \alpha}\right)_{\frac{\pi}{2}} =$ az előbbi érték $\underline{a}$ felcserélve $\underline{b}$ vel.

ezért ha $a=b$ az az quadratikus oszlopra:

$$\left(\frac{\partial F_1}{\partial \alpha}\right)_0 = \left(\frac{\partial F_1}{\partial \alpha}\right)_{\frac{\pi}{2}} = +4 f\sigma m b^2 \underline{l} \Big\{ (1+\underline{l})\,\mathrm{arc\,tg}\,\frac{\underline{c}}{(1+\underline{l})\sqrt{(1+\underline{l})^2+1+\underline{c}^2}} - (1-\underline{l})\,\mathrm{arc\,tg}\,\frac{\underline{c}}{(1-\underline{l})\sqrt{(1-\underline{l})^2+1+\underline{c}^2}}$$

$$+ \log \frac{\sqrt{(1+\underline{l})^2+1}}{\sqrt{(1-\underline{l})^2+1}} \cdot \frac{\underline{c}+\sqrt{(1-\underline{l})^2+1+\underline{c}^2}}{\underline{c}+\sqrt{(1+\underline{l})^2+1+\underline{c}^2}}$$

$$+ \underline{c} \log \frac{\sqrt{(1+\underline{l})^2+\underline{c}^2}}{\sqrt{(1-\underline{l})^2+\underline{c}^2}} \cdot \frac{1+\sqrt{(1-\underline{l})^2+1+\underline{c}^2}}{1+\sqrt{(1+\underline{l})^2+1+\underline{c}^2}} \Big\}$$

$$-4 f\sigma m b^2 \underline{l}^2 \Big\{ \mathrm{arc\,tg}\,\frac{(1-\underline{l})\underline{c}}{\sqrt{(1-\underline{l})^2+1+\underline{c}^2}} + \mathrm{arc\,tg}\,\frac{(1+\underline{l})\underline{c}}{\sqrt{(1+\underline{l})^2+1+\underline{c}^2}} \Big\}$$

$\underline{a}=1 \quad \underline{b}=1$ tehát $\underline{l}=0{,}857874$ re $\quad \underline{c}=1{,}965124$

a számi érték $\left(\frac{\partial F_1}{\partial \alpha_0}\right) = \left(\frac{\partial F_1}{\partial \alpha}\right)_{\frac{\pi}{2}} = f\sigma m b^2\, 1{,}390780$

$\underline{a}=1 \quad \underline{b}=1 \quad \underline{c}=1{,}965124 \quad l=0{,}3$

szám. ért. $\left(\frac{\partial F_1}{\partial \alpha}\right)_0 = \left(\frac{\partial F_1}{\partial \alpha}\right)_{\frac{\pi}{2}} = f\sigma m b^2\, 0{,}021564$

$\underline{a}=1 \quad \underline{b}=1 \quad c=1{,}965124 \quad l=1$

szám. ért. $\left(\frac{\partial F_1}{\partial \alpha}\right)_0 = \left(\frac{\partial F_1}{\partial \alpha}\right)_{\frac{\pi}{2}} = f\sigma m b^2\, 2{,}497424$

$a=1 \quad b=1 \quad c=1{,}965124 \quad l=0{,}6$

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

szám. ért. $\left(\frac{\partial F_1}{\partial \alpha}\right)_0 = \left(\frac{\partial F_1}{\partial \alpha}\right)_{\frac{\pi}{2}} = f\sigma m b^2\, 0{,}342072$

Parallelepipedikus lapra. 1915 Nov. $\frac{V}{3.}$

$\underline{a} = 1 \quad \underline{b} = 1 \quad \underline{c} = 1{,}965124$

$\underline{l} = 0{,}6$

$X_{\alpha=0} = +4{,}571405\, f\sigma\Delta a \qquad X_{\alpha=\pi} = +1{,}694191\, f\sigma\Delta a$

$y_{\alpha=\frac{\pi}{2}} = -0{,}903774\, f\sigma\Delta a \qquad y_{\alpha=+\frac{\pi}{2}} = +0{,}903774\, f\sigma\Delta a$

$\left(\frac{\partial X}{\partial\alpha}\right)_{\alpha=+\frac{\pi}{2}} = -1{,}123350\, f\sigma\Delta a \qquad \left(\frac{\partial X}{\partial\alpha}\right)_{\alpha=-\frac{\pi}{2}} = +1{,}123350\, f\sigma\Delta a$

$\left(\frac{\partial y}{\partial\alpha}\right)_{\alpha=0} = -1{,}814332\, f\sigma\Delta a \qquad \left(\frac{\partial y}{\partial\alpha}\right)_{\alpha=\pi} = +0{,}486293\, f\sigma\Delta a$

$\left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=0} = -3{,}831442\, f\sigma\Delta a\, mb \qquad \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=\pi} = +0{,}724739\, f\sigma\Delta a\, mb$

$\left(\frac{\partial F_1}{\partial\alpha}\right)_{\frac{\pi}{2}} = +1{,}216274\, f\sigma\Delta a\, mb \qquad \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=-\frac{\pi}{2}} = +1{,}216274\, f\sigma\Delta a\, mb$

$\left(\frac{\partial F}{\partial\alpha}\right)_0 = \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=0} + \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=\pi} = -3{,}106703\, f\sigma\Delta a\, mb$

$\left(\frac{\partial F}{\partial\alpha}\right)_{\frac{\pi}{2}} = \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=+\frac{\pi}{2}} + \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=-\frac{\pi}{2}} = +2{,}432548\, f\sigma\Delta a$

$\underline{a} = 1 \quad \underline{b} = 1 \quad \underline{c} = 1{,}965124 \quad \underline{l} = 0{,}3$

$X_{\alpha=0} = +3{,}526244\, f\sigma\Delta a \qquad X_{\alpha=\pi} = +2{,}133820\, f\sigma\Delta a$

$y_{\alpha=\frac{\pi}{2}} = -0{,}572557\, f\sigma\Delta a \qquad y_{\alpha=-\frac{\pi}{2}} = +0{,}572557\, f\sigma\Delta a$

$\left(\frac{\partial X}{\partial\alpha}\right)_{\alpha=+\frac{\pi}{2}} = -0{,}658619\, f\sigma\Delta a \qquad \left(\frac{\partial X}{\partial\alpha}\right)_{\alpha=-\frac{\pi}{2}} = +0{,}658619\, f\sigma\Delta a$

$\left(\frac{\partial y}{\partial\alpha}\right)_{\alpha=0} = -0{,}684129\, f\sigma\Delta a \qquad \left(\frac{\partial y}{\partial\alpha}\right)_{\alpha=+\pi} = +0{,}342484\, f\sigma\Delta a$

$\left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=0} = -1{,}263112\, f\sigma\Delta a \qquad \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=\pi} = +0{,}537401\, f\sigma\Delta a$

$\left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=\frac{\pi}{2}} = +0{,}351351\, f\sigma\Delta a \qquad \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=-\frac{\pi}{2}} = +0{,}351351\, f\sigma\Delta a$

$\frac{\partial F}{\partial\alpha}_0 = \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=0} + \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=\pi} = -0{,}725711\, f\sigma\Delta a$

$\left(\frac{\partial F}{\partial\alpha}\right)_{\frac{\pi}{2}} = \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=\frac{\pi}{2}} + \left(\frac{\partial F_1}{\partial\alpha}\right)_{\alpha=-\frac{\pi}{2}} = +0{,}702702\, f\sigma\Delta a$

Az oszlopra vonatkozólag $a = 44,73$ $b = 14,91$ $c = 29,3$ $\frac{a}{b} = 3$ $\underline{c} = 1,965124$ 1915 Nov. $\frac{XI}{4}$

Nagy oszlopos berendezésnél állandók kiszámítása

A pelz ~~[illegible]~~ által okozott forgatónyomaték $F_p$

$$\frac{1}{f\sigma}\frac{\partial F_p}{\partial \alpha} = -\cos 2\alpha\, K\Big\{+6,666870\Big(1-\frac{k}{K}-\frac{2}{K'}\int \varrho^2 dm\Big)+0,086304\frac{1}{K'}\frac{1}{b^2}\int l^4 dm \cos 2\varepsilon$$

$$-0,065312\frac{1}{K'}\frac{1}{b^4}\int l^6 dm \cos 2\varepsilon +0,000118\frac{1}{K'}\frac{1}{b^6}\int l^8 dm \cos 2\varepsilon\Big\}$$

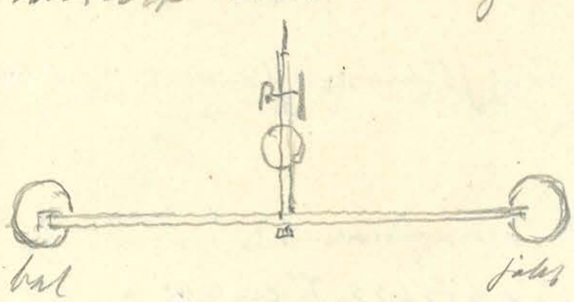
$$-\cos 4\alpha\, K\Big\{-0,353484\frac{1}{K'}\frac{1}{b^2}\int l^4 dm \cos 4\varepsilon + 0,005448\frac{1}{K'}\frac{1}{b^4}\int l^6 dm \cos 4\varepsilon$$

$$-0,000232\frac{1}{K'}\frac{1}{b^6}\int l^8 dm \cos 4\varepsilon\Big\}$$

$$-\cos 6\alpha\, K\Big\{-0,023532\frac{1}{K'}\frac{1}{b^4}\int l^6 dm \cos 6\varepsilon -0,000012\frac{1}{K'}\frac{1}{b^6}\int l^8 dm \cos 6\varepsilon\Big\}$$

$$-\cos 8\alpha\, K\Big\{-0,001376\frac{1}{K'}\frac{1}{b^6}\int l^8 dm \cos 8\varepsilon + \cdots\Big\}$$

A hatásos k a tükör és tükörtartó ~~tehetetlenségi~~ Momentuma $\int \varrho^2 dm$ ugyan a ~~[illegible]~~ rúd-tartó nélküli rúdra vonatkozik. A többi integrálok teljes összetételben az egész lengő rendszerre vonatkoznak ugyan – de e kisebb tagokban bizonyos elhanyagolásával elegendő ha azokat ~~[illegible]~~ a ~~[illegible]~~ rúd-tartó nélküli rúdra számítjuk ki. K a valódi, K' a számított tehetetlenségi momentum.




| | |
|---|---|
| A rúdtartó súlya tükrökkel | = 11,930 gr. |
| A rúd súlya | 7,203 |
| A jobb golyó súlya | 30,006 |
| A bal golyó súlya | 30,000 |
| Az egész felfüggesztés súlya | 79,139 |

Az egész rúd hossza = 24,25 c.
a függélytől golyóig 23,40 c.
A sárgarézrúd átmérője 0,35 c.

A jobb golyó átmérője 1,91 c.
A bal golyó átmérője 1,91 c.
a golyók középpontjainak távola = 25,58 C.

A rúdtartó lengésideje [illegible] = 20,0 c. ebből

$$\frac{k}{K} = \frac{20^2}{750,8^2} = 0,000709573$$

$K'$ és $\eta$ integrálok kiszámítása, egyelőre a kigyöngyített golyók helyett teljes golyókra számítunk, melyeknek tömege a kigyöngyített golyókkal egyenlő s középpontjuk az kigyöngyített golyók középpontjával összeesik.

$$K' = K_{golyó} + K_{rúd} + k = 60{,}006 \cdot \overline{12{,}79}^2 + 7{,}203 \cdot \frac{1}{3} \cdot \overline{12{,}125}^2 + 0{,}000709573\, K' + \frac{2}{5}\, 60{,}006 \cdot \overline{0{,}955}^2$$

A ~~[illegible]~~ K helyett K'-t írva, mert csak correctiós tagról van szó.

$$K' = 10198{,}139$$

Elhanyagolva a rúd vastagságát:

$$2\int \eta^2 dm = \frac{2}{5} \cdot 60{,}006 \cdot \overline{0{,}955}^2 = 21{,}890789$$

A golyó helyett középpontját és a rúd helyett tengelyét helyettesítve.

Legyen az egyes golyó tömege $= m = 30{,}003$, forgástengelytől $l = 12{,}79$

A rúd hosszegységének tömege $\mu = \frac{7{,}203}{24{,}25} = 0{,}297031$, $\lambda = 12{,}125$

$b = 14{,}91$ értékek.

$$\int l^4 dm \cos 2\varepsilon = 2ml^4 + \frac{2}{5}\mu\lambda^5 = 1636896 \qquad \frac{1}{K'}\frac{1}{b^2}\int l^4 dm \cos 2\varepsilon = 0{,}722017$$

$$\int l^6 dm \cos 2\varepsilon = 2ml^6 + \frac{2}{7}\mu\lambda^7 = 267253560 \qquad \frac{1}{K'}\frac{1}{b^4}\int l^6 dm \cos 2\varepsilon = 0{,}530265$$

$$\int l^8 dm \cos 2\varepsilon = 2ml^8 + \frac{2}{9}\mu\lambda^9 = 43642773000 \qquad \frac{1}{K'}\frac{1}{b^6}\int l^8 dm \cos 2\varepsilon = 0{,}389516$$

a golyó középpontjára és lineáris rúdra számítva $\cos n\varepsilon = 1$ és így $\int l^4 dm \cos 2\varepsilon = \int l^4 dm \cos 4\varepsilon$ és i. t. csak ezeket:

$$\frac{1}{f_0}\frac{\partial F_p}{\partial \alpha} = -6{,}662079\, K\cos 2\alpha - 0{,}057395\, K\cos 2\alpha + 0{,}252422\, K\cos 4\alpha + 0{,}012483\, K\cos 6\alpha + 0{,}000536\, K\cos 8\alpha.$$

$$\frac{1}{f_0}\left(\frac{\partial F_p}{\partial \alpha}\right)_0 = -6{,}454033\, K \qquad \frac{1}{f_0}\left(\frac{\partial F_p}{\partial \alpha}\right)_{\frac{\pi}{2}} = +6{,}959949\, K$$

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

$a = 14{,}91 \quad b = 14{,}91 \quad c = 29{,}1^3$

A Quadratikus antyo által okozott forgásmomentum $F_q$

Előző számításokból: az $l = 12{,}79$ forgássugárú golyókra (a midőn $\frac{l}{b} = \frac{1}{6} = 0{,}857874$)

helyettesítve a $\left(\frac{\partial F_q}{\partial \alpha}\right)_0$ értékére $= f\sigma m\, b^2 1{,}390780 = f\sigma . 60{,}006 . 14{,}91^2 . 1{,}390780 \pm f\sigma . 18552{,}76$

a rúd helyettesítése (grafikai úton számítva) $= \pm f\sigma . 297{,}20$

Tehát

$$\frac{1}{f\sigma}\left(\frac{\partial F_q}{\partial \alpha}\right)_0 = \frac{1}{f\sigma}\left(\frac{\partial F_q}{\partial \alpha}\right)_{\frac{\pi}{2}} = 18552{,}76 + 297{,}20 = 18849{,}96 = +1{,}848373\,K'$$

A belső $\Delta a$ vastagságú bc lap által okozott forgásmomentum $F_{l,b}$

$\underline{a} = 1 \quad \underline{b} = 1 \quad \underline{c} = 1{,}965124 \quad b = 14{,}91$

Előző számításokból és grafikai számításból a $\left(\frac{\partial F_{lb}}{\partial \alpha}\right)_0$ értékére helyettesítve

a két golyó $\quad -f\sigma . \Delta a . 6{,}666457 . 30{,}003 . 14{,}91 = -f\sigma \Delta a\ 2982{,}20$

rúd $\quad = -f\sigma \Delta a\ 83{,}98$

tehát

$$\frac{1}{f\sigma}\left(\frac{\partial F_{lb}}{\partial \alpha}\right)_0 = -\Delta a\ 2982{,}20 - \Delta a\ 83{,}98 = -\Delta a . 3067{,}18 = -0{,}300759\,\Delta a\, K'$$

ha $\Delta a = 0{,}13$ akkor $\frac{1}{f\sigma}\left(\frac{\partial F_{lb}}{\partial \alpha}\right)_0 = -0{,}039099\,K'$

Éppen így számítva a $\left(\frac{\partial F_{lb}}{\partial \alpha}\right)_{\frac{\pi}{2}}$ értékére helyettesítve:

a két golyó $- \quad + f\sigma \Delta a\ 4{,}059340 . 30{,}003 . 14{,}91 = + f\sigma \Delta a\ 1815{,}92$

a rúd $\quad = + f\sigma \Delta a . 64{,}33$

tehát

$$\frac{1}{f\sigma}\left(\frac{\partial F_{lb}}{\partial \alpha}\right)_{\frac{\pi}{2}} = +\Delta a\ 1815{,}92 + \Delta a\ 64{,}33 = +\Delta a\ 1880{,}25 = +0{,}184372\,\Delta a\, K'$$

ha $\Delta a = 0{,}13$ akkor $\frac{1}{f\sigma}\left(\frac{\partial F_{lb}}{\partial \alpha}\right)_{\frac{\pi}{2}} = +0{,}023968\,K'$

A Külső $\Delta a$ vastagságú bc lap által okozott forgásmomentum $F_{lk}$

Erre a szokásos formulát használjuk: lásd túloldalon

$$\frac{1}{f\sigma}\frac{\partial F_{Ik}}{\partial\alpha}=-\cos 2\alpha\, K\underline{\Delta a}\left\{+0{,}586402\left(1-\frac{k}{K}-\frac{2}{K'}\right)\int l^2 dm\right)+0{,}010272\frac{1}{K'}\frac{1}{b^2}\int l^4 dm\cos 2\varepsilon$$

$$+0{,}000060\frac{1}{b^4}\frac{1}{K'}\int l^6 dm\cos 2\varepsilon-0{,}000010\frac{1}{K'}\frac{1}{b^6}\int l^8 dm\cos 2\varepsilon\right\}$$

$$-\cos 4\alpha\, K\underline{\Delta a}\left\{+0{,}124144\frac{1}{K'}\frac{1}{b^2}\int l^4 dm\cos 4\varepsilon+0{,}001400\frac{1}{K'}\frac{1}{b^4}\int l^6 dm\cos 4\varepsilon\right.$$

$$\left.-0{,}000020\frac{1}{K'}\frac{1}{b^6}\int l^8 dm\cos 4\varepsilon\right\}$$

$$-\cos 6\alpha\, K\underline{\Delta a}\left\{+0{,}014574\frac{1}{K'}\frac{1}{b^4}\int l^6 dm\cos 6\varepsilon+0{,}000096\frac{1}{K'}\frac{1}{b^6}\int l^8 dm\cos 6\varepsilon\right\}$$

$$-\cos 8\alpha\, K\underline{\Delta a}\left\{+0{,}001104\frac{1}{K'}\frac{1}{b^6}\int l^8 dm\cos 8\varepsilon\right\}$$

ezekbe téve az integráloknak előbb talált értékeit kap:

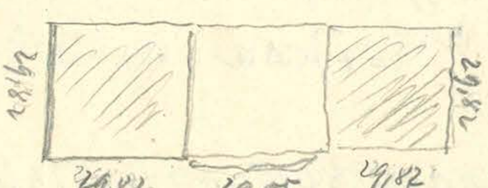
$$\frac{1}{f\sigma}\frac{\partial F_{Ik}}{\partial\alpha}=-0{,}585986\,K\underline{\Delta a}\cos 2\alpha-0{,}006176\,K\underline{\Delta a}\cos 2\alpha-0{,}090368\,K\underline{\Delta a}\cos 4\alpha$$

$$-0{,}007733\,K\underline{\Delta a}\cos 6\alpha-0{,}000430\,K.\underline{\Delta a}\cos 8\alpha.$$

s mint

$$\frac{1}{f\sigma}\left(\frac{\partial F_{Ik}}{\partial\alpha}\right)_0=-0{,}690693\,K\underline{\Delta a}\quad \text{és } \Delta a=0{,}13,\ \underline{\Delta a}=0{,}008719 \text{ -re}\quad \frac{1}{f\sigma}\left(\frac{\partial F_{Ik}}{\partial\alpha}\right)_{\alpha=0}=-0{,}006022\,K$$

$$\frac{1}{f\sigma}\left(\frac{\partial F_{Ik}}{\partial\alpha}\right)_{\frac{\pi}{2}}=+0{,}509097\,K\underline{\Delta a}\quad \text{és } \Delta a=0{,}13 \text{ m}\quad \frac{1}{f\sigma}\left(\frac{\partial F_{Ik}}{\partial\alpha}\right)_{\frac{\pi}{2}}=+0{,}004439\,K$$




az oszlopok egész magassága = 58,6

$a=44{,}73$ $\quad\underline{a}=3\quad$ $\underline{\Delta a}=0{,}008719$

$b=14{,}91$ $\quad\underline{b}=1$

$c=29{,}30$ $\quad\underline{c}=1{,}965124$

Legyen idő álomvastagokkal $T'$, azok nélkül $T$

| | $\frac{\partial F}{\partial\alpha}$ | $T_{\text{köré}}$ | $T'_{\text{köré}}$ | $\frac{1}{T_k^2}$ | $\frac{1}{T_k'^2}$ |
|---|---|---|---|---|---|
| $\alpha=0$ | $-8{,}269329\,K/\sigma$ | 742,82 | 641,079 | $1{,}81231\cdot10^{-6}$ | $2{,}43320\cdot10^{-6}$ |
| $\alpha=\frac{\pi}{2}$ | $+5{,}092047\,K/\sigma$ | 759,07 | 859,729 | $1{,}73554\cdot10^{-6}$ | $1{,}35293\cdot10^{-6}$ |

$f$-re számítás: tartási idő értékeinek $\alpha=0$ és $\alpha=\frac{\pi}{2}$ állásnál.

$$K\frac{\pi^2}{T_0'^2}-K\frac{\pi^2}{T_{\frac{\pi}{2}}'^2}-K\frac{\pi^2}{T_0^2}+K\frac{\pi^2}{T_{\frac{\pi}{2}}^2}=-\left(\frac{\partial F}{\partial\alpha}\right)_0+\left(\frac{\partial F}{\partial\alpha}\right)_{\frac{\pi}{2}}\quad \text{s így}$$

$$\pi^2\left(\frac{1}{T_0'^2}-\frac{1}{T_{\frac{\pi}{2}}'^2}-\frac{1}{T_0^2}+\frac{1}{T_{\frac{\pi}{2}}^2}\right)=13{,}361376\,f\sigma\quad \text{és ebből}$$

$$f\sigma=0{,}741252\cdot10^{-6}$$

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$$\sigma=\frac{1168817}{2.29{,}82.29{,}82.58{,}6}=11{,}215112\qquad f=66{,}09403\ 10^{-9}$$

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2ik számítás. csupán $\alpha=0$ állásból.

$$K\pi^2\left(\frac{1}{J_0'^2}-\frac{1}{J_0^2}\right)=-\left(\frac{\partial F}{\partial\alpha}\right)_0 \qquad \pi^2\left(\frac{1}{J_0'^2}-\frac{1}{J_0^2}\right)=+8{,}269329\,f\sigma$$

ebből

$$f\sigma=0{,}741044 \qquad f=66{,}07549$$

3ik számítás csupán $\alpha=\frac{\pi}{2}$ állásból

$$K\pi^2\left(\frac{1}{J'^2_{\frac{\pi}{2}}}-\frac{1}{J^2_{\frac{\pi}{2}}}\right)=-\left(\frac{\partial F}{\partial\alpha}\right)_{\frac{\pi}{2}} \qquad \pi^2\left(\frac{1}{J^2_{\frac{\pi}{2}}}-\frac{1}{J'^2_{\frac{\pi}{2}}}\right)=+5{,}092047\,f\sigma$$

ebből

$$f\sigma=0{,}741590 \qquad f=66{,}12417\cdot 10^{-9}.$$

---

4ik Kiszámítási mód $\Delta a$ és $f\sigma$ kiszámítása kétegyenletből

$$K\frac{\pi^2}{J_0'^2}-K\frac{\pi^2}{J_0^2}=-\left\{\left(\frac{\partial F_p}{\partial\alpha}\right)_0-\left(\frac{\partial F_g}{\partial\alpha}\right)_0-\left(\frac{\partial F_{lb}}{\partial\alpha}\right)_0+\left(\frac{\partial F_{lk}}{\partial\alpha}\right)_0\right\}$$

$$K\frac{\pi^2}{J'^2_{\frac{\pi}{2}}}-K\frac{\pi^2}{J^2_{\frac{\pi}{2}}}=-\left\{\left(\frac{\partial F_p}{\partial\alpha}\right)_{\frac{\pi}{2}}-\left(\frac{\partial F_g}{\partial\alpha}\right)_{\frac{\pi}{2}}-\left(\frac{\partial F_{lb}}{\partial\alpha}\right)_{\frac{\pi}{2}}+\left(\frac{\partial F_{lk}}{\partial\alpha}\right)_{\frac{\pi}{2}}\right\}$$

tehát $\pi^2\,0{,}62089\cdot10^{-6}=8{,}302406\,f\sigma-0{,}254435\,f\sigma\Delta a=6{,}127939\cdot10^{-6}$

$\pi^2\,0{,}38261\cdot10^{-6}=5{,}111576\,f\sigma-0{,}150227\,f\sigma\Delta a=3{,}776209\cdot10^{-6}$ 1,6242360

azaz

$$8{,}302406\,f\sigma-0{,}254435\,f\sigma\Delta a=6{,}127939$$
$$8{,}302406\,f\sigma-0{,}244004\,f\sigma\Delta a=\underline{6{,}133455}$$
$$0{,}005516 \;+)$$

ebből $f\sigma=0{,}754298\cdot10^{-6}$ és $f\sigma\Delta a=0{,}528808\cdot10^{6}$.

a +) különbség az értékek 6,127939 értékű eredménél kisebb részét teszi ki s így az észlelési hibák határainak közelébe esvén nem értékesíthető a számítás használhatatlan.

---

[illegible] !

Az [illegible] újabb helyesnek talált felállítása $(\Delta a=0)$ után tett észlelésekből kivett értékek:

| $T_0'$ | $T_{\frac{\pi}{2}}'$ |
|---|---|
| 641,084 | 860,797 |
| 641,090 | 860,930 |
| 641,084 | 860,697 |
| 641,069 | 860,797 |
| 641,126 | 860,836 |
| 641,084 | 860,977 |
| 641,056 | 861,080 |
| 641,176 | 861,003 |
| 641,226 | 861,041 |
| 641,169 | 861,430 |
| 641,204 | 861,368 |
| 641,241 | |
| Középértek = 641,142 | = 860,996 |

[illegible] $T_0 = 742,82$ $T_{\frac{\pi}{2}} = 759,07$

$$K\frac{\pi^2}{T_0'^2} - K\frac{\pi^2}{T_{\frac{\pi}{2}}'^2} - K\frac{\pi^2}{T_0^2} + K\frac{\pi^2}{T_{\frac{\pi}{2}}^2} = -\left\{\left(\frac{\partial F_\rho}{\partial \alpha}\right)_0 - \left(\frac{\partial F_\rho}{\partial \alpha}\right)_{\frac{\pi}{2}}\right\}$$

$$\pi^2\left(\frac{1}{T_0'^2} - \frac{1}{T_{\frac{\pi}{2}}'^2} - \frac{1}{T_0^2} + \frac{1}{T_{\frac{\pi}{2}}^2}\right) = +13,413982\, f\sigma \,.$$

$$\pi^2\left(1,08376 - 0,07677\right)10^{-6} = \pi^2 \cdot 1,00699\, 10^{-6} = 13,413982\, f\sigma$$

$$f\sigma = 0,740913\; 10^{-6}$$

$$f = 66,06381\; 10^{-9}.$$

$$\mathcal{L}_{ab} = dc\,\mathcal{L}'_{ab} \qquad \mathcal{L}_{bc} = da\,\mathcal{L}'_{bc} \qquad \mathcal{L}_{ca} = db\,\mathcal{L}'_{ca}$$

$$\frac{\partial^2 V}{\partial x^2} = -\Big|_{a_1}^{a_2} \frac{\partial \mathcal{L}'_{bc}}{\partial x} \qquad \frac{\partial^2 V}{\partial y^2} = -\Big|_{b_1}^{b_2} \frac{\partial \mathcal{L}'_{ca}}{\partial y}$$

$$\frac{\partial^4 V}{\partial x^4} = -\Big|_{a_1}^{a_2} \frac{\partial^3 \mathcal{L}'_{bc}}{\partial x^3} \qquad \frac{\partial^4 V}{\partial y^4} = -\Big|_{b_1}^{b} \frac{\partial^3 \mathcal{L}'_{ca}}{\partial y^3} \qquad \frac{\partial^4 V}{\partial x^2 \partial y^2} = -\Big|_{a_1}^{a_2} \frac{\partial^3 \mathcal{L}'_{bc}}{\partial x \partial y^2} = -\Big|_{b_1}^{b_2} \frac{\partial^3 \mathcal{L}'_{ca}}{\partial x^2 \partial y}$$

$$\frac{\partial^6 V}{\partial x^6} = -\Big|_{a_1}^{a} \frac{\partial^5 \mathcal{L}'_{bc}}{\partial x^5} \qquad \frac{\partial^6 V}{\partial y^6} = -\Big|_{b_1}^{b_2} \frac{\partial^5 \mathcal{L}'_{ca}}{\partial y^5} \qquad \frac{\partial^6 V}{\partial x^4 \partial y^2} = -\Big|_{a_1}^{a_2} \frac{\partial^5 \mathcal{L}'_{bc}}{\partial x^3 \partial y^2} = -\Big|_{b_1}^{b_2} \frac{\partial^5 \mathcal{L}'_{ca}}{\partial x^4 \partial y}$$

$$\frac{\partial^6 V}{\partial x^2 \partial y^4} = -\Big|_{a_1}^{a_2} \frac{\partial^5 \mathcal{L}'_{bc}}{\partial x \partial y^4} = -\Big|_{b_1}^{b_2} \frac{\partial^5 \mathcal{L}'_{ca}}{\partial x^2 \partial y^3}$$

$$\frac{\partial^8 V}{\partial x^8} = -\Big|_{a_1}^{a_2} \frac{\partial^7 \mathcal{L}'_{bc}}{\partial x^7} \qquad \frac{\partial^8 V}{\partial y^8} = -\Big|_{b_1}^{b_2} \frac{\partial^7 \mathcal{L}'_{ca}}{\partial y^7} \qquad \frac{\partial^8 V}{\partial x^4 \partial y^4} = -\Big|_{a_1}^{a_2} \frac{\partial^7 \mathcal{L}'_{bc}}{\partial x^3 \partial y^4} = -\Big|_{b_1}^{b_2} \frac{\partial^7 \mathcal{L}'_{ca}}{\partial x^4 \partial y^3}$$

$$\frac{\partial^8 V}{\partial x^6 \partial y^2} = -\Big|_{a_1}^{a_2} \frac{\partial^7 \mathcal{L}'_{bc}}{\partial x^5 \partial y^2} = -\Big|_{b_1}^{b_2} \frac{\partial^7 \mathcal{L}'_{ca}}{\partial x^6 \partial y}$$

$$\frac{\partial^8 V}{\partial x^2 \partial y^6} = -\Big|_{a_1}^{a_2} \frac{\partial^7 \mathcal{L}'_{bc}}{\partial x \partial y^6} = -\Big|_{b_1}^{b_2} \frac{\partial^7 \mathcal{L}'_{ca}}{\partial x^2 \partial y^5}$$

A túloldali értékekkel telt parallelepiped közepére

$$\underline{a} = 3 \qquad \underline{b} = 1 \qquad \underline{c} = 1{,}965124$$

$$\frac{F}{f\sigma} = -\sin 2\alpha \left\{ +3{,}333405 \int l^2 dm \cos 2\varepsilon + 0{,}043152 \frac{1}{b^2} \int l^4 dm \cos 2\varepsilon - 0{,}002656 \frac{1}{b^4} \int l^6 dm \cos 2\varepsilon + 0{,}000059 \frac{1}{b^6} \int l^8 dm \cos 2\varepsilon + \dots \right\}$$

$$- \sin 4\alpha \left\{ -0{,}088371 \frac{1}{b^2} \int l^4 dm \cos 4\varepsilon + 0{,}001362 \frac{1}{b^4} \int l^6 dm \cos 4\varepsilon - 0{,}000058 \frac{1}{b^6} \int l^8 dm \cos 4\varepsilon + \dots \right\}$$

$$- \sin 6\alpha \left\{ -0{,}003922 \frac{1}{b^4} \int l^6 dm \cos 6\varepsilon - 0{,}000002 \frac{1}{b^6} \int l^8 dm \cos 6\varepsilon + \dots \right\}$$

$$- \sin 8\alpha \left\{ -0{,}000172 \frac{1}{b^6} \int l^8 dm \cos 8\varepsilon + \dots \right\}$$

$$\cdot \; \dots\dots\dots\dots$$

A túloldali értékekkel $\Delta a$ vastagságú parallelepipedikus kéreg tengelyében

$$\frac{F}{f\sigma} = -\sin 2\alpha\, \underline{\Delta a} \left\{ +0{,}293201 \int l^2 dm \cos 2\varepsilon + 0{,}005136 \frac{1}{b^2} \int l^4 dm \cos 2\varepsilon + 0{,}000030 \frac{1}{b^4} \int l^6 dm \cos 2\varepsilon - 0{,}000005 \frac{1}{b^6} \int l^8 dm \cos 2\varepsilon + \dots \right\}$$

$$- \sin 4\alpha\, \underline{\Delta a} \left\{ +0{,}031036 \frac{1}{b^2} \int l^4 dm \cos 4\varepsilon + 0{,}000350 \frac{1}{b^4} \int l^6 dm \cos 4\varepsilon - 0{,}000005 \frac{1}{b^6} \int l^8 dm \cos 4\varepsilon + \dots \right\}$$

$$- \sin 6\alpha\, \underline{\Delta a} \left\{ +0{,}002419 \frac{1}{b^4} \int l^6 dm \cos 6\varepsilon + 0{,}000016 \frac{1}{b^6} \int l^8 dm \cos 6\varepsilon + \dots \right\}$$

$$- \sin 8\alpha\, \underline{\Delta a} \left\{ +0{,}000138 \frac{1}{b^6} \int l^8 dm \cos 8\varepsilon + \dots \right\}$$

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

| | $\underline{a}=3$, $\underline{b}=1$, $\underline{c}=2$ | $\underline{a}=3$, $\underline{b}=1$, $\underline{c}=1{,}965124$ | $\underline{a}=1$, $\underline{b}=1$, $\underline{c}=1{,}965124$ | $\underline{a}=1$, $\underline{b}=1$, $\underline{c}=\infty$ | | |
|---|---|---|---|---|---|---|
| $\frac{1}{f\sigma}\frac{\partial^2 V}{\partial x^2}$ | $-1{,}410574$ | $-1{,}393277$ | $-5{,}454503$ | $-6{,}283184$ | $\frac{1}{f\sigma}\frac{\partial^2 L_{bc}}{\partial x^2}$ | $-$ |
| $\frac{1}{f\sigma}\frac{\partial^2 V}{\partial y^2}$ | $-8{,}105581$ | $-8{,}060086$ | $-5{,}454503$ | $-6{,}283184$ | $\frac{1}{f\sigma}\frac{\partial^2 L_{bc}}{\partial y^2}$ | $-$ |
| $\frac{1}{f\sigma}\frac{\partial^4 V}{\partial x^4}$ | $-0{,}570508\frac{1}{b^2}$ | $-0{,}568936\frac{1}{b^2}$ | $-4{,}577827\frac{1}{b^2}$ | $-4\frac{1}{b^2}$ | $\frac{1}{f\sigma}\frac{\partial^4 L_{bc}}{\partial x^4}$ | $+$ |
| $\frac{1}{f\sigma}\frac{\partial^4 V}{\partial y^4}$ | $-1{,}55775\frac{1}{b^2}$ | $-1{,}604579\frac{1}{b^2}$ | $-4{,}577827\frac{1}{b^2}$ | $-4\frac{1}{b^2}$ | $\frac{1}{f\sigma}\frac{\partial^4 L_{bc}}{\partial y^4}$ | $+$ |
| $\frac{1}{f\sigma}\frac{\partial^4 V}{\partial x^2\partial y^2}$ | $+0{,}348203\frac{1}{b^2}$ | $+0{,}344714\frac{1}{b^2}$ | $+3{,}800534\frac{1}{b^2}$ | $+4\frac{1}{b^2}$ | $\frac{1}{f\sigma}\frac{\partial^4 L_{bc}}{\partial x^2\partial y^2}$ | $-$ |
| $\frac{1}{f\sigma}\frac{\partial^6 V}{\partial x^6}$ | $-0{,}589424\frac{1}{b^4}$ | $-0{,}594529\frac{1}{b^4}$ | $+0{,}993969\frac{1}{b^4}$ | $0$ | $\frac{1}{f\sigma}\frac{\partial^6 L_{bc}}{\partial x^6}$ | $+$ |
| $\frac{1}{f\sigma}\frac{\partial^6 V}{\partial y^6}$ | $+2{,}172789\frac{1}{b^4}$ | $+2{,}259187\frac{1}{b^4}$ | $+0{,}993969\frac{1}{b^4}$ | $0$ | $\frac{1}{f\sigma}\frac{\partial^6 L_{bc}}{\partial y^6}$ | $-$ |
| $\frac{1}{f\sigma}\frac{\partial^6 V}{\partial x^4\partial y^2}$ | $+0{,}444518\frac{1}{b^4}$ | $+0{,}442615\frac{1}{b^4}$ | $+0{,}241789\frac{1}{b^4}$ | $0$ | $\frac{1}{f\sigma}\frac{\partial^6 L_{bc}}{\partial x^4\partial y^2}$ | $-$ |
| $\frac{1}{f\sigma}\frac{\partial^6 V}{\partial x^2\partial y^4}$ | $-0{,}374385\frac{1}{b^4}$ | $-0{,}371272\frac{1}{b^4}$ | $+0{,}241789\frac{1}{b^4}$ | $0$ | $\frac{1}{f\sigma}\frac{\partial^6 L_{bc}}{\partial x^2\partial y^4}$ | $+$ |
| $\frac{1}{f\sigma}\frac{\partial^8 V}{\partial x^8}$ | $-0{,}925544\frac{1}{b^6}$ | $-0{,}940387\frac{1}{b^6}$ | $+119{,}246071\frac{1}{b^6}$ | $+120\frac{1}{b^6}$ | $\frac{1}{f\sigma}\frac{\partial^8 L_{bc}}{\partial x^8}$ | $+$ |
| $\frac{1}{f\sigma}\frac{\partial^8 V}{\partial y^8}$ | $-3{,}410976\frac{1}{b^6}$ | $-3{,}303087\frac{1}{b^6}$ | $+119{,}246071\frac{1}{b^6}$ | $+120\frac{1}{b^6}$ | $\frac{1}{f\sigma}\frac{\partial^8 L_{bc}}{\partial y^8}$ | $+$ |
| $\frac{1}{f\sigma}\frac{\partial^8 V}{\partial x^6\partial y^2}$ | $+0{,}949459\frac{1}{b^6}$ | $+0{,}950578\frac{1}{b^6}$ | $-120{,}477413\frac{1}{b^6}$ | $-120\frac{1}{b^6}$ | $\frac{1}{f\sigma}\frac{\partial^8 L_{bc}}{\partial x^6\partial y^2}$ | $-$ |
| $\frac{1}{f\sigma}\frac{\partial^8 V}{\partial x^4\partial y^4}$ | $-0{,}851773\frac{1}{b^6}$ | $-0{,}841582\frac{1}{b^6}$ | $+119{,}631577\frac{1}{b^6}$ | $+120\frac{1}{b^6}$ | $\frac{1}{f\sigma}\frac{\partial^8 L_{bc}}{\partial x^4\partial y^4}$ | $+$ |
| $\frac{1}{f\sigma}\frac{\partial^8 V}{\partial x^2\partial y^6}$ | $+0{,}769123\frac{1}{b^6}$ | $+0{,}763460\frac{1}{b^6}$ | $-120{,}477413\frac{1}{b^6}$ | $-120\frac{1}{b^6}$ | $\frac{1}{f\sigma}\frac{\partial^8 L_{bc}}{\partial x^2\partial y^6}$ | $-$ |

X

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

| | $\underline{a}=3$<br>$\underline{b}=1$<br>$\underline{c}=1{,}965124$ | $\underline{a}=1$<br>$\underline{b}=1$<br>$\underline{c}=1{,}965124$ | $\underline{a}=1$<br>$\underline{b}=1$<br>$\underline{c}=\infty$ |
|---|---|---|---|
| $\frac{1}{p_0}\frac{\partial^2 L_{bc}}{\partial x^2}$ | $+0{,}375276\,\underline{\Delta a}$ | $+2{,}291130\,\underline{\Delta a}$ | $+2\,\underline{\Delta a}$ |
| $\frac{1}{p_0}\frac{\partial^2 L_{bc}}{\partial y^2}$ | $-0{,}211126\,\underline{\Delta a}$ | $-1{,}623330\,\underline{\Delta a}$ | $-2\cdot\underline{\Delta a}$ |
| $\frac{1}{p_0}\frac{\partial^4 L_{bc}}{\partial x^4}$ | $+0{,}269340\frac{1}{b^2}\underline{\Delta a}$ | $+2{,}031419\frac{1}{b^2}\underline{\Delta a}$ | $+2\cdot\underline{\Delta a}\frac{1}{b^2}$ |
| $\frac{1}{p_0}\frac{\partial^4 L_{bc}}{\partial y^4}$ | $+0{,}146088\frac{1}{b^2}\underline{\Delta a}$ | $+1{,}758534\frac{1}{b^2}\underline{\Delta a}$ | $+2\cdot\underline{\Delta a}\frac{1}{b^2}$ |
| $\frac{1}{p_0}\frac{\partial^4 L_{bc}}{\partial x^2\partial y^2}$ | $-0{,}179051\frac{1}{b^2}\underline{\Delta a}$ | $-2{,}042000\frac{1}{b^2}\underline{\Delta a}$ | $-2\cdot\underline{\Delta a}\frac{1}{b^2}$ |
| $\frac{1}{p_0}\frac{\partial^6 L_{bc}}{\partial x^6}$ | $+0{,}363674\frac{1}{b^4}\underline{\Delta a}$ | $-12{,}978944\frac{1}{b^4}\underline{\Delta a}$ | $-12\,\underline{\Delta a}\frac{1}{b^2}$ |
| $\frac{1}{p_0}\frac{\partial^6 L_{bc}}{\partial y^6}$ | $-0{,}238782\frac{1}{b^4}\underline{\Delta a}$ | $+12{,}407651\frac{1}{b^4}\underline{\Delta a}$ | $+12\,\underline{\Delta a}\frac{1}{b^4}$ |
| $\frac{1}{p_0}\frac{\partial^6 L_{bc}}{\partial x^4\partial y^2}$ | $-0{,}310595\frac{1}{b^4}\underline{\Delta a}$ | $+11{,}924231\frac{1}{b^4}\underline{\Delta a}$ | $+12\,\underline{\Delta a}\frac{1}{b^4}$ |
| $\frac{1}{p_0}\frac{\partial^6 L_{bc}}{\partial x^2\partial y^4}$ | $+0{,}268448\frac{1}{b^4}\underline{\Delta a}$ | $-11{,}970946\frac{1}{b^4}\underline{\Delta a}$ | $-12\,\underline{\Delta a}\frac{1}{b^4}$ |
| $\frac{1}{p_0}\frac{\partial^8 L_{bc}}{\partial x^8}$ | $+0{,}600807\frac{1}{b^6}\underline{\Delta a}$ | $-172{,}036630\frac{1}{b^6}\underline{\Delta a}$ | $-180\,\underline{\Delta a}\frac{1}{b^6}$ |
| $\frac{1}{p_0}\frac{\partial^8 L_{bc}}{\partial y^8}$ | $+0{,}580704\frac{1}{b^6}\underline{\Delta a}$ | $-180{,}637790\frac{1}{b^6}\underline{\Delta a}$ | $-180\,\underline{\Delta a}\frac{1}{b^6}$ |
| $\frac{1}{p_0}\frac{\partial^8 L_{bc}}{\partial x^6\partial y^2}$ | $-0{,}756542\frac{1}{b^6}\underline{\Delta a}$ | $+180{,}794474\frac{1}{b^6}\underline{\Delta a}$ | $+180\,\underline{\Delta a}\frac{1}{b^6}$ |
| $\frac{1}{p_0}\frac{\partial^8 L_{bc}}{\partial x^4\partial y^4}$ | $+0{,}698389\frac{1}{b^6}\underline{\Delta a}$ | $-179{,}832695\frac{1}{b^6}\underline{\Delta a}$ | $-180\,\Delta a\frac{1}{b^6}$ |
| $\frac{1}{p_0}\frac{\partial^8 L_{bc}}{\partial x^2\partial y^6}$ | $-0{,}634877\frac{1}{b^6}\underline{\Delta a}$ | $+179{,}908006\frac{1}{b^6}\underline{\Delta a}$ | $+180\,\Delta a\frac{1}{b^6}$ |

(ab) lap.

$\frac{1}{f\sigma} L_{ab}$ = lap Potentialgr.

$$\frac{1}{f\sigma}\frac{\partial L_{ab}}{\partial x} = dc \Big|\Big|_{b_1\,a_1}^{b_2\,a_2} \log\sqrt{1+\frac{a^2}{c^2}}\cdot\frac{1+\sqrt{1+\frac{c^2}{b^2}}}{1+\sqrt{1+\frac{a^2}{b^2}+\frac{c^2}{b^2}}} = -dc\Big|\Big|\log(b+\sqrt{a^2+b^2+c^2})$$

$$\frac{1}{f\sigma}\frac{\partial L_{ab}}{\partial y} = dc \Big|\Big|_{b_1\,a_1}^{b_2\,a_2} \log\sqrt{1+\frac{b^2}{c^2}}\cdot\frac{1+\sqrt{1+\frac{c^2}{a^2}}}{1+\sqrt{1+\frac{b^2}{a^2}+\frac{c^2}{a^2}}} = -dc\Big|\Big|\log(a+\sqrt{a^2+b^2+c^2}$$

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$$\frac{1}{f\sigma}\frac{\partial^2 L_{ab}}{\partial x^2} = -dc\Big|\Big|_{b_1\,a_1}^{b_2\,a_2}\frac{ba}{a^2+c^2}\frac{1}{r}$$

$$\frac{1}{f\sigma}\frac{\partial^2 L_{ab}}{\partial x\partial y} = +dc\Big|\Big|_{b_1\,a_1}^{b_2\,a_2}\frac{1}{r}$$

$$\frac{1}{f\sigma}\frac{\partial^2 L_{ab}}{\partial y^2} = -dc\Big|\Big|_{b_1\,a_1}^{b_2\,a_2}\frac{ba}{b^2+c^2}\frac{1}{r}$$

$$\frac{1}{f\sigma}\frac{\partial^3 L_{ab}}{\partial x^3} = -dc\Big|\Big|_{b\,a} b\left\{(\varphi)_{10}\left(\frac{1}{r}\right)_{00}+\varphi_{00}\left(\frac{1}{r}\right)_{10}\right\} = -dc\Big|\Big| b\frac{\partial}{\partial x}\left(\varphi\frac{1}{r}\right) \qquad \varphi=\frac{a}{a^2+c^2}$$

$$\frac{1}{f\sigma}\frac{\partial^3 L_{ab}}{\partial x^2\partial y} = -dc\Big|\Big|_{b\,a}\cancel{\varphi_{00}\left\{-\left(\frac{1}{r}\right)_{00}+b\left(\frac{1}{r}\right)_{01}\right\}} = +dc\Big|\Big|_{b\,a}\left(\frac{1}{r}\right)_{10} = +dc\Big|\Big|\frac{\partial}{\partial x}\left(\frac{1}{r}\right)$$

$$\frac{1}{f\sigma}\frac{\partial^3 L_{ab}}{\partial x\partial y^2} = +dc\Big|\Big|_{b\,a}\left(\frac{1}{r}\right)_{01} = +dc\Big|\Big|\frac{\partial}{\partial y}\left(\frac{1}{r}\right)$$

$$\frac{1}{f\sigma}\frac{\partial^3 L_{ab}}{\partial y^3} = -dc\Big|\Big|_{b\,a} a\left\{(\varphi)_{01}\left(\frac{1}{r}\right)_{00}+(\varphi)_{00}\left(\frac{1}{r}\right)_{01}\right\} = -dc\Big|\Big| a\frac{\partial}{\partial y}\left(\varphi\frac{1}{r}\right) \qquad \varphi=\frac{b}{b^2+c^2}$$

$$\frac{\partial^n}{\partial x^n}\left(\varphi\frac{1}{r}\right) = \frac{1}{r}\frac{\partial^n\varphi}{\partial x^n} + n\frac{\partial}{\partial x}\frac{1}{r}\frac{\partial^{n-1}\varphi}{\partial x^{n-1}} + \frac{n(n-1)}{1.2}\frac{\partial^2}{\partial x^2}\frac{1}{r}\frac{\partial^{n-2}\varphi}{\partial x^{n-2}} + \frac{n(n-1)(n-2)}{1.2.3}\frac{\partial^3}{\partial x^3}\left(\frac{1}{r}\right)\frac{\partial^{n-3}\varphi}{\partial x^3} +$$

$$+\frac{n(n-1)(n-2)(n-3)}{1.2.3.4}\frac{\partial^4\frac{1}{r}}{\partial x^4}\frac{\partial^{n-4}\varphi}{\partial x^4}+\ldots$$

$$\frac{1}{f\sigma}\frac{\partial^4 L_{ab}}{\partial x^4} = -dc\int_{b_1}^{b}\int_{a_1}^{a} b\frac{\partial^2}{\partial x^2}\varphi\frac{1}{r} = -dc\iint b\left\{\left(\frac{1}{r}\right)_{00}(\varphi)_{20} + 2\left(\frac{1}{r}\right)_{10}\varphi_{10} + \left(\frac{1}{r}\right)_{20}\varphi_{00}\right\} \qquad \varphi = \frac{a}{a^2+c^2}$$

$$\frac{1}{f\sigma}\frac{\partial^4 L_{ab}}{\partial x^3\partial y} = +dc\iint\frac{\partial^2}{\partial x^2}\frac{1}{r} = +dc\iint\left(\frac{1}{r}\right)_{20}$$

$$\frac{1}{f\sigma}\frac{\partial^4 L_{ab}}{\partial x^2\partial y^2} = -dc\iint\varphi\frac{\partial^2}{\partial y^2}b\frac{1}{r} = -dc\iint\varphi\left\{-2\frac{\partial}{\partial y}\left(\frac{1}{r}\right) + b\frac{\partial^2}{\partial y^2}\left(\frac{1}{r}\right)\right\} \qquad \varphi = \frac{a}{a^2+c^2}$$

$$\frac{1}{f\sigma}\frac{\partial^4 L_{ab}}{\partial x\partial y^3} = +dc\iint\frac{\partial^2}{\partial y^2}\frac{1}{r} = +dc\iint\frac{\partial^2}{\partial y^2}\frac{1}{r}$$

$$\frac{1}{f\sigma}\frac{\partial^4 L_{ab}}{\partial y^4} = -dc\iint a\frac{\partial^2}{\partial y^2}\varphi\frac{1}{r} = -dc\iint a\left\{\frac{1}{r}\frac{\partial^2\varphi}{\partial y^2} + 2\frac{\partial}{\partial y}\frac{1}{r}\frac{\partial\varphi}{\partial y} + \frac{\partial^2\left(\frac{1}{r}\right)}{\partial y^2}\varphi\right\} \qquad \varphi = \frac{b}{b^2+c^2}$$

$\varphi = \frac{a}{a^2+c^2}$
$$\frac{1}{f\sigma}\frac{\partial^5 L_{ab}}{\partial x^5} = -dc\int_{b_1}\int_{a_1} b\frac{\partial^3}{\partial x^3}\varphi\frac{1}{r} = -dc\int_{b_1}\int_{a_1} b\left\{\frac{1}{r}\frac{\partial^3\varphi}{\partial x^3} + 3\frac{\partial\frac{1}{r}}{\partial x}\frac{\partial^2\varphi}{\partial x^2} + 3\frac{\partial^2\frac{1}{r}}{\partial x^2}\frac{\partial\varphi}{\partial x} + \frac{\partial^3\frac{1}{r}}{\partial x^3}\varphi\right\}$$

$$\frac{1}{f\sigma}\frac{\partial^5 L_{ab}}{\partial x^4\partial y} = +dc\iint\frac{\partial^3\frac{1}{r}}{\partial x^3} = +dc\iint\frac{\partial^3\frac{1}{r}}{\partial x^3}$$

$\varphi = \frac{b}{b^2+c^2}$
$$\frac{1}{f\sigma}\frac{\partial^5 L_{ab}}{\partial x^3\partial y^2} = -dc\iint\varphi\frac{\partial^3}{\partial x^3}a\frac{1}{r} = -dc\iint\varphi\left\{-3\frac{\partial^2\frac{1}{r}}{\partial x^2} + a\frac{\partial^3\frac{1}{r}}{\partial x^3}\right\}$$

$\varphi = \frac{a}{a^2+c^2}$
$$\frac{1}{f\sigma}\frac{\partial^5 L_{ab}}{\partial x^2\partial y^3} = -dc\iint\varphi\frac{\partial^3}{\partial y^3}b\frac{1}{r} = -dc\iint\varphi\left\{-3\frac{\partial^2\frac{1}{r}}{\partial y^2} + b\frac{\partial^3\frac{1}{r}}{\partial y^3}\right\}$$

$$\frac{1}{f\sigma}\frac{\partial^5 L_{ab}}{\partial x\partial y^4} = +dc\iint\frac{\partial^3\frac{1}{r}}{\partial y^3} = +dc\iint\frac{\partial^3\frac{1}{r}}{\partial y^3}$$

$\varphi = \frac{b}{b^2+c^2}$
$$\frac{1}{f\sigma}\frac{\partial^5 L_{ab}}{\partial y^5} = -dc\iint a\frac{\partial^3}{\partial y^3}\varphi\frac{1}{r} = -dc\iint a\left\{\frac{1}{r}\frac{\partial^3\varphi}{\partial y^3} + 3\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial^2\varphi}{\partial y^2} + 3\frac{\partial^2\frac{1}{r}}{\partial y^2}\frac{\partial\varphi}{\partial y} + \frac{\partial^3\frac{1}{r}}{\partial y^3}\varphi\right\}$$

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$\varphi = \frac{a}{a^2+c^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^6 L_{ab}}{\partial x^6} = -dc \iint_{ba} b \frac{\partial^4}{\partial x^4}\varphi \cdot \frac{1}{r} = -dc \iint b\left\{\frac{1}{r}\frac{\partial^4\varphi}{\partial x^4} + 4\frac{\partial \frac{1}{r}}{\partial x}\frac{\partial^3\varphi}{\partial x^3} + 6\frac{\partial^2\frac{1}{r}}{\partial x^2}\frac{\partial^2\varphi}{\partial x^2} + 4 \right.$

$\frac{1}{f\sigma}\frac{\partial^6 L_{ab}}{\partial x^5 \partial y} = +dc \iint \frac{\partial^4}{\partial x^4}\frac{1}{r} = +dc \iint \frac{\partial^4}{\partial x^4}\frac{1}{r}$

$\varphi = \frac{b}{b^2+c^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^6 L_{ab}}{\partial x^4 \partial y^2} = -dc \iint \varphi \frac{\partial^4}{\partial x^4} a \frac{1}{r} = -dc \iint \varphi\left\{-4\frac{\partial^3 \frac{1}{r}}{\partial x^3} + a\frac{\partial^4\frac{1}{r}}{\partial x^4}\right\}$

$\frac{1}{f\sigma}\frac{\partial^6 L_{ab}}{\partial x^3 \partial y^3} = +dc \iint \frac{\partial^4}{\partial x^2 \partial y^2}\left(\frac{1}{r}\right) = +dc \iint \frac{\partial^4 \frac{1}{r}}{\partial x^2 \partial y^2}$

$\varphi = \frac{a}{a^2+c^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^6 L_{ab}}{\partial x^2 \partial y^4} = -dc \iint \varphi \frac{\partial^4}{\partial y^4} b \cdot \frac{1}{r} = -dc \iint \varphi\left\{-4\frac{\partial^3\frac{1}{r}}{\partial y^3} + b\frac{\partial^4\frac{1}{r}}{\partial y^4}\right\}$

$\frac{1}{f\sigma}\frac{\partial^6 L_{ab}}{\partial x \partial y^5} = +dc \iint \frac{\partial^4}{\partial y^4}\frac{1}{r} = +dc \iint \frac{\partial^4}{\partial y^4}\frac{1}{r}$

$\varphi = \frac{b}{b^2+c^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^6 L_{ab}}{\partial y^6} = -dc \iint a \frac{\partial^4}{\partial y^4}\varphi \cdot \frac{1}{r} = -dc \iint a\left\{\frac{1}{r}\frac{\partial^4\varphi}{\partial y^4} + 4\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial^3\varphi}{\partial y^3} + 6\frac{\partial^2\frac{1}{r}}{\partial y^2}\frac{\partial^2\varphi}{\partial y^2} + \right.$

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$\varphi = \frac{a}{a^2+c^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^7 L_{ab}}{\partial x^7} = -dc \iint_{ba} b \frac{\partial^5}{\partial x^5}\varphi \cdot \frac{1}{r} = -dc \iint_{ba} b\left\{\frac{1}{r}\frac{\partial^5\varphi}{\partial x^5} + 5\frac{\partial\frac{1}{r}}{\partial x}\frac{\partial^4\varphi}{\partial x^4} + 10\frac{\partial^2\frac{1}{r}}{\partial x^2}\frac{\partial^3\varphi}{\partial x^3} + 10\frac{\partial^3\frac{1}{r}}{\partial x^3}\right.$

$\frac{1}{f\sigma}\frac{\partial^7 L_{ab}}{\partial x^6 \partial y} = +dc \iint \frac{\partial^5}{\partial x^5}\left(\frac{1}{r}\right) = +dc \iint \frac{\partial^5 \frac{1}{r}}{\partial x^5}$

$\varphi = \frac{b}{b^2+c^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^7 L_{ab}}{\partial x^5 \partial y^2} = -dc \iint \varphi \frac{\partial^5}{\partial x^5} a \frac{1}{r} = -dc \iint \varphi\left\{-5\frac{\partial^4\frac{1}{r}}{\partial x^4} + a\frac{\partial^5\frac{1}{r}}{\partial x^5}\right\}$

$\frac{1}{f\sigma}\frac{\partial^7 L_{ab}}{\partial x^4 \partial y^3} = +dc \iint \frac{\partial^5}{\partial x^3 \partial y^2}\left(\frac{1}{r}\right) = +dc \iint \frac{\partial^5\frac{1}{r}}{\partial x^3 \partial y^2}$

$\frac{1}{f\sigma}\frac{\partial^7 L_{ab}}{\partial x^3 \partial y^4} = +dc \iint \frac{\partial^5}{\partial x^2 \partial y^3}\left(\frac{1}{r}\right) = +dc \iint \frac{\partial^5\frac{1}{r}}{\partial x^2 \partial y^3}$

$\varphi = \frac{a}{a^2+c^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^7 L_{ab}}{\partial x^2 \partial y^5} = -dc \iint \varphi \frac{\partial^5}{\partial y^5} b \frac{1}{r} = -dc \iint \varphi\left\{-5\frac{\partial^4\frac{1}{r}}{\partial y^4} + b\frac{\partial^5\frac{1}{r}}{\partial y^5}\right\}$

$\frac{1}{f\sigma}\frac{\partial^7 L}{\partial x \partial y^6} = +dc \iint \frac{\partial^5}{\partial y^5}\left(\frac{1}{r}\right) = +dc \iint \frac{\partial^5\frac{1}{r}}{\partial y^5}$

$\varphi = \frac{b}{b^2+c^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^7 L}{\partial y^7} = -dc \iint a \frac{\partial^5}{\partial y^5}\varphi \frac{1}{r} = -dc \iint a\left\{\frac{1}{r}\frac{\partial^5\varphi}{\partial y^5} + 5\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial^4\varphi}{\partial y^4} + 10\frac{\partial^2\frac{1}{r}}{\partial y^2}\frac{\partial^3\varphi}{\partial y^3} + 10\frac{\partial^3}{\partial y}\right.$

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$$\ldots \frac{\partial^3 \frac{1}{r}}{\partial x^3} \frac{\partial \varphi}{\partial x} + \frac{\partial^4 \frac{1}{r}}{\partial x^4} \varphi \Big\}$$

$$\ldots + \frac{\partial^3 \frac{1}{r}}{\partial y^3} \frac{\partial \varphi}{\partial y} + \frac{\partial^4 \frac{1}{r}}{\partial y^4} \varphi \Big\}$$

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$$\ldots \frac{\partial^2 \varphi}{\partial x^2} + 5 \frac{\partial^4 \frac{1}{r}}{\partial x^4} \frac{\partial \varphi}{\partial x} + \frac{\partial^5 \frac{1}{r}}{\partial x^5} \varphi \Big\}$$

$$\ldots \frac{\partial^2 \varphi}{\partial y^2} + 5 \frac{\partial^4 \frac{1}{r}}{\partial y^4} \frac{\partial \varphi}{\partial y} + \frac{\partial^5 \frac{1}{r}}{\partial y^5} \varphi \Big\}$$

$\varphi = \frac{a}{a^2+c^2}$ $\qquad \frac{1}{f\sigma}\frac{\partial^8 \mathcal{L}_{ab}}{\partial x^8} = -dc \Big\|_{ba} b\frac{\partial^6}{\partial x^6}\varphi\cdot\frac{1}{r} = -dc \Big\| b\left\{\frac{1}{r}\frac{\partial^6\varphi}{\partial x^6} + 6\frac{\partial\frac{1}{r}}{\partial x}\frac{\partial^5\varphi}{\partial x^5} + 15\frac{\partial^2\frac{1}{r}}{\partial x^2}\frac{\partial^4\varphi}{\partial x^4} + 20\frac{\partial^3\frac{1}{r}}{\partial x^3}\frac{\partial^3\varphi}{\partial x^3} + 15\frac{\partial^4\frac{1}{r}}{\partial x^4}\frac{\partial^2\varphi}{\partial x^2} + 6\frac{\partial^5\frac{1}{r}}{\partial x^5}\frac{\partial\varphi}{\partial x} + \frac{\partial^6\frac{1}{r}}{\partial x^6}\varphi\right\}$

$$\frac{1}{f\sigma}\frac{\partial^8 \mathcal{L}_{ab}}{\partial x^7\partial y} = +dc\Big\|\frac{\partial^6\frac{1}{r}}{\partial x^6} = +dc\Big\|\frac{\partial^6\frac{1}{r}}{\partial x^6}$$

$\varphi = \frac{b}{b^2+c^2}$ $\qquad \frac{1}{f\sigma}\frac{\partial^8 \mathcal{L}_{ab}}{\partial x^6\partial y^2} = -dc\Big\|\varphi\frac{\partial^6}{\partial x^6}\left(a\frac{1}{r}\right) = -dc\Big\|\varphi\left\{-6\frac{\partial^5\frac{1}{r}}{\partial x^5} + a\frac{\partial^6\frac{1}{r}}{\partial x^6}\right\}$

$$\frac{1}{f\sigma}\frac{\partial^8 \mathcal{L}_{ab}}{\partial x^5\partial y^3} = +dc\Big\|\frac{\partial^6\frac{1}{r}}{\partial x^4\partial y^2} = -dc\Big\|\frac{\partial^6\frac{1}{r}}{\partial x^4\partial y^2}$$

$$\frac{1}{f\sigma}\frac{\partial^8 \mathcal{L}_{ab}}{\partial x^4\partial y^4} = +dc\Big\|\frac{\partial^6\frac{1}{r}}{\partial x^3\partial y^3} = +dc\Big\|\frac{\partial^6\frac{1}{r}}{\partial x^3\partial y^3}$$

$$\frac{1}{f\sigma}\frac{\partial^8 \mathcal{L}_{ab}}{\partial x^3\partial y^5} = +dc\Big\|\frac{\partial^6\frac{1}{r}}{\partial x^2\partial y^4} = +dc\Big\|\frac{\partial^6\frac{1}{r}}{\partial x^2\partial y^4}$$

$\varphi = \frac{a}{a^2+c^2}$ $\qquad \frac{1}{f\sigma}\frac{\partial^8 \mathcal{L}_{ab}}{\partial x^2\partial y^6} = -dc\Big\|\varphi\frac{\partial^6}{\partial y^6}\left(b\frac{1}{r}\right) = -dc\Big\|\varphi\left\{-6\frac{\partial^5\frac{1}{r}}{\partial y^5} + b\frac{\partial^6\frac{1}{r}}{\partial y^6}\right\}$

$$\frac{1}{f\sigma}\frac{\partial^8 \mathcal{L}_{ab}}{\partial x\partial y^7} = +dc\Big\|\frac{\partial^6\frac{1}{r}}{\partial y^6} = +dc\Big\|\frac{\partial^6\frac{1}{r}}{\partial y^6}$$

$\varphi = \frac{b}{b^2+c^2}$ $\qquad \frac{1}{f\sigma}\frac{\partial^8 \mathcal{L}_{ab}}{\partial y^8} = -dc\Big\|a\frac{\partial^6}{\partial y^6}\varphi\frac{1}{r} = -dc\Big\|a\left\{\frac{1}{r}\frac{\partial^6\varphi}{\partial y^6} + 6\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial^5\varphi}{\partial y^5} + 15\frac{\partial^2\frac{1}{r}}{\partial y^2}\frac{\partial^4\varphi}{\partial y^4} + 20\frac{\partial^3\frac{1}{r}}{\partial y^3}\frac{\partial^3\varphi}{\partial y^3} + 15\frac{\partial^4\frac{1}{r}}{\partial y^4}\frac{\partial^2\varphi}{\partial y^2} + 6\frac{\partial^5\frac{1}{r}}{\partial y^5}\frac{\partial\varphi}{\partial y} + \frac{\partial^6\frac{1}{r}}{\partial y^6}\varphi\right\}$

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bc lap.

$$\frac{1}{f\sigma}\frac{\partial L_{bc}}{\partial x} = da \Big\| \Big\|_{b_1\, c_1}^{b_2\, c_2} \operatorname{arctg} \frac{bc}{a\sqrt{a^2+b^2+c^2}}$$

$$\frac{1}{f\sigma}\frac{\partial L_{bc}}{\partial y} = da \Big\| \Big\|_{b_1\, c_1}^{b_2\, c_2} \log \sqrt{1+\frac{b^2}{a^2}}\; \frac{1+\sqrt{1+\frac{b^2}{c^2}}}{1+\sqrt{1+\frac{a^2}{c^2}+\frac{b^2}{c^2}}} = -da \Big\|\Big\| \log\left(c+\sqrt{a^2+b^2+c^2}\right)$$

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$$\frac{1}{f\sigma}\frac{\partial^2 L_{bc}}{\partial x^2} = + da \Big\| \Big\|_{b_1\, c_1}^{b_2\, c_2} \left(\frac{bc}{(a^2+c^2)} + \frac{bc}{(a^2+b^2)}\right)\frac{1}{r} = + da \Big\|\Big\| \cancel{bc}$$

$$\frac{1}{f\sigma}\frac{\partial^2 L_{bc}}{\partial x \partial y} = - da \Big\| \Big\|_{b_1\, c_1}^{b_2\, c_2} \frac{ac}{a^2+b^2}\,\frac{1}{r}$$

$$\frac{1}{f\sigma}\frac{\partial^2 L_{bc}}{\partial y^2} = - da \Big\| \Big\|_{b_1\, c_1}^{b_2\, c_2} \frac{bc}{a^2+b^2}\,\frac{1}{r}.$$

$$\frac{1}{f\sigma}\frac{\partial^3 L_{bc}}{\partial x^3} = + da \Big\|\Big\|_{b\, c} bc \frac{\partial}{\partial x}(\psi+\psi')\frac{1}{r} = + da \Big\|\Big\| bc\left\{\frac{1}{r}\left(\frac{\partial \psi}{\partial x}+\frac{\partial \psi'}{\partial x}\right) + (\psi+\psi')\frac{\partial \frac{1}{r}}{\partial x}\right\} \qquad \psi = \frac{1}{a^2+b^2} \quad \psi' = \frac{1}{a^2+c^2}$$

$$\frac{1}{f\sigma}\frac{\partial^3 L_{bc}}{\partial x^2 \partial y} = - da \Big\|\Big\| c\frac{\partial}{\partial x}\varphi\frac{1}{r} = - da \Big\|\Big\| c\left\{\frac{1}{r}\frac{\partial \varphi}{\partial x} + \varphi\frac{\partial \frac{1}{r}}{\partial x}\right\} \qquad \varphi = \frac{a}{a^2+b^2}$$

$$\frac{1}{f\sigma}\frac{\partial^3 L_{bc}}{\partial x \partial y^2} = - da \Big\|\Big\| ac\frac{\partial}{\partial y}\psi\frac{1}{r} = - da \Big\|\Big\| ac\left\{\frac{1}{r}\frac{\partial \psi}{\partial y} + \psi\frac{\partial \frac{1}{r}}{\partial y}\right\} \qquad \psi = \frac{1}{a^2+b^2}$$

$$\frac{1}{f\sigma}\frac{\partial^3 L_{bc}}{\partial y^3} = - da \Big\|\Big\| c\frac{\partial}{\partial y}\varphi\frac{1}{r} = - da \Big\|\Big\| c\left\{\frac{1}{r}\frac{\partial \varphi}{\partial y} + \varphi\frac{\partial \frac{1}{r}}{\partial y}\right\} \qquad \varphi = \frac{b}{a^2+b^2}$$

bc

$$\varphi = \frac{1}{a^2+b^2} \quad \varphi' = \frac{1}{a^2+c^2} \qquad \frac{1}{f_0}\frac{\partial^4 L_{bc}}{\partial x^4} = +da \iint_{cb} bc \frac{\partial^2}{\partial x^2}(\varphi+\varphi')\frac{1}{r} = +da \iint bc\left\{\frac{1}{r}\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi'}{\partial x^2}\right) + 2\frac{\partial \frac{1}{r}}{\partial x}\left(\frac{\partial\varphi}{\partial x}+\frac{\partial\varphi'}{\partial x}\right) + \frac{\partial^2\frac{1}{r}}{\partial x^2}(\varphi+\varphi')\right\}$$

$$\varphi = \frac{a}{a^2+b^2} \qquad \frac{1}{f_0}\frac{\partial^4 L_b}{\partial x^3 \partial y} = -da \iint c \frac{\partial^2}{\partial x^2}\varphi \cdot \frac{1}{r} = -da \iint c\left\{\frac{1}{r}\frac{\partial^2\varphi}{\partial x^2} + 2\frac{\partial\frac{1}{r}}{\partial x}\frac{\partial\varphi}{\partial x} + \frac{\partial^2\frac{1}{r}}{\partial x^2}\varphi\right\}$$

$$\varphi = \frac{1}{a^2+b^2} \qquad \frac{1}{f_0}\frac{\partial^4 L}{\partial x^2 \partial y^2} = -da \iint_{cb} bc \frac{\partial^2}{\partial x^2}\varphi \frac{1}{r} = -da \iint bc\left\{\frac{1}{r}\frac{\partial^2\varphi}{\partial x^2} + 2\frac{\partial\frac{1}{r}}{\partial x}\frac{\partial\varphi}{\partial x} + \frac{\partial^2\frac{1}{r}}{\partial x^2}\varphi\right\}$$

$$\varphi = \frac{1}{a^2+b^2} \qquad \frac{1}{f_0}\frac{\partial^4 L}{\partial x \partial y^3} = -da \iint ac \frac{\partial^2}{\partial y^2}\varphi \frac{1}{r} = -da \iint ac\left\{\frac{1}{r}\frac{\partial^2\varphi}{\partial y^2} + 2\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial\varphi}{\partial y} + \frac{\partial^2\frac{1}{r}}{\partial y^2}\varphi\right\}$$

$$\varphi = \frac{b}{a^2+b^2} \qquad \frac{1}{f_0}\frac{\partial^4 L}{\partial y^4} = -da \iint c \frac{\partial^2}{\partial y^2}\varphi \frac{1}{r} = -da \iint c\left\{\frac{1}{r}\frac{\partial^2\varphi}{\partial y^2} + 2\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial\varphi}{\partial y} + \frac{\partial^2\frac{1}{r}}{\partial y^2}\varphi\right\}$$

$$\varphi = \frac{1}{a^2+b^2} \quad \varphi' = \frac{1}{a^2+c^2} \qquad \frac{1}{f_0}\frac{\partial^5 L}{\partial x^5} = +da \iint_{cb} bc \frac{\partial^3}{\partial x^3}(\varphi+\varphi')\frac{1}{r} = +da \iint bc\left\{\frac{1}{r}\left(\frac{\partial^3\varphi}{\partial x^3}+\frac{\partial^3\varphi'}{\partial x^3}\right) + 3\frac{\partial\frac{1}{r}}{\partial x}\left(\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi'}{\partial x^2}\right) + 3\frac{\partial^2\frac{1}{r}}{\partial x^2}\left(\frac{\partial\varphi}{\partial x}+\frac{\partial\varphi'}{\partial x}\right) + \frac{\partial^3\frac{1}{r}}{\partial x^3}(\varphi+\varphi')\right\}$$

$$\varphi = \frac{a}{a^2+b^2} \qquad \frac{1}{f_0}\frac{\partial^5 L}{\partial x^4 \partial y} = -da \iint c \frac{\partial^3}{\partial x^3}\varphi \frac{1}{r} = -da \iint c\left\{\frac{1}{r}\frac{\partial^3\varphi}{\partial x^3} + 3\frac{\partial\frac{1}{r}}{\partial x}\frac{\partial^2\varphi}{\partial x^2} + 3\frac{\partial^2\frac{1}{r}}{\partial x^2}\frac{\partial\varphi}{\partial x} + \frac{\partial^3\frac{1}{r}}{\partial x^3}\varphi\right\}$$

$$\varphi = \frac{1}{a^2+b^2} \qquad \frac{1}{f_0}\frac{\partial^5 L}{\partial x^3 \partial y^2} = -da \iint bc \frac{\partial^3}{\partial x^3}\varphi \frac{1}{r} = -da \iint bc\left\{\frac{1}{r}\frac{\partial^3\varphi}{\partial x^3} + 3\frac{\partial\frac{1}{r}}{\partial x}\frac{\partial^2\varphi}{\partial x^2} + 3\frac{\partial^2\frac{1}{r}}{\partial x^2}\frac{\partial\varphi}{\partial x} + \frac{\partial^3\frac{1}{r}}{\partial x^3}\varphi\right\}$$

$$\varphi = \frac{b}{a^2+b^2} \quad \varphi' = \frac{b}{a^2+c^2} \qquad \frac{1}{f_0}\frac{\partial^5 L}{\partial x^2 \partial y^3} = +da \iint c \frac{\partial^3}{\partial y^3}(\varphi+\varphi')\frac{1}{r} = +da \iint c\left\{\frac{1}{r}\frac{\partial^3\varphi}{\partial y^3} + 3\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial^2\varphi}{\partial y^2} + 3\frac{\partial^2\frac{1}{r}}{\partial y^2}\left(\frac{\partial\varphi}{\partial y} - \frac{1}{a^2+c^2}\right) + \frac{\partial^3\frac{1}{r}}{\partial y^3}\left(\varphi + \frac{b}{a^2+c^2}\right)\right\}$$

$$\varphi = \frac{1}{a^2+b^2} \qquad \frac{1}{f_0}\frac{\partial^5 L}{\partial x \partial y^4} = -da \iint ac \frac{\partial^3}{\partial y^3}\varphi \frac{1}{r} = -da \iint ac\left\{\frac{1}{r}\frac{\partial^3\varphi}{\partial y^3} + 3\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial^2\varphi}{\partial y^2} + 3\frac{\partial^2\frac{1}{r}}{\partial y^2}\frac{\partial\varphi}{\partial y} + \frac{\partial^3\frac{1}{r}}{\partial y^3}\varphi\right\}$$

$$\varphi = \frac{b}{a^2+b^2} \qquad \frac{1}{f_0}\frac{\partial^5 L}{\partial y^5} = -da \iint c \frac{\partial^3}{\partial y^3}\varphi \frac{1}{r} = -da \iint c\left\{\frac{1}{r}\frac{\partial^3\varphi}{\partial y^3} + 3\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial^2\varphi}{\partial y^2} + 3\frac{\partial^2\frac{1}{r}}{\partial y^2}\frac{\partial\varphi}{\partial y} + \frac{\partial^3\frac{1}{r}}{\partial y^3}\varphi\right\}$$

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$bc.$

$$\varphi=\frac{1}{a^2+b^2}\quad \varphi'=\frac{1}{a^2+c^2}\qquad \frac{1}{f\sigma}\frac{\partial^6 L}{\partial x^6}=+da\iint bc\frac{\partial^4}{\partial x^4}(\varphi+\varphi')\frac{1}{r}=+da\iint bc\left\{\frac{1}{r}\left(\frac{\partial^4\varphi}{\partial x^4}+\frac{\partial^4\varphi'}{\partial x^4}\right)+4\frac{\partial\frac{1}{r}}{\partial x}\left(\frac{\partial^3\varphi}{\partial x^3}+\frac{\partial^3\varphi'}{\partial x^3}\right)\right.$$

$$\varphi=\frac{a}{a^2+b^2}\qquad \frac{1}{f\sigma}\frac{\partial^6 L}{\partial x^5\partial y}=-da\iint c\frac{\partial^4}{\partial x^4}\varphi\frac{1}{r}=-da\iint c\left\{\frac{1}{r}\frac{\partial^4\varphi}{\partial x^4}+4\frac{\partial\frac{1}{r}}{\partial x}\frac{\partial^3\varphi}{\partial x^3}+6\frac{\partial^2\frac{1}{r}}{\partial x^2}\frac{\partial^2\varphi}{\partial x^2}+4\right.$$

$$\varphi=\frac{1}{a^2+b^2}\qquad \frac{1}{f\sigma}\frac{\partial^6 L}{\partial x^4\partial y^2}=-da\iint bc\frac{\partial^4}{\partial x^4}\varphi\cdot\frac{1}{r}=-da\iint bc\left\{\frac{1}{r}\frac{\partial^4\varphi}{\partial x^4}+4\frac{\partial\frac{1}{r}}{\partial x}\frac{\partial^3\varphi}{\partial x^3}+6\frac{\partial^2\frac{1}{r}}{\partial x^2}\frac{\partial^2\varphi}{\partial x^2}+4\right.$$

$$\varphi=\frac{a}{a^2+b^2}\qquad \frac{1}{f\sigma}\frac{\partial^6 L}{\partial x^3\partial y^3}=+da\iint c\frac{\partial^4}{\partial x^2\partial y^2}\varphi\frac{1}{r}=-da\iint c\left\{\frac{1}{r}\frac{\partial^4\varphi}{\partial x^2\partial y^2}+2\frac{\partial\frac{1}{r}}{\partial x}\frac{\partial^3\varphi}{\partial x\partial y^2}+2\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial^3\varphi}{\partial x^2\partial y}+\right.$$

$$\varphi=\frac{b}{a^2+b^2},\ \varphi'=\frac{b}{a^2+c^2}\qquad \frac{1}{f\sigma}\frac{\partial^6 L}{\partial x^2\partial y^4}=+da\iint c\frac{\partial^4}{\partial y^4}(\varphi+\varphi')\frac{1}{r}=+da\iint c\left\{\frac{1}{r}\frac{\partial^4\varphi}{\partial y^4}+4\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial^3\varphi}{\partial y^3}+6\frac{\partial^2\frac{1}{r}}{\partial y^2}\frac{\partial^2\varphi}{\partial y^2}+\right.$$

$$\varphi=\frac{1}{a^2+b^2}\qquad \frac{1}{f\sigma}\frac{\partial^6 L}{\partial x\partial y^5}=-da\iint ac\frac{\partial^4}{\partial y^4}\varphi\frac{1}{r}=-da\iint ac\left\{\frac{1}{r}\frac{\partial^4\varphi}{\partial y^4}+4\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial^3\varphi}{\partial y^3}+6\frac{\partial^2\frac{1}{r}}{\partial y^2}\frac{\partial^2\varphi}{\partial y^2}+\right.$$

$$\varphi=\frac{b}{a^2+b^2}\qquad \frac{1}{f\sigma}\frac{\partial^6 L}{\partial y^6}=-da\iint c\frac{\partial^4}{\partial y^4}\varphi\frac{1}{r}=-da\iint c\left\{\frac{1}{r}\frac{\partial^4\varphi}{\partial y^4}+4\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial^3\varphi}{\partial y^3}+6\frac{\partial^2\frac{1}{r}}{\partial y^2}\frac{\partial^2\varphi}{\partial y^2}+\right.$$

$$\varphi=\frac{1}{a^2+b^2}\quad \varphi'=\frac{1}{a^2+c^2}\qquad \frac{1}{f\sigma}\frac{\partial^7 L}{\partial x^7}=+da\iint bc\frac{\partial^5}{\partial x^5}(\varphi+\varphi')\frac{1}{r}=+da\iint bc\left\{\frac{1}{r}\left(\frac{\partial^5\varphi}{\partial x^5}+\frac{\partial^5\varphi'}{\partial x^5}\right)+5\frac{\partial\frac{1}{r}}{\partial x}\left(\frac{\partial^4\varphi}{\partial x^4}+\right.\right.$$

$$\varphi=\frac{a}{a^2+b^2}\qquad \frac{1}{f\sigma}\frac{\partial^7 L}{\partial x^6\partial y}=-da\iint c\frac{\partial^5}{\partial x^5}\varphi\frac{1}{r}=-da\iint c\left\{\frac{1}{r}\frac{\partial^5\varphi}{\partial x^5}+5\frac{\partial\frac{1}{r}}{\partial x}\frac{\partial^4\varphi}{\partial x^4}+10\frac{\partial^2\frac{1}{r}}{\partial x^2}\right.$$

$$\varphi=\frac{1}{a^2+b^2}\qquad \frac{1}{f\sigma}\frac{\partial^7 L}{\partial x^5\partial y^2}=-da\iint bc\frac{\partial^5}{\partial x^5}\varphi\cdot\frac{1}{r}=-da\iint bc\left\{\frac{1}{r}\frac{\partial^5\varphi}{\partial x^5}+5\frac{\partial\frac{1}{r}}{\partial x}\frac{\partial^4\varphi}{\partial x^4}+10\frac{\partial^2\frac{1}{r}}{\partial x^2}\right.$$

$$\varphi=\frac{a}{a^2+b^2}\qquad \frac{1}{f\sigma}\frac{\partial^7 L}{\partial x^4\partial y^3}=-da\iint c\frac{\partial^5}{\partial x^3\partial y^2}\varphi\frac{1}{r}=-da\iint c\left\{\frac{1}{r}\frac{\partial^5\varphi}{\partial x^3\partial y^2}+3\frac{\partial\frac{1}{r}}{\partial x}\frac{\partial^4\varphi}{\partial x^2\partial y^2}+2\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial^4\varphi}{\partial x^3\partial y}\right.$$

$$\varphi=\frac{a}{a^2+b^2}\qquad \frac{1}{f\sigma}\frac{\partial^7 L}{\partial x^3\partial y^4}=-da\iint c\frac{\partial^5}{\partial x^2\partial y^3}\varphi\frac{1}{r}=-da\iint c\left\{\frac{1}{r}\frac{\partial^5\varphi}{\partial x^2\partial y^3}+2\frac{\partial\frac{1}{r}}{\partial x}\frac{\partial^4\varphi}{\partial x\partial y^3}+3\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial^4\varphi}{\partial x^2\partial y^2}\right.$$

$$\varphi=\frac{b}{a^2+b^2}\quad \varphi'=\frac{b}{a^2+c^2}\qquad \frac{1}{f\sigma}\frac{\partial^7 L}{\partial x^2\partial y^5}=+da\iint c\frac{\partial^5}{\partial y^5}(\varphi+\varphi')\frac{1}{r}=+da\iint c\left\{\frac{1}{r}\frac{\partial^5\varphi}{\partial y^5}+5\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial^4\varphi}{\partial y^4}+10\frac{\partial^2\frac{1}{r}}{\partial y^2}\frac{\partial^3\varphi}{\partial y^3}\right.$$

$$\varphi=\frac{1}{a^2+b^2}\qquad \frac{1}{f\sigma}\frac{\partial^7 L}{\partial x\partial y^6}=-da\iint ac\frac{\partial^5}{\partial y^5}\varphi\frac{1}{r}=-da\iint ac\left\{\frac{1}{r}\frac{\partial^5\varphi}{\partial y^5}+5\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial^4\varphi}{\partial y^4}+10\frac{\partial^2\frac{1}{r}}{\partial y^2}\right.$$

$$\varphi=\frac{b}{a^2+b^2}\qquad \frac{1}{f\sigma}\frac{\partial^7 L}{\partial y^7}=-da\iint c\frac{\partial^5}{\partial y^5}\varphi\frac{1}{r}=-da\iint c\left\{\frac{1}{r}\frac{\partial^5\varphi}{\partial y^5}+5\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial^4\varphi}{\partial y^4}+10\frac{\partial^2\frac{1}{r}}{\partial y^2}\right.$$

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$$+ 6\frac{\partial^2 \frac{1}{r}}{\partial x^2}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi'}{\partial x^2}\right) + 4\frac{\partial^3 \frac{1}{r}}{\partial x^3}\left(\frac{\partial \psi}{\partial x} + \frac{\partial \psi'}{\partial x}\right) + \frac{\partial^4 \frac{1}{r}}{\partial x^4}(\psi + \psi')\Big\}$$

$$\frac{\partial^3 \frac{1}{r}}{\partial x^3}\frac{\partial \varphi}{\partial x} + \frac{\partial^4 \frac{1}{r}}{\partial x^4}\varphi\Big\}$$

$$\frac{\partial^3 \frac{1}{r}}{\partial x^3}\frac{\partial \psi}{\partial x} + \frac{\partial^4 \frac{1}{r}}{\partial x^4}\psi\Big\}$$

$$\frac{\partial^2 \frac{1}{r}}{\partial x^2}\frac{\partial^2 \varphi}{\partial y^2} + 4\frac{\partial^2 \frac{1}{r}}{\partial x \partial y}\frac{\partial^2 \varphi}{\partial x \partial y} + \frac{\partial^2 \frac{1}{r}}{\partial y^2}\frac{\partial^2 \varphi}{\partial x^2} + 2\frac{\partial^3 \frac{1}{r}}{\partial x^2 \partial y}\frac{\partial \varphi}{\partial y} + 2\frac{\partial^3 \frac{1}{r}}{\partial x \partial y^2}\frac{\partial \varphi}{\partial x} + \frac{\partial^4 \frac{1}{r}}{\partial x^2 \partial y^2}\varphi\Big\}$$

$$4\frac{\partial^3 \frac{1}{r}}{\partial y^3}\left(\frac{\partial \varphi}{\partial y} - \frac{1}{a^2+c^2}\right) + \frac{\partial^4 \frac{1}{r}}{\partial y^4}\left(\varphi + \frac{b}{a^2+c^2}\right)\Big\} = -\frac{1}{45}\frac{\partial^6 L}{\partial y^6} + da \iint c\left\{-\frac{4}{a^2+c^2}\frac{\partial^3 \frac{1}{r}}{\partial y^3} + \frac{b}{a^2+c^2}\frac{\partial^4 \frac{1}{r}}{\partial y^4}\right\}$$

$$4\frac{\partial^3 \frac{1}{r}}{\partial y^3}\frac{\partial \psi}{\partial y} + \frac{\partial^4 \frac{1}{r}}{\partial y^4}\psi\Big\}$$

$$4\frac{\partial^3 \frac{1}{r}}{\partial y^3}\frac{\partial \varphi}{\partial y} + \frac{\partial^4 \frac{1}{r}}{\partial y^4}\varphi\Big\}$$

$$\frac{\partial^4 \psi'}{\partial x^4}\Big) + 10\frac{\partial^2 \frac{1}{r}}{\partial x^2}\left(\frac{\partial^3 \psi}{\partial x^3} + \frac{\partial^3 \psi'}{\partial x^3}\right) + 10\frac{\partial^3 \frac{1}{r}}{\partial x^3}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi'}{\partial x^2}\right) + 5\frac{\partial^4 \frac{1}{r}}{\partial x^4}\left(\frac{\partial \psi}{\partial x} + \frac{\partial \psi'}{\partial x}\right) + \frac{\partial^5 \frac{1}{r}}{\partial x^5}(\psi + \psi')\Big\}$$

$$\frac{\partial^3 \varphi}{\partial x^3} + 10\frac{\partial^3 \frac{1}{r}}{\partial x^3}\frac{\partial^2 \varphi}{\partial x^2} + 5\frac{\partial^4 \frac{1}{r}}{\partial x^4}\frac{\partial \varphi}{\partial x} + \frac{\partial^5 \frac{1}{r}}{\partial x^5}\varphi\Big\}$$

$$\frac{\partial^3 \psi}{\partial x^3} + 10\frac{\partial^3 \frac{1}{r}}{\partial x^3}\frac{\partial^2 \psi}{\partial x^2} + 5\frac{\partial^4 \frac{1}{r}}{\partial x^4}\frac{\partial \psi}{\partial x} + \frac{\partial^5 \frac{1}{r}}{\partial x^5}\psi\Big\}$$

$$+3\frac{\partial^2 \frac{1}{r}}{\partial x^2}\frac{\partial^3 \varphi}{\partial x \partial y^2} + 6\frac{\partial^2 \frac{1}{r}}{\partial x \partial y}\frac{\partial^3 \varphi}{\partial x^2 \partial y} + \frac{\partial^2 \frac{1}{r}}{\partial y^2}\frac{\partial^3 \varphi}{\partial x^3} + \frac{\partial^3 \frac{1}{r}}{\partial x^3}\frac{\partial^2 \varphi}{\partial y^2} + 6\frac{\partial^3 \frac{1}{r}}{\partial x^2 \partial y}\frac{\partial^2 \varphi}{\partial x \partial y} + 3\frac{\partial^3 \frac{1}{r}}{\partial x \partial y^2}\frac{\partial^2 \varphi}{\partial x^2} + \cancel{\frac{\partial \frac{1}{r}}{\partial y}\frac{\partial^2 \varphi}{\partial x^2}} + 2\frac{\partial^4 \frac{1}{r}}{\partial x^3 \partial y}\frac{\partial \varphi}{\partial y} + 3\frac{\partial^4 \frac{1}{r}}{\partial x^2 \partial y^2}\frac{\partial \varphi}{\partial x} + \frac{\partial^5 \frac{1}{r}}{\partial x^3 \partial y^2}\varphi\Big\}$$

$$+\frac{\partial^2 \frac{1}{r}}{\partial x^2}\frac{\partial^3 \varphi}{\partial y^3} + 6\frac{\partial^2 \frac{1}{r}}{\partial x \partial y}\frac{\partial^3 \varphi}{\partial x \partial y^2} + 3\frac{\partial^2 \frac{1}{r}}{\partial y^2}\frac{\partial^3 \varphi}{\partial x^2 \partial y} + 3\frac{\partial^3 \frac{1}{r}}{\partial x^2 \partial y}\frac{\partial^2 \varphi}{\partial y^2} + 6\frac{\partial^3 \frac{1}{r}}{\partial x \partial y^2}\frac{\partial^2 \varphi}{\partial x \partial y} + \frac{\partial^3 \frac{1}{r}}{\partial y^3}\frac{\partial^2 \varphi}{\partial x^2} + 3\frac{\partial^4 \frac{1}{r}}{\partial x^2 \partial y^2}\frac{\partial \varphi}{\partial y} + 2\frac{\partial^4 \frac{1}{r}}{\partial x \partial y^3}\frac{\partial \varphi}{\partial x} + \frac{\partial^5 \frac{1}{r}}{\partial x^2 \partial y^3}\varphi\Big\}$$

$$+ 10\frac{\partial^3 \frac{1}{r}}{\partial y^3}\frac{\partial^2 \varphi}{\partial y^2} + 5\frac{\partial^4 \frac{1}{r}}{\partial y^4}\left(\frac{\partial \varphi}{\partial y} - \frac{1}{a^2+c^2}\right) + \frac{\partial^5 \frac{1}{r}}{\partial y^5}\left(\varphi + \frac{b}{a^2+c^2}\right)\Big\}$$

$$\frac{\partial^3 \psi}{\partial y^3} + 10\frac{\partial^3 \frac{1}{r}}{\partial y^3}\frac{\partial^2 \psi}{\partial y^2} + 5\frac{\partial^4 \frac{1}{r}}{\partial y^4}\frac{\partial \psi}{\partial y} + \frac{\partial^5 \frac{1}{r}}{\partial y^5}\psi\Big\}$$

$$\frac{\partial^3 \varphi}{\partial y^3} + 10\frac{\partial^3 \frac{1}{r}}{\partial y^3}\frac{\partial^2 \varphi}{\partial y^2} + 5\frac{\partial^4 \frac{1}{r}}{\partial y^4}\frac{\partial \varphi}{\partial y} + \frac{\partial^5 \frac{1}{r}}{\partial y^5}\varphi\Big\}$$

# Ca lap.

$$\frac{1}{f\sigma}\frac{\partial L_{ca}}{\partial x} = db \Big|_{c_1}^{c_2}\Big|_{a_1}^{a_2} \log\sqrt{1+\frac{a^2}{b^2}}\cdot\frac{1+\sqrt{1+\frac{b^2}{c^2}}}{1+\sqrt{1+\frac{a^2}{c^2}+\frac{b^2}{c^2}}} = -db \Big|_{c_1}^{c_2}\Big|_{a_1}^{a_2} \log\left(c+\sqrt{a^2+b^2+c^2}\right)$$

$$\frac{1}{f\sigma}\frac{\partial L_{ca}}{\partial y} = db \Big|_{c_1}^{c_2}\Big|_{a_1}^{a_2} \operatorname{arctg}\frac{ac}{b\sqrt{a^2+b^2+c^2}}$$

$$\frac{1}{f\sigma}\frac{\partial^2 L_{ca}}{\partial x^2} = -db \Big|_{c_1}^{c_2}\Big|_{a_1}^{a_2} \frac{ac}{a^2+b^2}\frac{1}{r}.$$

$$\frac{1}{f\sigma}\frac{\partial^2 L_{ca}}{\partial x\,\partial y} = -db \Big|_{c_1}^{c_2}\Big|_{a_1}^{a_2} \frac{bc}{a^2+b^2}\frac{1}{r}.$$

$$\frac{1}{f\sigma}\frac{\partial^2 L_{ca}}{\partial y^2} = db \Big|_{c_1}^{c_2}\Big|_{a_1}^{a_2} \left(\frac{ac}{b^2+c^2}+\frac{ac}{a^2+b^2}\right)\frac{1}{r}.$$

$$\frac{1}{f\sigma}\frac{\partial^3 L}{\partial x^3} = -db \Big|\Big| c\frac{\partial}{\partial x}\varphi\frac{1}{r} \qquad \varphi = \frac{a}{a^2+b^2}$$

$$\underline{\frac{1}{f\sigma}\frac{\partial^3 L}{\partial x^2\,\partial y}} = -db \Big|\Big| bc\frac{\partial}{\partial x}\psi\frac{1}{r} = -db \Big|\Big| bc\left\{\frac{1}{r}\frac{\partial\psi}{\partial x}+\frac{\partial\frac{1}{r}}{\partial x}\psi\right\} \qquad \psi = \frac{1}{a^2+b^2}$$

$$\frac{1}{f\sigma}\frac{\partial^3 L}{\partial x\,\partial y^2} = -db \Big|\Big| c\frac{\partial}{\partial y}\varphi\frac{1}{r} \qquad \varphi = \frac{b}{a^2+b^2}$$

$$\underline{\frac{1}{f\sigma}\frac{\partial^3 L}{\partial y^3}} = +db \Big|\Big| ac\frac{\partial}{\partial y}(\psi+\psi')\frac{1}{r} = +db \Big|\Big| ac\left\{\frac{1}{r}\left(\frac{\partial\psi}{\partial y}+\frac{\partial\psi'}{\partial y}\right)+\frac{\partial\frac{1}{r}}{\partial y}(\psi+\psi')\right\} \qquad \psi = \frac{1}{a^2+b^2}\quad \psi' = \frac{1}{b^2+c^2}$$

Qa

$\varphi = \frac{a}{a^2+b^2}$ $\frac{1}{f\sigma}\frac{\partial^4 L}{\partial x^4} = -db \| c\frac{\partial^2}{\partial x^2}\varphi\frac{1}{r} = -db \| c\left\{\frac{1}{r}\frac{\partial^2\varphi}{\partial x^2} + 2\frac{\partial\frac{1}{r}}{\partial x}\frac{\partial\varphi}{\partial x} + \frac{\partial^2\frac{1}{r}}{\partial x^2}\varphi\right\}$

$\psi = \frac{1}{a^2+b^2}$ $\frac{1}{f\sigma}\frac{\partial^4 L}{\partial x^3\partial y} = -db \| bc\frac{\partial^2}{\partial x^2}\psi\frac{1}{r}$

$\psi = \frac{1}{a^2+b^2}$ $\frac{1}{f\sigma}\frac{\partial^4 L}{\partial x^2\partial y^2} = -db \| ac\frac{\partial^2}{\partial y^2}\psi\frac{1}{r} = -db \| ac\left\{\frac{1}{r}\frac{\partial^2\psi}{\partial y^2} + 2\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial\psi}{\partial y} + \frac{\partial^2\frac{1}{r}}{\partial y^2}\psi\right\}$

$\varphi = \frac{b}{a^2+b^2}$ $\frac{1}{f\sigma}\frac{\partial^4 L}{\partial x\partial y^3} = -db \| c\frac{\partial^2}{\partial y^2}\varphi\frac{1}{r}$

$\psi = \frac{1}{a^2+b^2}$ $\psi' = \frac{1}{b^2+c^2}$ $\frac{1}{f\sigma}\frac{\partial^4 L}{\partial y^4} = +db \| ac\frac{\partial^2}{\partial y^2}(\psi+\psi')\frac{1}{r} = +db \| ac\left\{\frac{1}{r}\left(\frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi'}{\partial y^2}\right) + 2\frac{\partial\frac{1}{r}}{\partial y}\left(\frac{\partial\psi}{\partial y} + \frac{\partial\psi'}{\partial y}\right) + \frac{\partial^2\frac{1}{r}}{\partial y^2}(\psi+\psi')\right\}$

$\varphi = \frac{a}{a^2+b^2}$ $\frac{1}{f\sigma}\frac{\partial^5 L}{\partial x^5} = -db \| c\frac{\partial^3}{\partial x^3}\varphi\frac{1}{r}$

$\psi = \frac{1}{a^2+b^2}$ $\frac{1}{f\sigma}\frac{\partial^5 L}{\partial x^4\partial y} = -db \| bc\frac{\partial^3}{\partial x^3}\psi\frac{1}{r} = -db \| bc\left\{\frac{1}{r}\frac{\partial^3\psi}{\partial x^3} + 3\frac{\partial\frac{1}{r}}{\partial x}\frac{\partial^2\psi}{\partial x^2} + 3\frac{\partial^2\frac{1}{r}}{\partial x^2}\frac{\partial\psi}{\partial x} + \frac{\partial^3\frac{1}{r}}{\partial x^3}\psi\right\}$

$\varphi = \frac{a}{a^2+b^2}$ $\varphi' = \frac{a}{b^2+c^2}$ $\frac{1}{f\sigma}\frac{\partial^5 L}{\partial x^3\partial y^2} = +db \| c\frac{\partial^3}{\partial x^3}(\varphi+\varphi')\frac{1}{r} =$

$\psi = \frac{1}{a^2+b^2}$ $\frac{1}{f\sigma}\frac{\partial^5 L}{\partial x^2\partial y^3} = -db \| ac\frac{\partial^3}{\partial y^3}\psi\frac{1}{r} = -db \| ac\left\{\frac{1}{r}\frac{\partial^3\psi}{\partial y^3} + 3\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial^2\psi}{\partial y^2} + 3\frac{\partial^2\frac{1}{r}}{\partial y^2}\frac{\partial\psi}{\partial y} + \frac{\partial^3\frac{1}{r}}{\partial y^3}\psi\right\}$

$\varphi = \frac{b}{a^2+b^2}$ $\frac{1}{f\sigma}\frac{\partial^5 L}{\partial x\partial y^4} = -db \| c\frac{\partial^3}{\partial y^3}\varphi\frac{1}{r}$

$\psi = \frac{1}{a^2+b^2}$ $\psi' = \frac{1}{b^2+c^2}$ $\frac{1}{f\sigma}\frac{\partial^5 L}{\partial y^5} = +db \| ac\frac{\partial^3}{\partial y^3}(\psi+\psi')\frac{1}{r} = +db \| ac\left\{\frac{1}{r}\left(\frac{\partial^3\psi}{\partial y^3} + \frac{\partial^3\psi'}{\partial y^3}\right) + 3\frac{\partial\frac{1}{r}}{\partial y}\left(\frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi'}{\partial y^2}\right) + 3\frac{\partial^2\frac{1}{r}}{\partial y^2}\left(\frac{\partial\psi}{\partial y} + \frac{\partial\psi'}{\partial y}\right) + \frac{\partial^3\frac{1}{r}}{\partial y^3}(\psi+\psi')\right\}$

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Ca

$\varphi = \dfrac{a}{a^2+b^2}$ $\quad \dfrac{1}{f^6}\dfrac{\partial^8 \mathcal{L}}{\partial x^8} = -db \| c \dfrac{\partial^6}{\partial x^6}\varphi\dfrac{1}{r} = -db\|c\Big\{\dfrac{1}{r}\dfrac{\partial^6\varphi}{\partial x^6} + 6\dfrac{\partial\frac{1}{r}}{\partial x}\dfrac{\partial^5\varphi}{\partial x^5} + 15\dfrac{\partial^2\frac{1}{r}}{\partial x^2}\dfrac{\partial^4\varphi}{\partial x^4} + 20$

$\psi = \dfrac{1}{a^2+b^2}$ $\quad \dfrac{1}{f^6}\dfrac{\partial^8 \mathcal{L}}{\partial x^7\partial y} = -db\| bc\dfrac{\partial^6}{\partial x^6}\psi\dfrac{1}{r}$

$\varphi = \dfrac{a}{a^2+b^2}$ $\varphi' = \dfrac{a}{b^2+c^2}$ $\quad \dfrac{1}{f^6}\dfrac{\partial^8 \mathcal{L}}{\partial x^6\partial y^2} = +db\|c\dfrac{\partial^6}{\partial x^6}(\varphi+\varphi')\dfrac{1}{r} = +db\|c\Big\{\dfrac{1}{r}\dfrac{\partial^6\varphi}{\partial x^6} + 6\dfrac{\partial\frac{1}{r}}{\partial x}\dfrac{\partial^5\varphi}{\partial x^5} + 15\dfrac{\partial^2\frac{1}{r}}{\partial x^2}\dfrac{\partial^4\varphi}{\partial x^4} + 20\dfrac{\partial^3\frac{1}{r}}{\partial x^3}$

$\varphi = \dfrac{b}{a^2+b^2}$ $\quad \dfrac{1}{f^6}\dfrac{\partial^8 \mathcal{L}}{\partial x^5\partial y^3} = -db\|c\dfrac{\partial^6}{\partial x^4\partial y^2}\varphi\dfrac{1}{r}$

$\varphi = \dfrac{b}{a^2+b^2}$ $\quad \dfrac{1}{f^6}\dfrac{\partial^8 \mathcal{L}}{\partial x^4\partial y^4} = -db\|c\dfrac{\partial^6}{\partial x^3\partial y^3}\varphi\dfrac{1}{r} = -db\|c\Big\{\dfrac{1}{r}\dfrac{\partial^6\varphi}{\partial x^3\partial y^3} + 3\dfrac{\partial\frac{1}{r}}{\partial x}\dfrac{\partial^5\varphi}{\partial x^2\partial y^3} + 3\dfrac{\partial\frac{1}{r}}{\partial y}\dfrac{\partial^5\varphi}{\partial x^3\partial y^2} +$

$\varphi = \dfrac{b}{a^2+b^2}$ $\quad \dfrac{1}{f^6}\dfrac{\partial^8 \mathcal{L}}{\partial x^3\partial y^5} = -db\|c\dfrac{\partial^6}{\partial x^2\partial y^4}\varphi\dfrac{1}{r}$

$\psi = \dfrac{1}{a^2+b^2}$ $\quad \dfrac{1}{f^6}\dfrac{\partial^8 \mathcal{L}}{\partial x^2\partial y^6} = -db\|ac\dfrac{\partial^6}{\partial y^6}\psi\dfrac{1}{r} = -db\|ac\Big\{\dfrac{1}{r}\dfrac{\partial^6\psi}{\partial y^6} + 6\dfrac{\partial\frac{1}{r}}{\partial y}\dfrac{\partial^5\psi}{\partial y^5} + 15\dfrac{\partial^2\frac{1}{r}}{\partial y^2}\dfrac{\partial^4\psi}{\partial y^4}$

$\varphi = \dfrac{b}{a^2+b^2}$ $\quad \dfrac{1}{f^6}\dfrac{\partial^8 \mathcal{L}}{\partial x\partial y^7} = -db\|c\dfrac{\partial^6}{\partial y^6}\varphi\dfrac{1}{r}$

$\psi = \dfrac{1}{a^2+b^2}$ $\psi' = \dfrac{1}{a^2+c^2}$ $\quad \dfrac{1}{f^6}\dfrac{\partial^8 \mathcal{L}}{\partial y^8} = +db\|ac\dfrac{\partial^6}{\partial y^6}(\psi+\psi')\dfrac{1}{r} = +db\|ac\Big\{\dfrac{1}{r}\Big(\dfrac{\partial^6\psi}{\partial y^6}+\dfrac{\partial^6\psi'}{\partial y^6}\Big) + 6\dfrac{\partial\frac{1}{r}}{\partial y}\Big(\dfrac{\partial^5\psi}{\partial y^5}+\dfrac{\partial^5\psi}{\partial y^5}$

$$\frac{\partial^{m+n}\varphi\frac{1}{r}}{\partial x^m\partial y^n} = \frac{1}{r}\frac{\partial^{m+n}\varphi}{\partial x^m\partial y^n} + \binom{m}{1}\frac{\partial\frac{1}{r}}{\partial x}\frac{\partial^{m+n-1}\varphi}{\partial x^{m-1}\partial y^n} + \binom{n}{1}\frac{\partial\frac{1}{r}}{\partial y}\frac{\partial^{m+n-1}\varphi}{\partial x^m\partial y^{n-1}} + \binom{m}{2}\frac{\partial^2\frac{1}{r}}{\partial x^2}\frac{\partial^{m+n-2}\varphi}{\partial x^{m-2}\partial y^n} + \binom{m}{1}\binom{n}{1}\frac{\partial^2\frac{1}{r}}{\partial x\partial y}$$

$$+ \binom{m}{3}\frac{\partial^3\frac{1}{r}}{\partial x^3}\frac{\partial^{m+n-3}\varphi}{\partial x^{m-3}\partial y^n} + \binom{m}{2}\binom{n}{1}\frac{\partial^3\frac{1}{r}}{\partial x^2\partial y}\frac{\partial^{m+n}\varphi}{\partial x^{m-}}$$

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$$+ \binom{m}{4}\frac{\partial^4\frac{1}{r}}{\partial x^4}\frac{\partial^{m+n-4}\varphi}{\partial x^{m-4}\partial y^n} + \binom{m}{3}\binom{n}{1}\frac{\partial^4\frac{1}{r}}{\partial x^3\partial y}\frac{\partial}{\partial x^m}$$

$$+ \binom{m}{5}\frac{\partial^5\frac{1}{r}}{\partial x^5}\frac{\partial^{m+n-5}\varphi}{\partial x^{m-5}\partial y^n} + \binom{m}{4}\binom{n}{1}\frac{\partial^5\frac{1}{r}}{\partial x^4\partial y}\frac{\partial}{\partial x^m}$$

$$+ \binom{m}{6}\frac{\partial^6\frac{1}{r}}{\partial x^6}\frac{\partial^{m+n-6}\varphi}{\partial x^{m-6}\partial y^n} + \binom{m}{5}\binom{n}{1}\frac{\partial^6\frac{1}{r}}{\partial x^5\partial y}\frac{\partial^{m+n}\varphi}{\partial x^{m-5}\partial y}$$

$$\frac{\partial^3 \frac{1}{r}}{\partial x^3}\frac{\partial^3 \varphi}{\partial x^3} + 15\frac{\partial^4 \frac{1}{r}}{\partial x^4}\frac{\partial^2 \varphi}{\partial x^2} + 6\frac{\partial^5 \frac{1}{r}}{\partial x^5}\frac{\partial \varphi}{\partial x} + \frac{\partial^6 \frac{1}{r}}{\partial x^6}\varphi \Big\}$$

$$\frac{\partial^3 \varphi}{\partial x^3} + 15\frac{\partial^4 \frac{1}{r}}{\partial x^4}\frac{\partial^2 \varphi}{\partial x^2} + 6\frac{\partial^5 \frac{1}{r}}{\partial x^5}\left(\frac{\partial \varphi}{\partial x} - \frac{1}{b^2+c^2}\right) + \frac{\partial^6 \frac{1}{r}}{\partial x^6}\left(\varphi + \frac{a}{b^2+c^2}\right) = -\frac{1}{\sqrt{5}}\frac{\partial^8 L}{\partial x^8} + db \,\Big/\!\Big/\, c\Big\{-\frac{6}{b^2+c^2}\frac{\partial^5 \frac{1}{r}}{\partial x^5} + \frac{a}{b^2+c^2}\frac{\partial^6 \frac{1}{r}}{\partial x^6}\Big\}$$

$$3\frac{\partial^2 \frac{1}{r}}{\partial x^2}\frac{\partial^4 \varphi}{\partial x \partial y^3} + 9\frac{\partial^2 \frac{1}{r}}{\partial x \partial y}\frac{\partial^4 \varphi}{\partial x^2 \partial y^2} + 3\frac{\partial^2 \frac{1}{r}}{\partial y^2}\frac{\partial^4 \varphi}{\partial x^3 \partial y} + \frac{\partial^3 \frac{1}{r}}{\partial x^3}\frac{\partial^3 \varphi}{\partial y^3} + 9\frac{\partial^3 \frac{1}{r}}{\partial x^2 \partial y}\frac{\partial^3 \varphi}{\partial x \partial y^2} + 9\frac{\partial^3 \frac{1}{r}}{\partial x \partial y^2}\frac{\partial^3 \varphi}{\partial x^2 \partial y} + \frac{\partial^3 \frac{1}{r}}{\partial y^3}\frac{\partial^3 \varphi}{\partial x^3} + 3\frac{\partial^4 \frac{1}{r}}{\partial x^3 \partial y}\frac{\partial^2 \varphi}{\partial y^2} +$$

$$+ 9\frac{\partial^4 \frac{1}{r}}{\partial x^2 \partial y^2}\frac{\partial^2 \varphi}{\partial x \partial y} + 3\frac{\partial^4 \frac{1}{r}}{\partial x \partial y^3}\frac{\partial^2 \varphi}{\partial x^2} + 3\frac{\partial^5 \frac{1}{r}}{\partial x^3 \partial y^2}\frac{\partial \varphi}{\partial y} + 3\frac{\partial^5 \frac{1}{r}}{\partial x^2 \partial y^3}\frac{\partial \varphi}{\partial x} + \frac{\partial^6 \frac{1}{r}}{\partial x^3 \partial y^3}\varphi \Big\}$$

$$+ 20\frac{\partial^3 \frac{1}{r}}{\partial y^3}\frac{\partial^3 \psi}{\partial y^3} + 15\frac{\partial^4 \frac{1}{r}}{\partial y^4}\frac{\partial^2 \psi}{\partial y^2} + 6\frac{\partial^5 \frac{1}{r}}{\partial y^5}\frac{\partial \psi}{\partial y} + \frac{\partial^6 \frac{1}{r}}{\partial y^6}\psi \Big\}$$

$$\Big) + 15\frac{\partial^2 \frac{1}{r}}{\partial y^2}\left(\frac{\partial^4 \psi}{\partial y^4} + \frac{\partial^4 \psi'}{\partial y^4}\right) + 20\frac{\partial^3 \frac{1}{r}}{\partial y^3}\left(\frac{\partial^3 \psi}{\partial y^3} + \frac{\partial^3 \psi'}{\partial y^3}\right) + 15\frac{\partial^4 \frac{1}{r}}{\partial y^4}\left(\frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi'}{\partial y^2}\right) + 6\frac{\partial^5 \frac{1}{r}}{\partial y^5}\left(\frac{\partial \psi}{\partial y} + \frac{\partial \psi'}{\partial y}\right) + \frac{\partial^6 \frac{1}{r}}{\partial y^6}(\psi + \psi') \Big\}$$

$$\frac{\partial^{m+n-2} \varphi}{\partial x^{m-1}\partial y^{n-1}} + \binom{n}{2}\frac{\partial^2 \frac{1}{r}}{\partial y^2}\frac{\partial^{m+n-2} \varphi}{\partial x^{m}\partial y^{n-2}} +$$

$$\frac{\cdots^{-3}}{\cdots\partial y^{n-1}} + \binom{m}{1}\binom{n}{2}\frac{\partial^3 \frac{1}{r}}{\partial x \partial y^2}\frac{\partial^{m+n-3} \varphi}{\partial x^{m-1}\partial y^{n-2}} + \binom{n}{3}\frac{\partial^3 \frac{1}{r}}{\partial y^3}\frac{\partial^{m+n-3} \varphi}{\partial x^{m}\partial y^{n-3}} +$$

$$\frac{\partial^{m+n-4} \varphi}{\partial x^{m-3}\partial y^{n-1}} + \binom{m}{2}\binom{n}{2}\frac{\partial^4 \frac{1}{r}}{\partial x^2 \partial y^2}\frac{\partial^{m+n-4} \varphi}{\partial x^{m-2}\partial y^{n-2}} + \binom{m}{1}\binom{n}{3}\frac{\partial^4 \frac{1}{r}}{\partial x \partial y^3}\frac{\partial^{m+n-4} \varphi}{\partial x^{m-1}\partial y^{n-3}} + \binom{n}{4}\frac{\partial^4 \frac{1}{r}}{\partial y^4}\frac{\partial^{m+n-4} \varphi}{\partial x^{m}\partial y^{n-4}} +$$

$$\frac{\partial^{m+n-5} \varphi}{\partial x^{m-4}\partial y^{n-1}} + \binom{m}{3}\binom{n}{2}\frac{\partial^5 \frac{1}{r}}{\partial x^3 \partial y^2}\frac{\partial^{m+n-5} \varphi}{\partial x^{m-3}\partial y^{n-2}} + \binom{m}{2}\binom{n}{3}\frac{\partial^5 \frac{1}{r}}{\partial x^2 \partial y^3}\frac{\partial^{m+n-5} \varphi}{\partial x^{m-2}\partial y^{n-3}} + \binom{m}{1}\binom{n}{4}\frac{\partial^5 \frac{1}{r}}{\partial x \partial y^4}\frac{\partial^{m+n-5} \varphi}{\partial x^{m-1}\partial y^{n-4}} + \binom{n}{5}\frac{\partial^5 \frac{1}{r}}{\partial y^5}\frac{\partial^{m+n-5} \varphi}{\partial x^{m}\partial y^{n-5}} +$$

$$\frac{\cdots^{-6}}{\cdots y^{n-1}} + \binom{m}{4}\binom{n}{2}\frac{\partial^6 \frac{1}{r}}{\partial x^4 \partial y^2}\frac{\partial^{m+n-6} \varphi}{\partial x^{m-4}\partial y^{n-2}} + \binom{m}{3}\binom{n}{3}\frac{\partial^6 \frac{1}{r}}{\partial x^3 \partial y^3}\frac{\partial^{m+n-6} \varphi}{\partial x^{m-3}\partial y^{n-3}} + \binom{m}{2}\binom{n}{4}\frac{\partial^6 \frac{1}{r}}{\partial x^2 \partial y^4}\frac{\partial^{m+n-6} \varphi}{\partial x^{m-2}\partial y^{n-4}} + \binom{m}{1}\binom{n}{5}\frac{\partial^6 \frac{1}{r}}{\partial x \partial y^5}\frac{\partial^{m+n-6} \varphi}{\partial x^{m-1}\partial y^{n-5}} + \binom{n}{6}\frac{\partial^6 \frac{1}{r}}{\partial y^6}\frac{\partial^{m+n-6} \varphi}{\partial x^{m}\partial y^{n-6}} + \cdots$$

Ca.

$\varphi = \frac{a}{a^2+b^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^6 L}{\partial x^6} = -db \| c \frac{\partial^4}{\partial x^4}\varphi \frac{1}{r} = -db \| c \left\{ \frac{1}{r}\frac{\partial^4 \varphi}{\partial x^4} + 4\frac{\partial \frac{1}{r}}{\partial x}\frac{\partial^3 \varphi}{\partial x^3} + 6 \frac{\partial^2 \frac{1}{r}}{\partial x^2}\frac{\partial^2 \varphi}{\partial x^2} + 4 \frac{\partial^3 \frac{1}{r}}{\partial x^3} \right.$

$\psi = \frac{1}{a^2+b^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^6 L}{\partial x^5 \partial y} = -db \| bc \frac{\partial^4}{\partial x^4}\psi \frac{1}{r}$

$\varphi = \frac{a}{a^2+b^2}$ $\varphi' = \frac{a}{b^2+c^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^6 L}{\partial x^4 \partial y^2} = +db \| c \frac{\partial^4}{\partial x^4}(\varphi+\varphi')\frac{1}{r} = +db \| c \left( \frac{1}{r}\frac{\partial^4 \varphi}{\partial x^4} + 4\frac{\partial \frac{1}{r}}{\partial x}\frac{\partial^3 \varphi}{\partial x^3} + 6\frac{\partial^2 \frac{1}{r}}{\partial x^2}\frac{\partial^2 \varphi}{\partial x^2} + 4\frac{\partial^3 \frac{1}{r}}{\partial x^3} \right. ($

$\varphi = \frac{b}{a^2+b^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^6 L}{\partial x^3 \partial y^3} = -db \| c \frac{\partial^4}{\partial x^2 \partial y^2}\varphi \frac{1}{r}$

$\psi = \frac{1}{a^2+b^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^6 L}{\partial x^2 \partial y^4} = -db \| ac \frac{\partial^4}{\partial y^4}\psi \frac{1}{r} = -db \| ac \left\{ \frac{1}{r}\frac{\partial^4 \psi}{\partial y^4} + 4\frac{\partial \frac{1}{r}}{\partial y}\frac{\partial^3 \psi}{\partial y^3} + 6\frac{\partial^2 \frac{1}{r}}{\partial y^2}\frac{\partial^2 \psi}{\partial y^2} + 4 \right.$

$\varphi = \frac{b}{a^2+b^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^6 L}{\partial x \partial y^5} = -db \| c \frac{\partial^4}{\partial y^4}\varphi \frac{1}{r}$

$\psi = \frac{1}{a^2+b^2}$ $\psi' = \frac{1}{b^2+c^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^6 L}{\partial y^6} = +db \| ac \frac{\partial^4}{\partial y^4}(\psi+\psi')\frac{1}{r} = +db \| ac \left\{ \frac{1}{r}\left(\frac{\partial^4 \psi}{\partial y^4} + \frac{\partial^4 \psi'}{\partial y^4}\right) + 4\frac{\partial \frac{1}{r}}{\partial y}\left(\frac{\partial^3 \psi}{\partial y^3} + \frac{\partial^3 \psi'}{\partial y^3}\right) \right.$

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$\varphi = \frac{a}{a^2+b^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^7 L}{\partial x^7} = -db \| c \frac{\partial^5}{\partial x^5}\varphi \frac{1}{r}$

$\psi = \frac{1}{a^2+b^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^7 L}{\partial x^6 \partial y} = -db \| bc \frac{\partial^5}{\partial x^5}\psi \frac{1}{r} = -db \| bc \left\{ \frac{1}{r}\frac{\partial^5 \psi}{\partial x^5} + 5\frac{\partial \frac{1}{r}}{\partial x}\frac{\partial^4 \psi}{\partial x^4} + 10\frac{\partial^2 \frac{1}{r}}{\partial x^2}\frac{\partial^3 \psi}{\partial x^3} + 10\frac{\partial^3 \frac{1}{r}}{\partial x^3} \right.$

$\varphi = \frac{a}{a^2+b^2}$ $\varphi' = \frac{a}{b^2+c^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^7 L}{\partial x^5 \partial y^2} = +db \| c \frac{\partial^5}{\partial x^5}(\varphi+\varphi')\frac{1}{r} =$

$\varphi = \frac{b}{a^2+b^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^7 L}{\partial x^4 \partial y^3} = -db \| c \frac{\partial^5}{\partial x^3 \partial y^2}\varphi \frac{1}{r} = -db \| c \left\{ \frac{1}{r}\frac{\partial^5 \varphi}{\partial x^3 \partial y^2} + 3\frac{\partial \frac{1}{r}}{\partial x}\frac{\partial^4 \varphi}{\partial x^2 \partial y^2} + 2\frac{\partial \frac{1}{r}}{\partial y}\frac{\partial^4 \varphi}{\partial x^3 \partial y} + 3\frac{\partial^2 \frac{1}{r}}{\partial x^2}\frac{\partial^3 \varphi}{\partial x \partial y^2} + \right.$

$\varphi = \frac{b}{a^2+b^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^7 L}{\partial x^3 \partial y^4} = -db \| c \frac{\partial^5}{\partial x^2 \partial y^3}\varphi \frac{1}{r}$

$\psi = \frac{1}{a^2+b^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^7 L}{\partial x^2 \partial y^5} = -db \| ac \frac{\partial^5}{\partial y^5}\psi \frac{1}{r} = -db \| ac \left\{ \frac{1}{r}\frac{\partial^5 \psi}{\partial y^5} + 5\frac{\partial \frac{1}{r}}{\partial y}\frac{\partial^4 \psi}{\partial y^4} + 10\frac{\partial^2 \frac{1}{r}}{\partial y^2}\frac{\partial^3 \psi}{\partial y^3} + 10\frac{\partial^3 \frac{1}{r}}{\partial y^3} \right.$

$\varphi = \frac{b}{a^2+b^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^7 L}{\partial x \partial y^6} = -db \| c \frac{\partial^5}{\partial y^5}\varphi \frac{1}{r}$

$\psi = \frac{1}{a^2+b^2}$ $\psi' = \frac{1}{b^2+c^2}$ $\quad \frac{1}{f\sigma}\frac{\partial^7 L}{\partial y^7} = +db \| ac \frac{\partial^5}{\partial y^5}(\psi+\psi')\frac{1}{r} = +db \| ac \left\{ \frac{1}{r}\left(\frac{\partial^5 \psi}{\partial y^5} + \frac{\partial^5 \psi'}{\partial y^5}\right) + 5\frac{\partial \frac{1}{r}}{\partial y}\left(\frac{\partial^4 \psi}{\partial y^4} + \frac{\partial^4 \psi'}{\partial y^4}\right) + 10 \right.$

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$$\ldots + 4\frac{\partial^3\frac{1}{r}}{\partial x^3}\frac{\partial\varphi}{\partial x} + \frac{\partial^4\frac{1}{r}}{\partial x^4}\varphi\Big\}$$

$$\ldots 4\frac{\partial^3\frac{1}{r}}{\partial x^3}\left(\frac{\partial\varphi}{\partial x} - \frac{1}{b^2+c^2}\right) + \frac{\partial^4\frac{1}{r}}{\partial x^4}\left(\varphi + \frac{a}{b^2+c^2}\right)\Big\} = -\frac{1}{15}\frac{\partial^6\frac{1}{r}}{\partial x^6} + db \,\|\, c\left\{-\frac{4}{b^2+c^2}\frac{\partial^3\frac{1}{r}}{\partial x^3} + \frac{a}{b^2+c^2}\frac{\partial^4\frac{1}{r}}{\partial x^4}\right\}$$

$$\ldots\frac{\partial^2\psi}{\partial y^2} + 4\frac{\partial^3\frac{1}{r}}{\partial y^3}\frac{\partial\psi}{\partial y} + \frac{\partial^4\frac{1}{r}}{\partial y^4}\psi\Big\}$$

$$\ldots + \frac{\partial^3\psi'}{\partial y^3}\Big) + 6\frac{\partial^2\frac{1}{r}}{\partial y^2}\left(\frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi'}{\partial y^2}\right) + 4\frac{\partial^3\frac{1}{r}}{\partial y^3}\left(\frac{\partial\psi}{\partial y} + \frac{\partial\psi'}{\partial y}\right) + \frac{\partial^4\frac{1}{r}}{\partial y^4}(\psi+\psi')\Big\}.$$

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$$\ldots + 10\frac{\partial^3\frac{1}{r}}{\partial x^3}\frac{\partial^2\psi}{\partial x^2} + 5\frac{\partial^4\frac{1}{r}}{\partial x^4}\frac{\partial\psi}{\partial x} + \frac{\partial^5\frac{1}{r}}{\partial x^5}\psi\Big\}$$

$$\ldots\frac{\partial^2\frac{1}{r}}{\partial x^2}\frac{\partial^3\varphi}{\partial x\partial y^2} + 6\frac{\partial^2\frac{1}{r}}{\partial x\partial y}\frac{\partial^3\varphi}{\partial x^2\partial y} + \frac{\partial^2\frac{1}{r}}{\partial y^2}\frac{\partial^3\varphi}{\partial x^3} + \frac{\partial^3\frac{1}{r}}{\partial x^3}\frac{\partial^2\varphi}{\partial y^2} + 6\frac{\partial^3\frac{1}{r}}{\partial x^2\partial y}\frac{\partial^2\varphi}{\partial x\partial y} + 3\frac{\partial^3\frac{1}{r}}{\partial x\partial y^2}\frac{\partial^2\varphi}{\partial x^2} + \cancel{\frac{\partial^3\frac{1}{r}}{\partial y^3}\frac{\partial^4\varphi}{\partial x^3}} + 2\frac{\partial^4\frac{1}{r}}{\partial x^3\partial y}\frac{\partial\varphi}{\partial y} + 3\frac{\partial^4\frac{1}{r}}{\partial x^2\partial y^2}\frac{\partial\varphi}{\partial x} + \frac{\partial^5\frac{1}{r}}{\partial x^3\partial y^2}\varphi\Big\}$$

$$\ldots\frac{\partial^4\psi}{\partial y^3} + 10\frac{\partial^3\frac{1}{r}}{\partial y^3}\frac{\partial^2\psi}{\partial y^2} + 5\frac{\partial^4\frac{1}{r}}{\partial y^4}\frac{\partial\psi}{\partial y} + \frac{\partial^5\frac{1}{r}}{\partial y^5}\psi\Big\}$$

$$\ldots\frac{\partial^4\psi'}{\partial y^4}\Big) + 10\frac{\partial^2\frac{1}{r}}{\partial y^2}\left(\frac{\partial^3\psi}{\partial y^3} + \frac{\partial^3\psi'}{\partial y^3}\right) + 10\frac{\partial^3\frac{1}{r}}{\partial y^3}\left(\frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi'}{\partial y^2}\right) + 5\frac{\partial^4\frac{1}{r}}{\partial y^4}\left(\frac{\partial\psi}{\partial y} + \frac{\partial\psi'}{\partial y}\right) + \frac{\partial^5\frac{1}{r}}{\partial y^5}(\psi+\psi')\Big\}$$

$\psi = \frac{1}{a^2+c^2}$ $\quad \underline{a} = \frac{a}{b}$ $\quad \underline{c} = \frac{c}{b}$ $\quad \underline{\Psi} = \frac{1}{\underline{a}^2+\underline{c}^2}$

$\psi = \frac{1}{b^2}\underline{\Psi}$ $\quad$ ψ differ. hányadosai

| | | |
|---|---|---|
| $\frac{\partial}{\partial x}$ | $+\frac{1}{b^3}\, 2\underline{a}\underline{\Psi}^2$ | |
| $\frac{\partial}{\partial x^2}$ | $+\frac{1}{b^4}\, 2\underline{\Psi}^3(3\underline{a}^2-\underline{c}^2)$ | |
| $\frac{\partial}{\partial x^3}$ | $+\frac{1}{b^5}\, 24\underline{a}\underline{\Psi}^4(\underline{a}^2-\underline{c}^2)$ | |
| $\frac{\partial}{\partial x^4}$ | $+\frac{1}{b^6}\, 24\underline{\Psi}^5(5\underline{a}^4-10\underline{a}^2\underline{c}^2+\underline{c}^4)$ | |
| $\frac{\partial}{\partial x^5}$ | $+\frac{1}{b^7}\, 240\underline{a}\underline{\Psi}^6(3\underline{a}^4-10\underline{a}^2\underline{c}^2+3\underline{c}^4)$ | |
| $\frac{\partial}{\partial x^6}$ | $+\frac{1}{b^8}\, 720\underline{\Psi}^7(7\underline{a}^6-35\underline{a}^4\underline{c}^2+21\underline{a}^2\underline{c}^4-\underline{c}^6)$ | |

$\varphi = \frac{a}{a^2+c^2}$ $\quad \psi = \frac{1}{a^2+c^2}$

$\varphi = b.\underline{a}\psi$ $\quad$ φ diff. hányadosai

| | |
|---|---|
| $\frac{\partial}{\partial x}$ | $+\, b.\underline{a}\frac{\partial\psi}{\partial x} - \psi$ |
| $\frac{\partial}{\partial x^2}$ | $+\, b.\underline{a}\frac{\partial^2\psi}{\partial x^2} - 2\frac{\partial\psi}{\partial x}$ |
| $\frac{\partial}{\partial x^3}$ | $+\, b.\underline{a}\frac{\partial^3\psi}{\partial x^3} - 3\frac{\partial^2\psi}{\partial x^2}$ |
| $\frac{\partial}{\partial x^4}$ | $+\, b.\underline{a}\frac{\partial^4\psi}{\partial x^4} - 4\frac{\partial^3\psi}{\partial x^3}$ |
| $\frac{\partial}{\partial x^5}$ | $+\, b.\underline{a}\frac{\partial^5\psi}{\partial x^5} - 5\frac{\partial^4\psi}{\partial x^4}$ |
| $\frac{\partial}{\partial x^6}$ | $+\, b.\underline{a}\frac{\partial^6\psi}{\partial x^6} - 6\frac{\partial^5\psi}{\partial x^5}$ |

$\varphi =$

$\varphi =$

$\psi = \frac{1}{b^2+c^2}$ $\quad \underline{a} = \frac{a}{b}$ $\quad \underline{c} = \frac{c}{b}$ $\quad \underline{\Psi} = \frac{1}{1+\underline{c}^2}$

$\psi = \frac{1}{b^2}\underline{\Psi}$

| | |
|---|---|
| $\frac{\partial}{\partial y}$ | $+\frac{1}{b^3}\, 2\underline{\Psi}^2$ |
| $\frac{\partial}{\partial y^2}$ | $+\frac{1}{b^4}\, 2\underline{\Psi}^3(3-\underline{c}^2)$ |
| $\frac{\partial}{\partial y^3}$ | $+\frac{1}{b^5}\, 24\underline{\Psi}^4(1-\underline{c}^2)$ |
| $\frac{\partial}{\partial y^4}$ | $+\frac{1}{b^6}\, 24\underline{\Psi}^5(5-10\underline{c}^2+\underline{c}^4)$ |
| $\frac{\partial}{\partial y^5}$ | $+\frac{1}{b^7}\, 240\underline{\Psi}^6(3-10\underline{c}^2+3\underline{c}^4)$ |
| $\frac{\partial}{\partial y^6}$ | $+\frac{1}{b^8}\, 720\underline{\Psi}^7(7-35\underline{c}^2+21\underline{c}^4-\underline{c}^6)$ |

$\varphi = \frac{a}{b^2+c^2}$ $\quad \psi = \frac{1}{b^2+c^2}$

$\varphi = b\underline{a}\psi$

| | |
|---|---|
| $\frac{\partial}{\partial y}$ | $+\, b.\underline{a}\frac{\partial\psi}{\partial y}$ |
| $\frac{\partial}{\partial y^2}$ | $+\, b.\underline{a}\frac{\partial^2\psi}{\partial y^2}$ |
| $\frac{\partial}{\partial y^3}$ | $+\, b.\underline{a}\frac{\partial^3\psi}{\partial y^3}$ |
| $\frac{\partial}{\partial y^4}$ | $+\, b.\underline{a}\frac{\partial^4\psi}{\partial y^4}$ |
| $\frac{\partial}{\partial y^5}$ | $+\, b.\underline{a}\frac{\partial^5\psi}{\partial y^5}$ |
| $\frac{\partial}{\partial y^6}$ | $+\, b.\underline{a}\frac{\partial^6\psi}{\partial y^6}$ |

$\varphi =$

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$$\varphi = \frac{b}{a^2+c^2} \qquad \psi = \frac{1}{a^2+c^2}$$

$$\varphi = b\psi$$

$$+\,b.\frac{\partial\psi}{\partial x}$$

$$+\,b.\frac{\partial^2\psi}{\partial x^2}$$

$$+\,b.\frac{\partial^3\psi}{\partial x^3}$$

$$+\,b.\frac{\partial^4\psi}{\partial x^4}$$

$$+\,b.\frac{\partial^5\psi}{\partial x^5}$$

$$+\,b.\frac{\partial^6\psi}{\partial x^6}$$

$$\varphi = \frac{b}{b^2+c^2} \qquad \psi = \frac{1}{b^2+c^2}$$

$$\varphi = b\psi$$

$$+\,b.\frac{\partial\psi}{\partial y} - \psi$$

$$+\,b.\frac{\partial^2\psi}{\partial y^2} - 2\frac{\partial\psi}{\partial y}$$

$$+\,b.\frac{\partial^3\psi}{\partial y^3} - 3\frac{\partial^2\psi}{\partial y^2}$$

$$+\,b.\frac{\partial^4\psi}{\partial y^4} - 4\frac{\partial^3\psi}{\partial y^4}$$

$$+\,b.\frac{\partial^5\psi}{\partial y^5} - 5\frac{\partial^4\psi}{\partial y^4}$$

$$+\,b\,\frac{\partial^6\psi}{\partial y^6} - 6\frac{\partial^5\psi}{\partial y^5}$$

$$\psi = \frac{1}{a^2+b^2} \qquad \frac{a}{b} = \underline{a}$$

$$\psi = \frac{1}{b^2}\Psi \qquad \psi = \frac{1}{b^2}\frac{1}{1+\underline{a}^2} \qquad \frac{1}{1+\underline{a}^2} = \Psi$$

$$\frac{\partial\psi}{\partial x} = 2a\psi^2 = \frac{1}{b^3}2\underline{a}\Psi^2$$

$$\frac{\partial^2\psi}{\partial x^2} = -2\psi^2 + 8a^2\psi^3 = 2\psi^3(3a^2-b^2)$$

$$\frac{\partial^3\psi}{\partial x^3} = -24a\psi^3 + 48a^3\psi^4 = 24a\psi^4(a^2-b^2)$$

$$\frac{\partial^4\psi}{\partial x^4} = +24\psi^3 - 288a^2\psi^4 + 384a^4\psi^5 = 24\psi^5(5a^4-10a^2b^2+b^4)$$

$$\frac{\partial^5\psi}{\partial x^5} = +720a\psi^4 - 3840a^3\psi^5 + 3840a^5\psi^6 = 240a\psi^6(3a^4-10a^2b^2+3b^4)$$

$$\frac{\partial^6\psi}{\partial x^6} = -720\psi^4 + 17280a^2\psi^5 - 57600a^4\psi^6 + 46080a^6\psi^7$$
$$= 720\psi^7(7a^6-35a^4b^2+21a^2b^4-b^6)$$

$$\frac{\partial\psi}{\partial y} = 2b\psi^2$$

$$\frac{\partial^2\psi}{\partial y^2} = -2\psi^2 + 8b^2\psi^3$$

$$\frac{\partial^3\psi}{\partial y^3} = -24b\psi^3 + 48b^3\psi^4$$

$$\frac{\partial^4\psi}{\partial y^4} = +24\psi^3 - 288b^2\psi^4 + 384b^4\psi^5$$

$$\frac{\partial^5\psi}{\partial y^5} = +720b\psi^4 - 3840b^3\psi^5 + 3840b^5\psi^6$$

$$\frac{\partial^6\psi}{\partial y^6} = -720\psi^4 + 17280b^2\psi^5 - 57600b^4\psi^6 + 46080b^6\psi^7$$

$$\varphi = \frac{a}{a^2+b^2}$$

$$\varphi = b\underline{a}\Psi \qquad b\left(\underline{a}\frac{\partial\Psi}{\partial\underline{a}} - \Psi_0\right)$$

$$\frac{\partial\varphi}{\partial x} = a\frac{\partial\psi}{\partial x} - \psi$$

$$\frac{\partial^2\varphi}{\partial x^2} = a\frac{\partial^2\psi}{\partial x^2} - 2\frac{\partial\psi}{\partial x}$$

$$\frac{\partial^3\varphi}{\partial x^3} = a\frac{\partial^3\psi}{\partial x^3} - 3\frac{\partial^2\psi}{\partial x^2}$$

$$\frac{\partial^4\varphi}{\partial x^4} = a\frac{\partial^4\psi}{\partial x^4} - 4\frac{\partial^3\psi}{\partial x^3}$$

$$\frac{\partial^5\varphi}{\partial x^5} = a\frac{\partial^5\psi}{\partial x^5} - 5\frac{\partial^4\psi}{\partial x^4}$$

$$\frac{\partial^6\varphi}{\partial x^6} = a\frac{\partial^6\psi}{\partial x^6} - 6\frac{\partial^5\psi}{\partial x^5}$$

$$\frac{\partial\varphi}{\partial y} = a\frac{\partial\psi}{\partial y}$$

$$\frac{\partial^2\varphi}{\partial y^2} = a\frac{\partial^2\psi}{\partial y^2}$$

$$\frac{\partial^3\varphi}{\partial y^3} = a\frac{\partial^3\psi}{\partial y^3}$$

$$\frac{\partial^4\varphi}{\partial y^4} = a\frac{\partial^4\psi}{\partial y^4}$$

$$\frac{\partial^5\varphi}{\partial y^5} = a\frac{\partial^5\psi}{\partial y^5}$$

$$\frac{\partial^6\varphi}{\partial y^6} = a\frac{\partial^6\psi}{\partial y^6}$$

$$\varphi = \frac{a}{a^2+b^2}$$

$$\frac{\partial^2\varphi}{\partial x\partial y} = -2b\psi^2(1-4a^2\psi) = 2b\psi^3(3a^2-b^2)$$

$$\frac{\partial^3\varphi}{\partial x\partial y^2} = -6\psi^2(1-8a^2b^2\psi^2) = -6\psi^4(a^4-6a^2b^2+b^4)$$

$$\frac{\partial^3\varphi}{\partial x^2\partial y} = -24.ab\psi^3(1-2a^2\psi) = 24ab\psi^4(a^2-b^2)$$

$$\frac{\partial^4\varphi}{\partial x\partial y^3} = -24.b\psi^3(1+4a^2\psi-16a^2b^2\psi^2) = -24b\psi^5(5a^4-10a^2b^2+b^4)$$

$$\frac{\partial^4\varphi}{\partial x^2\partial y^2} = -24a\psi^3(1+4b^2\psi-16a^2b^2\psi^2) = -24a\psi^5(a^4-10a^2b^2+5b^4)$$

$$\frac{\partial^4\varphi}{\partial x^3\partial y} = +24b\psi^3(1-12a^2\psi+16a^4\psi^2) = +24b\psi^5(5a^4-10a^2b^2+b^4)$$

$$\frac{\partial^5\varphi}{\partial x^2\partial y^3} = -240ab\psi^4(3-16a^2b^2\psi^2) = -240ab\psi^6(3a^4-10a^2b^2+3b^4)$$

$$\frac{\partial^5\varphi}{\partial x^3\partial y^2} = -120\psi^3(1-2b^2\psi+16a^2b^2\psi^2-32a^4b^2\psi^3) = -120\psi^6(a^6-15a^4b^2+15a^2b^4-b^6)$$

$$\frac{\partial^6\varphi}{\partial x^3\partial y^3} = +720b\psi^4(1-8a^2\psi-16a^2b^2\psi^2+64a^4b^2\psi^3) = -720b\psi^7(7a^6-35a^4b^2+21a^2b^4-b^6)$$

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$$\varphi = \frac{b}{a^2+b^2}$$

| | | |
|---|---|---|
| $\frac{\partial\varphi}{\partial x} = b\frac{\partial\psi}{\partial x}$ | $\varphi = \frac{a}{a^2+b^2}$ | $\frac{\partial\varphi}{\partial x} =$ |
| $\frac{\partial^2\varphi}{\partial x^2} = b\frac{\partial^2\psi}{\partial x^2}$ | " | $\frac{\partial^2\varphi}{\partial x^2} =$ |
| $\frac{\partial^3\varphi}{\partial x^3} = b\frac{\partial^3\psi}{\partial x^3}$ | " | $\frac{\partial^3\varphi}{\partial x^3} =$ |
| $\frac{\partial^4\varphi}{\partial x^4} = b\frac{\partial^4\psi}{\partial x^4}$ | " | $\frac{\partial^4\varphi}{\partial x^4} =$ |
| $\frac{\partial^5\varphi}{\partial x^5} = b\frac{\partial^5\psi}{\partial x^5}$ | " | $\frac{\partial^5\varphi}{\partial x^5} =$ |
| $\frac{\partial^6\varphi}{\partial x^6} = b\frac{\partial^6\psi}{\partial x^6}$ | " | $\frac{\partial^6\varphi}{\partial x^6} =$ |
| $\frac{\partial\varphi}{\partial y} = b\frac{\partial\psi}{\partial y} - y$ | $\varphi = \frac{b}{a^2+b^2}$ | $\frac{\partial\varphi}{\partial y} =$ |
| $\frac{\partial^2\varphi}{\partial y^2} = b\frac{\partial^2\psi}{\partial y^2} - 2\frac{\partial\psi}{\partial y}$ | " | $\frac{\partial^2\varphi}{\partial y^2} =$ |
| $\frac{\partial^3\varphi}{\partial y^3} = b\frac{\partial^3\psi}{\partial y^3} - 3\frac{\partial^2\psi}{\partial y^2}$ | " | $\frac{\partial^3\varphi}{\partial y^3} =$ |
| $\frac{\partial^4\varphi}{\partial y^4} = b\cdot\frac{\partial^4\psi}{\partial y^4} - 4\frac{\partial^3\psi}{\partial y^3}$ | " | $\frac{\partial^4\varphi}{\partial y^4} =$ |
| $\frac{\partial^5\varphi}{\partial y^5} = b\cdot\frac{\partial^5\psi}{\partial y^5} - 5\frac{\partial^4\psi}{\partial y^4}$ | " | $\frac{\partial^5\varphi}{\partial y^5} =$ |
| $\frac{\partial^6\varphi}{\partial y^6} = b\cdot\frac{\partial^6\psi}{\partial y^6} - 6\frac{\partial^5\psi}{\partial y^5}$ | " | $\frac{\partial^6\varphi}{\partial y^6} =$ |

$$\varphi = \frac{b}{a^2+b^2}$$

$\frac{\partial^2\varphi}{\partial x\partial y} = 2a\psi^3(3b^2-a^2)$

$\frac{\partial^3\varphi}{\partial x\partial y^2} = +24ab\psi^4(b^2-a^2)$

$\frac{\partial^3\varphi}{\partial x^2\partial y} = -6\psi^4(b^4-6a^2b^2+a^4)$

$\frac{\partial^4\varphi}{\partial x\partial y^3} = +24a\psi^5(5b^4-10a^2b^2+a^4)$

$\frac{\partial^4\varphi}{\partial x^2\partial y^2} = -24b\psi^5(b^4-10a^2b^2+5a^4)$

$\frac{\partial^4\varphi}{\partial x^3\partial y} = -24a\psi^5(5b^4-10a^2b^2+a^4)$

$\frac{\partial^5\varphi}{\partial x^2\partial y^3} = -120\psi^6(b^6-15a^2b^4+15b^2a^4-a^6)$ ~

$\frac{\partial^5\varphi}{\partial x^3\partial y^2} = -240ab\psi^6(3b^4-10a^2b^2+3a^4)$

$\frac{\partial^6\varphi}{\partial x^3\partial y^3} = -720a\psi^7(7b^6-35a^2b^4+21b^2a^4-a^6)$ ~

$\psi = \frac{1}{a^2+b^2}$, $\underline{a} = \frac{a}{b}$ $\underline{\psi} = \frac{1}{1+\underline{a}^2}$ | $\varphi = \frac{a}{a^2+b^2}$ | $\psi = \frac{1}{a^2+b^2}$ | $\varphi$

$\psi = \frac{1}{b^2}\underline{\psi}$ $\psi$ differenciálhányadosai | $\varphi = b.\underline{a}.\psi$ $\varphi$ differenciálhányadosai | $\varphi$

| | | |
|---|---|---|
| $\frac{\partial}{\partial x}$ | $+\frac{1}{b^3}2\underline{a}\underline{\psi}^2$ | $+b.\underline{a}\frac{\partial\psi}{\partial x} - \psi$ |
| $\frac{\partial^2}{\partial x^2}$ | $+\frac{1}{b^4}\cdot 2\cdot\underline{\psi}^3(3\underline{a}^2-1)$ | $+b.\underline{a}\frac{\partial^2\psi}{\partial x^2} - 2\frac{\partial\psi}{\partial x}$ |
| $\frac{\partial^3}{\partial x^3}$ | $+\frac{1}{b^5}24\underline{a}\underline{\psi}^4(\underline{a}^2-1)$ | $+b.\underline{a}\frac{\partial^3\psi}{\partial x^3} - 3\frac{\partial^2\psi}{\partial x^2}$ |
| $\frac{\partial^4}{\partial x^4}$ | $+\frac{1}{b^6}\cdot 24\underline{\psi}^5(5\underline{a}^4-10\underline{a}^2+1)$ | $+b.\underline{a}\frac{\partial^4\psi}{\partial x^4} - 4\frac{\partial^3\psi}{\partial x^3}$ |
| $\frac{\partial^5}{\partial x^5}$ | $+\frac{1}{b^7}240\underline{a}\underline{\psi}^6(3\underline{a}^4-10\underline{a}^2+3)$ | $+b.\underline{a}\frac{\partial^5\psi}{\partial x^5} - 5\frac{\partial^4\psi}{\partial x^4}$ |
| $\frac{\partial^6}{\partial x^6}$ | $+\frac{1}{b^8}720\underline{\psi}^7(7\underline{a}^6-35\underline{a}^4+21\underline{a}^2-1)$ | $+b.\underline{a}\frac{\partial^6\psi}{\partial x^6} - 6\frac{\partial^5\psi}{\partial x^5}$ |
| $\frac{\partial}{\partial y}$ | $+\frac{1}{b^3}2\underline{\psi}^2$ | $+b.\underline{a}\frac{\partial\psi}{\partial y}$ |
| $\frac{\partial^2}{\partial y^2}$ | $+\frac{1}{b^4}\cdot 2\underline{\psi}^3(3-\underline{a}^2)$ | $+b.\underline{a}\frac{\partial^2\psi}{\partial y^2}$ |
| $\frac{\partial^3}{\partial y^3}$ | $+\frac{1}{b^5}\cdot 24\underline{\psi}^4(1-\underline{a}^2)$ | $+b.\underline{a}\frac{\partial^3\psi}{\partial y^3}$ |
| $\frac{\partial^4}{\partial y^4}$ | $+\frac{1}{b^6}\cdot 24\underline{\psi}^5(5-10\underline{a}^2+\underline{a}^4)$ | $+b.\underline{a}\frac{\partial^4\psi}{\partial y^4}$ |
| $\frac{\partial^5}{\partial y^5}$ | $+\frac{1}{b^7}\cdot 240\underline{\psi}^6(3-10\underline{a}^2+3\underline{a}^4)$ | $+b.\underline{a}\frac{\partial^5\psi}{\partial y^5}$ |
| $\frac{\partial^6}{\partial y^6}$ | $+\frac{1}{b^8}\cdot 720\underline{\psi}^7(7-35\underline{a}^2+21\underline{a}^4-\underline{a}^6)$ | $+b.\underline{a}\frac{\partial^6\psi}{\partial y^6}$ |
| $\frac{\partial^2}{\partial x\partial y}$ | $\cancel{+\frac{1}{b^3}2\underline{\psi}^3(3\underline{a}^2-1)}$ | $+\frac{1}{b^3}2\underline{\psi}^3(3\underline{a}^2-1)$ |
| $\frac{\partial^3}{\partial x\partial y^2}$ | $\cancel{-\frac{1}{b^4}6\underline{\psi}^4(\underline{a}^4-6\underline{a}^2+1)}$ | $-\frac{1}{b^4}6\underline{\psi}^4(\underline{a}^4-6\underline{a}^2+1)$ |
| $\frac{\partial^3}{\partial x^2\partial y}$ | $\cancel{+\frac{1}{b^4}24\underline{a}\underline{\psi}^4(\underline{a}^2-1)}$ | $+\frac{1}{b^4}24\underline{a}\underline{\psi}^4(\underline{a}^2-1)$ |
| $\frac{\partial^4}{\partial x\partial y^3}$ | $\cancel{-\frac{1}{b^5}24\underline{\psi}^5(5\underline{a}^4-10\underline{a}^2+1)}$ | $-\frac{1}{b^5}24\underline{\psi}^5(5\underline{a}^4-10\underline{a}^2+1)$ |
| $\frac{\partial^4}{\partial x^2\partial y^2}$ | $\cancel{-\frac{1}{b^5}24\underline{a}\underline{\psi}^5(\underline{a}^4-10\underline{a}^2+5)}$ | $-\frac{1}{b^5}24\underline{a}\psi^5(\underline{a}^4-10\underline{a}^2+5)$ |
| $\frac{\partial^4}{\partial x^3\partial y}$ | $\cancel{+\frac{1}{b^5}\cdot 24\underline{\psi}^5(5\underline{a}^4-10\underline{a}^2+1)}$ | $+\frac{1}{b^5}24\underline{\psi}^5(5\underline{a}^4-10\underline{a}^2+1)$ |
| $\frac{\partial^5}{\partial x^2\partial y^3}$ | $\cancel{-\frac{1}{b^6}240\underline{a}\underline{\psi}^6(3\underline{a}^4-10\underline{a}^2+3)}$ | $-\frac{1}{b^6}240\underline{a}\underline{\psi}^6(3\underline{a}^4-10\underline{a}^2+3)$ |
| $\frac{\partial^5}{\partial x^3\partial y^2}$ | $\cancel{-\frac{1}{b^6}120\psi^6(\underline{a}^6}$ | $-\frac{1}{b^6}120\underline{\psi}^6(\underline{a}^6-15\underline{a}^4+15\underline{a}^2-1)$ |
| $\frac{\partial^6}{\partial x^3\partial y^3}$ | | $-\frac{1}{b^7}720\underline{\psi}^7(7\underline{a}^6-35\underline{a}^4+21\underline{a}^2-1)$ |

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| | | |
|---|---|---|
| $\varphi = \dfrac{b}{a^2+b^2}$ $\qquad \psi = \dfrac{1}{a^2+b^2}$ | $\dfrac{1}{r} = \dfrac{1}{\sqrt{a^2+b^2+c^2}}$ $\quad \underline{a} = \dfrac{a}{b}$ $\quad \underline{c} = \dfrac{c}{b}$ $\quad \dfrac{1}{\underline{r}} = \dfrac{1}{\sqrt{\underline{a}^2+1+\underline{c}^2}}$ | |
| $\varphi = b.\psi$ $\quad \varphi$ differenciál hányadosai. | $\dfrac{1}{r} = \dfrac{1}{b}\dfrac{1}{\underline{r}}$ $\quad \dfrac{1}{r}$ differenciál hányadosai | |
| $+b.\dfrac{\partial\psi}{\partial x}$ | $+\dfrac{1}{b^2}\dfrac{\underline{a}}{\underline{r}^3}$ | $\dfrac{\partial}{\partial x}$ |
| $+b.\dfrac{\partial^2\psi}{\partial x^2}$ | $+\dfrac{1}{b^3}\left(-\dfrac{1}{\underline{r}^3}+3\underline{a}^2\dfrac{1}{\underline{r}^5}\right)$ | $\dfrac{\partial^2}{\partial x^2}$ |
| $+b.\dfrac{\partial^3\psi}{\partial x^3}$ | $+\dfrac{1}{b^4}\left(-9.\underline{a}\dfrac{1}{\underline{r}^5}+15\underline{a}^3\dfrac{1}{\underline{r}^7}\right)$ | $\dfrac{\partial^3}{\partial x^3}$ |
| $+b.\dfrac{\partial^4\psi}{\partial x^4}$ | $+\dfrac{1}{b^5}\left(9\dfrac{1}{\underline{r}^5}-90\underline{a}^2\dfrac{1}{\underline{r}^7}+105\underline{a}^4\dfrac{1}{\underline{r}^9}\right)$ | $\dfrac{\partial^4}{\partial x^4}$ |
| $+b.\dfrac{\partial^5\psi}{\partial x^5}$ | $+\dfrac{1}{b^6}\left(225\underline{a}\dfrac{1}{\underline{r}^7}-1050\underline{a}^3\dfrac{1}{\underline{r}^9}+945\underline{a}^5\dfrac{1}{\underline{r}^{11}}\right)$ | $\dfrac{\partial^5}{\partial x^5}$ |
| $+b.\dfrac{\partial^6\psi}{\partial x^6}$ | $+\dfrac{1}{b^7}\left(-225\dfrac{1}{\underline{r}^7}+4725\underline{a}^2\dfrac{1}{\underline{r}^9}-14175\underline{a}^4\dfrac{1}{\underline{r}^{11}}+10395\underline{a}^6\dfrac{1}{\underline{r}^{13}}\right.$ | $\dfrac{\partial^6}{\partial x^6}$ |
| $+b.\dfrac{\partial\psi}{\partial y}-\psi$ | $+\dfrac{1}{b^2}\cdot\dfrac{1}{\underline{r}^3}$ | $\dfrac{\partial}{\partial y}$ |
| $+b.\dfrac{\partial^2\psi}{\partial y^2}-2\dfrac{\partial\psi}{\partial y}$ | $+\dfrac{1}{b^3}\left(-\dfrac{1}{\underline{r}^3}+3\dfrac{1}{\underline{r}^5}\right)$ | $\dfrac{\partial^2}{\partial y^2}$ |
| $+b.\dfrac{\partial^3\psi}{\partial y^3}-3\dfrac{\partial^2\psi}{\partial y^2}$ | $+\dfrac{1}{b^4}\left(-9\dfrac{1}{\underline{r}^5}+15\dfrac{1}{\underline{r}^7}\right)$ | $\dfrac{\partial^3}{\partial y^3}$ |
| $+b.\dfrac{\partial^4\psi}{\partial y^4}-4\dfrac{\partial^3\psi}{\partial y^3}$ | $+\dfrac{1}{b^5}\left(9\dfrac{1}{\underline{r}^5}-90\dfrac{1}{\underline{r}^7}+105\dfrac{1}{\underline{r}^9}\right)$ | $\dfrac{\partial^4}{\partial y^4}$ |
| $+b.\dfrac{\partial^5\psi}{\partial y^5}-5\dfrac{\partial^4\psi}{\partial y^4}$ | $+\dfrac{1}{b^6}\left(225\dfrac{1}{\underline{r}^7}-1050\dfrac{1}{\underline{r}^9}+945\dfrac{1}{\underline{r}^{11}}\right.$ | $\dfrac{\partial^5}{\partial y^5}$ |
| $+b.\dfrac{\partial^6\psi}{\partial y^6}-6\dfrac{\partial^5\psi}{\partial y^5}$ | $+\dfrac{1}{b^7}\left(-225\dfrac{1}{\underline{r}^7}+4725\dfrac{1}{\underline{r}^9}-14175\dfrac{1}{\underline{r}^{11}}+10395\dfrac{1}{\underline{r}^{13}}\right)$ | $\dfrac{\partial^6}{\partial y^6}$ |
| $+\dfrac{1}{b^3}\cdot 2\underline{a}\underline{\psi}^3(3-\underline{a}^2)$ | $+\dfrac{1}{b^3}3\underline{a}\dfrac{1}{\underline{r}^5}$ | |
| $+\dfrac{1}{b^4}\cdot 24\underline{a}\underline{\psi}^4(1-\underline{a}^2)$ | $+\dfrac{1}{b^4}\left(-3\underline{a}\dfrac{1}{\underline{r}^5}+15\underline{a}\dfrac{1}{\underline{r}^7}\right)$ | |
| $-\dfrac{1}{b^4}\cdot 6\underline{\psi}^4(1-6\underline{a}^2+\underline{a}^4)$ | $+\dfrac{1}{b^4}\left(-3\dfrac{1}{\underline{r}^5}+15\underline{a}^2\dfrac{1}{\underline{r}^7}\right)$ | |
| $+\dfrac{1}{b^5}\cdot 24\underline{a}\underline{\psi}^5(5-10\underline{a}^2+\underline{a}^4)$ | $+\dfrac{1}{b^5}\left(-45\underline{a}\dfrac{1}{\underline{r}^7}+105\underline{a}\dfrac{1}{\underline{r}^9}\right)$ | |
| $-\dfrac{1}{b^5}\cdot 24\underline{\psi}^5(1-10\underline{a}^2+5\underline{a}^4)$ | $+\dfrac{1}{b^5}\left(+3\dfrac{1}{\underline{r}^5}-15(\underline{a}^2+1)\dfrac{1}{\underline{r}^7}+105\underline{a}^2\dfrac{1}{\underline{r}^9}\right)$ | |
| $-\dfrac{1}{b^5}24\underline{a}\underline{\psi}^5(5-10\underline{a}^2+\underline{a}^4)$ | $+\dfrac{1}{b^5}\left(-45\underline{a}\dfrac{1}{\underline{r}^7}+105\underline{a}^3\dfrac{1}{\underline{r}^9}\right)$ | |
| $-\dfrac{1}{b^6}\cdot 120\underline{\psi}^6(1-15\underline{a}^2+15\underline{a}^4-\underline{a}^6)$ | $+\dfrac{1}{b^6}\left(+45\dfrac{1}{\underline{r}^7}-105(3\underline{a}^2+1)\dfrac{1}{\underline{r}^9}+945\underline{a}^2\dfrac{1}{\underline{r}^{11}}\right)$ | |
| $-\dfrac{1}{b^6}\cdot 240\underline{a}\underline{\psi}^6(3-10\underline{a}^2+3\underline{a}^4)$ | $+\dfrac{1}{b^6}\left(+45\underline{a}\dfrac{1}{\underline{r}^7}-105\underline{a}(3+\underline{a}^2)\dfrac{1}{\underline{r}^9}+945\underline{a}^3\dfrac{1}{\underline{r}^{11}}\right)$ | |
| $-\dfrac{1}{b^7}\cdot 720\underline{a}\underline{\psi}^7(7-35\underline{a}^2+21\underline{a}^4-\underline{a}^6)$ | $+\dfrac{1}{b^7}\left(+945\underline{a}\dfrac{1}{\underline{r}^9}-2835\dfrac{\underline{a}(\underline{a}^2+1)}{\underline{r}^{11}}+10395\underline{a}^3\dfrac{1}{\underline{r}^{13}}\right)$ | |

$\underline{a = 1}$ $\underline{c} = \frac{586}{298,2} = 1,965124$

| | $\frac{1}{r}$ | $\varphi=\frac{1}{a^2+b^2}$ | $\varphi=\frac{a}{a^2+b^2}$ | $\varphi=\frac{b}{a^2+b^2}$ | $\varphi=\frac{1}{b^2+c^2}$ |
|---|---|---|---|---|---|
| | $\frac{1}{r}=\frac{1}{b}0{,}413035$ | $\varphi=\frac{1}{b^2}0{,}5$ | $\varphi=\frac{1}{b}\cdot 0{,}5$ | $\varphi=\frac{1}{b}0{,}5$ | $\varphi=\frac{1}{b^2}0{,}205689$ |
| $\frac{\partial}{\partial x}$ | $=+0{,}070463\frac{1}{b^2}$ | $+0{,}5\frac{1}{b^3}$ | $0$ | $+\frac{1}{b^2}0{,}5$ | |
| $\frac{\partial^2}{\partial x^2}$ | $=-0{,}034401\frac{1}{b^3}$ | $+0{,}5\frac{1}{b^4}$ | $-\frac{1}{b^3}0{,}5$ | $+\frac{1}{b^3}0{,}5$ | |
| $\frac{\partial^3}{\partial x^3}$ | $=-0{,}077426\frac{1}{b^4}$ | $0$ | $-\frac{1}{b^4}1{,}5$ | $0$ | |
| $\frac{\partial^4}{\partial x^4}$ | $=-0{,}039644\frac{1}{b^5}$ | $-3\frac{1}{b^6}$ | $-\frac{1}{b^5}3$ | $-\frac{1}{b^5}3$ | |
| $\frac{\partial^5}{\partial x^5}$ | $=+0{,}150471\frac{1}{b^6}$ | $-15\frac{1}{b^7}$ | $0$ | $-\frac{1}{b^6}15$ | |
| $\frac{\partial^6}{\partial x^6}$ | $=+0{,}451454\frac{1}{b^7}$ | $-45\frac{1}{b^8}$ | $+\frac{1}{b^7}45$ | $-\frac{1}{b^7}45$ | |
| $\frac{\partial}{\partial y}$ | $+\frac{1}{b^2}0{,}070463$ | $+\frac{1}{b^3}0{,}5$ | $+\frac{1}{b^2}\cdot 0{,}5$ | $0$ | $+\frac{1}{b^3}0{,}084616$ |
| $\frac{\partial^2}{\partial y^2}$ | $-\frac{1}{b^3}0{,}034401$ | $+\frac{1}{b^4}0{,}5$ | $+\frac{1}{b^3}\cdot 0{,}5$ | $-\frac{1}{b^3}0{,}5$ | $-\frac{1}{b^4}0{,}014998$ |
| $\frac{\partial^3}{\partial y^3}$ | $-\frac{1}{b^4}0{,}077426$ | $0$ | $0$ | $-\frac{1}{b^4}1{,}5$ | $-\frac{1}{b^5}0{,}122936$ |
| $\frac{\partial^4}{\partial y^4}$ | $-\frac{1}{b^5}0{,}039644$ | $-\frac{1}{b^6}3$ | $-\frac{1}{b^5}3$ | $-\frac{1}{b^5}3$ | $-\frac{1}{b^6}0{,}165275$ |
| $\frac{\partial^5}{\partial y^5}$ | $+\frac{1}{b^6}0{,}150471$ | $-\frac{1}{b^7}15$ | $-\frac{1}{b^6}15$ | $0$ | $+\frac{1}{b^7}0{,}165780$ |
| $\frac{\partial^6}{\partial y^6}$ | $+\frac{1}{b^7}0{,}451454$ | $-\frac{1}{b^8}45$ | $-\frac{1}{b^7}45$ | $+\frac{1}{b^7}45$ | $+\frac{1}{b^8}1{,}429047$ |
| $\frac{\partial^2}{\partial x\partial y}$ | $+\frac{1}{b^3}0{,}036062$ | | $+\frac{1}{b^3}0{,}5$ | $+\frac{1}{b^3}0{,}5$ | |
| $\frac{\partial^3}{\partial x\partial y^2}$ | $-\frac{1}{b^4}0{,}005301$ | | $+\frac{1}{b^4}1{,}5$ | $0$ | |
| $\frac{\partial^3}{\partial x^2\partial y}$ | $-\frac{1}{b^4}0{,}005301$ | | $0$ | $+\frac{1}{b^4}1{,}5$ | |
| $\frac{\partial^4}{\partial x\partial y^3}$ | $-\frac{1}{b^5}0{,}055549$ | | $+\frac{1}{b^5}\cdot 3$ | $-\frac{1}{b^5}3$ | |
| $\frac{\partial^4}{\partial x^2\partial y^2}$ | $+\frac{1}{b^5}0{,}011274$ | | $+\frac{1}{b^5}\cdot 3$ | $+\frac{1}{b^5}3$ | |
| $\frac{\partial^4}{\partial x^3\partial y}$ | $-\frac{1}{b^5}0{,}055549$ | | $-\frac{1}{b^5}\cdot 3$ | $+\frac{1}{b^5}3$ | |
| $\frac{\partial^5}{\partial x^2\partial y^3}$ | $+\frac{1}{b^6}0{,}001747$ | | $+\frac{1}{b^6}15$ | $0$ | |
| $\frac{\partial^5}{\partial x^3\partial y^2}$ | $+\frac{1}{b^6}0{,}001747$ | | $0$ | $+\frac{1}{b^6}15$ | |
| $\frac{\partial^5}{\partial x^3\partial y^3}$ | $+\frac{1}{b^6}0{,}098042$ | | $+\frac{1}{b^7}45$ | $+\frac{1}{b^7}45$ | |

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

| | | | | |
|---|---|---|---|---|
| $\ldots+c^2$ | $\varphi = \frac{a}{b^2+c^2}$ | $\varphi = \frac{b}{b^2+c^2}$ | $\varphi = \frac{1}{a^2+c^2}$ | $\varphi = \frac{a}{a^2+c^2}$ |
| …5689 | | | $\varphi = \frac{1}{b^2}\,0,205689$ | $\varphi = \frac{1}{b}\,0,205689$ |
| | | | $+\frac{1}{b^3}\,0,084616$ | $-\frac{1}{b^2}\,0,121073$ |
| | | | $-\frac{1}{b^4}\,0,014998$ | $-\frac{1}{b^3}\,0,184230$ |
| | | | $-\frac{1}{b^5}\,0,122936$ | $-\frac{1}{b^4}\,0,077942$ |
| | | | $-\frac{1}{b^6}\,0,165275$ | $+\frac{1}{b^5}\,0,326469$ |
| | | | $+\frac{1}{b^7}\,0,165780$ | $+\frac{1}{b^6}\,0,992155$ |
| | | | $+\frac{1}{b^8}\,1,429047$ | $+\frac{1}{b^7}\,0,434367$ |
| …4616 | | $-\frac{1}{b^2}\,0,121073$ | | |
| …4998 | | $-\frac{1}{b^3}\,0,184230$ | | |
| …2936 | | $-\frac{1}{b^4}\,0,077942$ | | |
| …5275 | | $+\frac{1}{b^5}\,0,326469$ | | |
| …65780 | | $+\frac{1}{b^6}\,0,992155$ | | |
| …29047 | | $+\frac{1}{b^7}\,0,434367$ | | |

$\underline{a} = 3$ $\underline{c} = \frac{586}{298,2} = 1,965124$

| | | $\psi = \frac{1}{a^2+b^2}$ | $\psi = \frac{a}{a^2+b^2}$ | $\psi = \frac{b}{a^2+b^2}$ | $\psi = \frac{1}{b^2+c^2}$ |
|---|---|---|---|---|---|
| | $\frac{1}{r} = \frac{1}{b}\,0{,}268591$ | $\psi = \frac{1}{b^2}\,0{,}100000$ | $\psi = +\frac{1}{b}\,0{,}300000$ | $\psi = +0{,}100000$ | $\psi = \frac{1}{b^2}\,0{,}205689$ |
| $\frac{\partial}{\partial x}$ | $+\frac{1}{b^2}\,0{,}058128$ | $+\frac{1}{b^3}\,0{,}060000$ | $+\frac{1}{b^2}\,0{,}080000$ | $+\frac{1}{b^2}\,0{,}060000$ | |
| $\frac{\partial^2}{\partial x^2}$ | $+\frac{1}{b^3}\,0{,}018365$ | $+\frac{1}{b^4}\,0{,}052000$ | $+\frac{1}{b^3}\,0{,}036000$ | $+\frac{1}{b^3}\,0{,}052000$ | |
| $\frac{\partial^3}{\partial x^3}$ | $+\frac{1}{b^4}\,0{,}003099$ | $+\frac{1}{b^5}\,0{,}057600$ | $+\frac{1}{b^4}\,0{,}016800$ | $+\frac{1}{b^4}\,0{,}057600$ | |
| $\frac{\partial^4}{\partial x^4}$ | $-\frac{1}{b^5}\,0{,}007229$ | $+\frac{1}{b^6}\,0{,}075840$ | $-\frac{1}{b^5}\,0{,}002880$ | $+\frac{1}{b^5}\,0{,}075840$ | |
| $\frac{\partial^5}{\partial x^5}$ | $-\frac{1}{b^6}\,0{,}017657$ | $+\frac{1}{b^7}\,0{,}112320$ | $-\frac{1}{b^6}\,0{,}032240$ | $+\frac{1}{b^6}\,0{,}112320$ | |
| $\frac{\partial^6}{\partial x^6}$ | $-\frac{1}{b^7}\,0{,}028899$ | $+\frac{1}{b^8}\,0{,}176832$ | $-\frac{1}{b^7}\,0{,}143424$ | $+\frac{1}{b^7}\,0{,}176832$ | |
| $\frac{\partial}{\partial y}$ | $+\frac{1}{b^2}\,0{,}019376$ | $+\frac{1}{b^3}\,0{,}020000$ | $+\frac{1}{b^2}\,0{,}060000$ | $-\frac{1}{b^2}\,0{,}080000$ | $+\frac{1}{b^3}\,0{,}084616$ |
| $\frac{\partial^2}{\partial y^2}$ | $-\frac{1}{b^3}\,0{,}015182$ | $-\frac{1}{b^4}\,0{,}012000$ | $-\frac{1}{b^3}\,0{,}036000$ | $-\frac{1}{b^3}\,0{,}052000$ | $-\frac{1}{b^4}\,0{,}014998$ |
| $\frac{\partial^3}{\partial y^3}$ | $-\frac{1}{b^4}\,0{,}011068$ | $-\frac{1}{b^5}\,0{,}019200$ | $-\frac{1}{b^4}\,0{,}057600$ | $+\frac{1}{b^4}\,0{,}016800$ | $-\frac{1}{b^5}\,0{,}122965$ |
| $\frac{\partial^4}{\partial y^4}$ | $+\frac{1}{b^5}\,0{,}004269$ | $-\frac{1}{b^6}\,0{,}000960$ | $-\frac{1}{b^5}\,0{,}002880$ | $+\frac{1}{b^5}\,0{,}075840$ | $-\frac{1}{b^6}\,0{,}165275$ |
| $\frac{\partial^5}{\partial y^5}$ | $+\frac{1}{b^6}\,0{,}015547$ | $+\frac{1}{b^7}\,0{,}037440$ | $+\frac{1}{b^6}\,0{,}112320$ | $+\frac{1}{b^6}\,0{,}042240$ | $+\frac{1}{b^7}\,0{,}165781$ |
| $\frac{\partial^6}{\partial y^6}$ | $+\frac{1}{b^7}\,0{,}004609$ | $+\frac{1}{b^8}\,0{,}047808$ | $+\frac{1}{b^7}\,0{,}143424$ | $-\frac{1}{b^7}\,0{,}176732$ | $+\frac{1}{b^8}\,1{,}429050$ |
| $\frac{\partial^2}{\partial x \partial y}$ | $+\frac{1}{b^3}\,0{,}012581$ | | $+\frac{1}{b^3}\,0{,}052000$ | $-\frac{1}{b^3}\,0{,}036000$ | |
| $\frac{\partial^3}{\partial x \partial y^2}$ | $-\frac{1}{b^4}\,0{,}008043$ | | $-\frac{1}{b^4}\,0{,}016800$ | $-\frac{1}{b^4}\,0{,}057600$ | |
| $\frac{\partial^3}{\partial x^2 \partial y}$ | $+\frac{1}{b^4}\,0{,}009420$ | | $+\frac{1}{b^4}\,0{,}057600$ | $-\frac{1}{b^4}\,0{,}016800$ | |
| $\frac{\partial^4}{\partial x \partial y^3}$ | $-\frac{1}{b^5}\,0{,}011322$ | | $-\frac{1}{b^5}\,0{,}075840$ | $-\frac{1}{b^5}\,0{,}002880$ | |
| $\frac{\partial^4}{\partial x^2 \partial y^2}$ | $-\frac{1}{b^5}\,0{,}004058$ | | $+\frac{1}{b^5}\,0{,}002880$ | $-\frac{1}{b^5}\,0{,}075840$ | |
| $\frac{\partial^4}{\partial x^3 \partial y}$ | $+\frac{1}{b^5}\,0{,}007010$ | | $+\frac{1}{b^5}\,0{,}075840$ | $+\frac{1}{b^5}\,0{,}002880$ | |
| $\frac{\partial^5}{\partial x^2 \partial y^3}$ | $-\frac{1}{b^6}\,0{,}012386$ | | $-\frac{1}{b^6}\,0{,}112320$ | $-\frac{1}{b^6}\,0{,}042240$ | |
| $\frac{\partial^5}{\partial x^3 \partial y^2}$ | $-\frac{1}{b^6}\,0{,}000495$ | | $+\frac{1}{b^6}\,0{,}042240$ | $-\frac{1}{b^6}\,0{,}112320$ | |
| $\frac{\partial^6}{\partial x^3 \partial y^3}$ | $-\frac{1}{b^7}\,0{,}013385$ | | $-\frac{1}{b^7}\,0{,}176832$ | $-\frac{1}{b^7}\,0{,}143424$ | |

MAGYAR TUDOMÁNYOS AKADÉMIA KÖNYVTÁRA

| | | | | | |
|---|---|---|---|---|---|
| $= \frac{1}{b^2+c^2}$ | $\varphi = \frac{a}{b^2+c^2}$ | $\varphi = \frac{b}{b^2+c^2}$ | $\psi = \frac{1}{a^2+c^2}$ | $\varphi = \frac{a}{a^2+c^2}$ | $\varphi = \frac{b}{a^2+c^2}$ |
| 0,205689 | $\varphi = \frac{1}{b}$ 0,205689 | $\varphi =$ | $\psi = \frac{1}{b^2}$ 0,077750 | $\varphi = \frac{1}{b}$ 0,233150 | $\varphi =$ |
| | | | $+ \frac{1}{b^3}$ 0,036270 | $+ \frac{1}{b^2}$ 0,031060 | |
| | | | $+ \frac{1}{b^4}$ 0,021750 | $- \frac{1}{b^3}$ 0,007290 | |
| | | | $+ \frac{1}{b^5}$ 0,013519 | $- \frac{1}{b^4}$ 0,024693 | |
| | | | $+ \frac{1}{b^6}$ 0,004934 | $- \frac{1}{b^5}$ 0,039758 | |
| | | | $- \frac{1}{b^7}$ 0,009514 | $- \frac{1}{b^6}$ 0,053212 | |
| | | | $- \frac{1}{b^8}$ 0,038138 | $- \frac{1}{b^7}$ 0,057330 | |
| 084616 | | $- \frac{1}{b^2}$ 0,121073 | | | |
| 014998 | | $- \frac{1}{b^3}$ 0,184230 | | | |
| 22965 | | $- \frac{1}{b^4}$ 0,077971 | | | |
| 65275 | | $+ \frac{1}{b^5}$ 0,326585 | | | |
| 65781 | | $+ \frac{1}{b^6}$ 0,992156 | | | |
| 29050 | | $+ \frac{1}{b^7}$ 0,434364 | | | |

$a = 3b$ $\quad b. \neq 1.b$ $\quad c = 2.b$

$\underline{a} = 3b$ $\quad \underline{c} = 2$

| | | $\varphi = \frac{1}{a^2+b^2}$ | $\varphi = \frac{a}{a^2+b^2}$ | $\varphi = \frac{b}{a^2+b^2}$ | $\varphi = \frac{1}{b^2+c^2}$ |
|---|---|---|---|---|---|
| | $\frac{1}{r} = \frac{1}{b}\,0{,}267261$ | $\varphi = \frac{1}{b^2}\,0{,}100000$ | $\varphi = +\frac{1}{b}\,0{,}300000$ | $\varphi = +\frac{1}{b}\,0{,}100000$ | $\varphi = +\frac{1}{b^2}\,0{,}200000$ |
| $\frac{\partial}{\partial x}$ | $+\frac{1}{b^2}\,0{,}057270$ | $+\frac{1}{b^3}\,0{,}060000$ | $+\frac{1}{b^2}\,0{,}080000$ | $+\frac{1}{b^2}\,0{,}060000$ | |
| $\frac{\partial^2}{\partial x^2}$ | $+\frac{1}{b^3}\,0{,}017727$ | $+\frac{1}{b^4}\,0{,}052000$ | $+\frac{1}{b^3}\,0{,}036000$ | $+\frac{1}{b^3}\,0{,}052000$ | |
| $\frac{\partial^3}{\partial x^3}$ | $+\frac{1}{b^4}\,0{,}002630$ | $+\frac{1}{b^5}\,0{,}057600$ | $+\frac{1}{b^4}\,0{,}016800$ | $+\frac{1}{b^4}\,0{,}057600$ | |
| $\frac{\partial^4}{\partial x^4}$ | $-\frac{1}{b^5}\,0{,}007452$ | $+\frac{1}{b^6}\,0{,}075840$ | $-\frac{1}{b^5}\,0{,}002880$ | $+\frac{1}{b^5}\,0{,}075840$ | |
| $\frac{\partial^5}{\partial x^5}$ | $-\frac{1}{b^6}\,0{,}017377$ | $+\frac{1}{b^7}\,0{,}112320$ | $-\frac{1}{b^6}\,0{,}032240$ | $+\frac{1}{b^6}\,0{,}112320$ | |
| $\frac{\partial^6}{\partial x^6}$ | $-\frac{1}{b^7}\,0{,}027652$ | $+\frac{1}{b^8}\,0{,}176832$ | $-\frac{1}{b^7}\,0{,}143424$ | $+\frac{1}{b^7}\,0{,}176832$ | |
| $\frac{\partial}{\partial y}$ | $+\frac{1}{b^2}\,0{,}019090$ | $+\frac{1}{b^3}\,0{,}020000$ | $+\frac{1}{b^2}\,0{,}060000$ | $-\frac{1}{b^2}\,0{,}080000$ | $+\frac{1}{b^3}\,0{,}080000$ |
| $\frac{\partial^2}{\partial y^2}$ | $-\frac{1}{b^3}\,0{,}014999$ | $-\frac{1}{b^4}\,0{,}012000$ | $-\frac{1}{b^3}\,0{,}036000$ | $-\frac{1}{b^3}\,0{,}052000$ | $-\frac{1}{b^4}\,0{,}016000$ |
| $\frac{\partial^3}{\partial y^3}$ | $-\frac{1}{b^4}\,0{,}010871$ | $-\frac{1}{b^5}\,0{,}019200$ | $-\frac{1}{b^4}\,0{,}057600$ | $+\frac{1}{b^4}\,0{,}016800$ | $-\frac{1}{b^5}\,0{,}115200$ |
| $\frac{\partial^4}{\partial y^4}$ | $+\frac{1}{b^5}\,0{,}004237$ | $-\frac{1}{b^6}\,0{,}000960$ | $-\frac{1}{b^5}\,0{,}002880$ | $+\frac{1}{b^5}\,0{,}075840$ | $-\frac{1}{b^6}\,0{,}145920$ |
| $\frac{\partial^5}{\partial y^5}$ | $+\frac{1}{b^6}\,0{,}015079$ | $+\frac{1}{b^7}\,0{,}037440$ | $+\frac{1}{b^6}\,0{,}112320$ | $+\frac{1}{b^6}\,0{,}042240$ | $+\frac{1}{b^7}\,0{,}168960$ |
| $\frac{\partial^6}{\partial y^6}$ | $+\frac{1}{b^7}\,0{,}004282$ | $+\frac{1}{b^8}\,0{,}047808$ | $+\frac{1}{b^7}\,0{,}143424$ | $-\frac{1}{b^7}\,0{,}176732$ | $+\frac{1}{b^8}\,1{,}281024$ |
| $\frac{\partial^2}{\partial x \partial y}$ | $+\frac{1}{b^3}\,0{,}012272$ | | $+\frac{1}{b^3}\,0{,}052000$ | $-\frac{1}{b^3}\,0{,}036000$ | |
| $\frac{\partial^3}{\partial x \partial y^2}$ | $-\frac{1}{b^4}\,0{,}007889$ | | $-\frac{1}{b^4}\,0{,}016800$ | $-\frac{1}{b^4}\,0{,}057600$ | |
| $\frac{\partial^3}{\partial x^2 \partial y}$ | $+\frac{1}{b^4}\,0{,}009058$ | | $+\frac{1}{b^4}\,0{,}057600$ | $-\frac{1}{b^4}\,0{,}016800$ | |
| $\frac{\partial^4}{\partial x \partial y^3}$ | $-\frac{1}{b^5}\,0{,}010957$ | | $-\frac{1}{b^5}\,0{,}075840$ | $-\frac{1}{b^5}\,0{,}002880$ | |
| $\frac{\partial^4}{\partial x^2 \partial y^2}$ | $-\frac{1}{b^5}\,0{,}003945$ | | $+\frac{1}{b^5}\,0{,}002880$ | $-\frac{1}{b^5}\,0{,}075840$ | |
| $\frac{\partial^4}{\partial x^3 \partial y}$ | $+\frac{1}{b^5}\,0{,}006574$ | | $+\frac{1}{b^5}\,0{,}075840$ | $+\frac{1}{b^5}\,0{,}002880$ | |
| $\frac{\partial^5}{\partial x^2 \partial y^3}$ | $-\frac{1}{b^6}\,0{,}011844$ | | $-\frac{1}{b^6}\,0{,}112320$ | $-\frac{1}{b^6}\,0{,}042240$ | |
| $\frac{\partial^5}{\partial x^3 \partial y^2}$ | $-\frac{1}{b^6}\,0{,}000470$ by6 | | $+\frac{1}{b^6}\,0{,}042240$ | $-\frac{1}{b^6}\,0{,}112320$ | |
| $\frac{\partial^6}{\partial x^3 \partial y^3}$ | $-\frac{1}{b^7}\,0{,}012579$ | | $-\frac{1}{b^7}\,0{,}176832$ | $-\frac{1}{b^7}\,0{,}143424$ | |

MAGYAR
TUDOMÁNYOS AKADÉMIA
KÖNYVTÁRA

| $\frac{1}{b^2+c^2}$ | $\varphi = \frac{a}{b^2+c^2}$ | $\varphi = \frac{b}{b^2+c^2}$ | $\varphi = \frac{1}{a^2+c^2}$ | $\varphi = \frac{a}{a^2+c^2}$ | $\varphi = \frac{b}{a^2+c^2}$ |
|---|---|---|---|---|---|
| $\frac{1}{b^2}0,200000$ | $\varphi = +\frac{1}{b}0,200000$ | $\varphi =$ | $\varphi = \frac{1}{b^2}0,076923$ | $\varphi = \frac{1}{b}0,230769$ | $\varphi =$ |
| | | | $+\frac{1}{b^3}0,035503$ | $+\frac{1}{b^2}0,029586$ | |
| | | | $+\frac{1}{b^4}0,020938$ | $-\frac{1}{b^3}0,008192$ | |
| | | | $+\frac{1}{b^5}0,012605$ | $-\frac{1}{b^4}0,024999$ | |
| | | | $+\frac{1}{b^6}0,003943$ | $-\frac{1}{b^5}0,038591$ | |
| | | | $-\frac{1}{b^7}0,010292_6$ | $-\frac{1}{b^6}0,050594$ | |
| | | | $-\frac{1}{b^8}0,037601_8$ | $-\frac{1}{b^7}0,051048$ | |
| ,080000 | | $-\frac{1}{b^2}0,120000$ | | | |
| 016000 | | $-\frac{1}{b^3}0,176000$ | | | |
| 115200 | | $-\frac{1}{b^4}0,067200$ | | | |
| 145920 | | $-\frac{1}{b^5}0,314880$ | | | |
| 68960 | | $+\frac{1}{b^6}0,898560$ | | | |
| 281024 | | $+\frac{1}{b^7}0,267264$ | | | |