Ms 5100/16. Gravitáció 1915.

1915

Junius 22 ikén

a Könyvtári antalógiát ellettes

Oszlopos és közre vonatkozó

számítások.

$$\frac{1}{f\sigma\,dm}V = -\frac{1}{\sqrt{r^2+\rho^2+c^2-2\rho r\cos\alpha}}$$

$$k=\frac{2\rho r}{r^2+\rho^2+c^2}$$

V

$\cos\alpha = 1-\sin^2\alpha.$

$$r^2+\rho^2+c^2-2\rho r+2\rho r\sin^2\alpha$$

$$1+\frac{2\rho r}{r^2+\rho^2-2\rho r+c^2}\sin^2\alpha$$

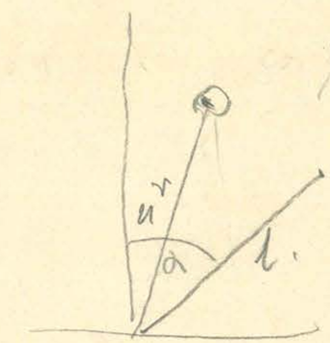
$$\frac{1}{f\sigma\,dm}V = -\frac{1}{\sqrt{r^2+\rho^2+c^2}}\frac{1}{\sqrt{1-k\cos\alpha}}$$

$$\frac{1}{\sqrt{1-k\cos\alpha}} = 1+\tfrac{1}{2}k\cos\alpha+\frac{1.3}{2.4}k^2\cos^2\alpha+\frac{1.3.5}{2.4.6}k^3\cos^3\alpha+\frac{1.3.5.7}{2.4.6.8}k^4\cos^4\alpha+\cdots$$

$$\cos^2\alpha = \frac{1}{2}\cos 2\cancel{\alpha}+\frac{1}{2}.$$

$$\cos^3\alpha = \frac{1}{2^2}\cos 3\cancel{\alpha}+\frac{1}{2^2}\cdot\frac{3}{1}\cos\varphi$$

$\alpha=\varphi$



$$\cos^4\varphi = \frac{1}{2^3}\cos 4\varphi+\frac{1}{2^3}4\cos 2\varphi+\frac{1}{2^3}\frac{1}{2}\frac{4.3}{1.2}$$

$$\cos^5\varphi = \frac{1}{2^4}\cos 5\varphi+\frac{1}{2^4}5\cos 3\varphi+\frac{1}{2^4}\cdot\frac{5.4}{1.2}\cos\varphi$$

$$\cos^6\varphi = \frac{1}{2^5}\cos 6\varphi+\frac{1}{2^5}6\cos 4\varphi+\frac{1}{2^5}\frac{6.5}{1.2}\cos 2\varphi+\frac{1}{2^5}\cdot\frac{1}{2}\frac{6.5.4}{1.2.3}$$

$$\cos^7\varphi = \frac{1}{2^6}\cos 7\varphi+\frac{1}{2^6}7\cos 5\varphi+\frac{7.6}{1.2}\cdot\frac{1}{2^6}\cos 3\varphi+\frac{1}{2^6}\cdot\frac{7.6.5}{1.2.3}\cos\varphi$$

$$\cos^8\varphi = \frac{1}{2^7}\cos 8\varphi+\frac{1}{2^7}8\cos 6\varphi+\frac{1}{2^7}\cdot\frac{8.7}{1.2}\cos 4\varphi+\frac{1}{2^7}\frac{8.7.6}{1.2.3}\cos 2\varphi+\frac{1}{2^7}\cdot\frac{1}{2}\frac{8.7.6.5}{1.2.3.4}$$

$$\cos^9\varphi = \frac{1}{2^8}\cos 9\varphi+\frac{1}{2^8}9\cos 7\varphi+\frac{1}{2^8}\cdot\frac{9.8}{1.2}\cos 5\varphi+\frac{1}{2^8}\cdot\frac{9.8.7}{1.2.3}\cos 3\varphi+\frac{1}{2^8}\cdot\frac{9.8.7.6}{1.2.3.4}\cos\varphi$$

$$\cos^{10}\varphi = \frac{1}{2^9}\cos 10\varphi+\frac{1}{2^9}10\cos 8\varphi+\frac{1}{2^9}\cdot\frac{10.9}{1.2}\cos 6\varphi+\frac{1}{2^9}\cdot\frac{10.9.8}{1.2.3}\cos 4\varphi+\frac{1}{2^9}\cdot\frac{10.9.8.7}{1.2.3.4}\cos 2\varphi+\frac{1}{2^9}\cdot\frac{1}{2}\frac{10.9.8.7.6}{1.2.3.4.5}$$

$$\frac{1}{\sqrt{1-k\cos\varphi}} =$$

$\cos(\alpha - u) = \cos\alpha\cos u + \sin\alpha\sin u$

$\cos\alpha x$ $\cos 2\alpha$ $-\frac{1}{\sqrt{\sigma}}\, V =$

1

$\frac{1}{2^0}\,\frac{1\cdot 1}{2\cdot 1}k$

$\frac{1}{2^2}\,\frac{1.3.5}{2.4.6}\cdot\frac{3}{1}\,k^3$

$\frac{1}{2^4}\cdot\frac{1.3.5.7.9}{2.4.6.8.10}\,\frac{5.4}{1.2}\,k^5$

$\frac{1}{2^6}\,\frac{1\cdots 13}{2\cdots 14}\cdot\frac{7.6.5}{1.2.3}\,k^7$

$$\frac{1}{\sqrt{1-k\cos\alpha}} = 1$$

$\frac{1.3}{2.4} = \frac{3}{8}$

$\frac{1\ldots 5}{2.6} = \frac{5}{16}$

$\frac{1\ldots 7}{2\ldots 8} = \frac{35}{128}$

$\frac{1\ldots 9}{2\ldots 10} = \frac{63}{256}$

$\frac{1-11}{2-12} = \frac{231}{1024}$

$\frac{1-13}{2-14} = \frac{429}{2048}$

$\frac{1-15}{2-16} = \frac{6435}{32768}$

$\frac{1-17}{2-18} = \frac{12155}{65536}$

$\frac{1-19}{2-20} = \frac{46189}{262144}$

$\frac{3}{16}$	$\frac{3}{8}\cdot\frac{35}{128} = \frac{105}{1024}$	$\frac{5}{16}\cdot\frac{231}{1024} = \frac{1155}{16384}$	$\frac{35}{128}\cdot\frac{6435}{32768} = \frac{225225}{4194304}$
$\frac{1}{2}$	$\frac{3}{4}\cdot\frac{5}{16} = \frac{15}{64}$	$\frac{5}{8}\cdot\frac{63}{256} = \frac{315}{2048}$	$\frac{35}{64}\cdot\frac{429}{2048} = \frac{15015}{131072}$
$\frac{3}{16}$	$\frac{1}{2}\cdot\frac{35}{128} = \frac{35}{256}$	$\frac{15}{32}\cdot\frac{231}{1024} = \frac{3465}{32768}$	$\frac{7}{16}\cdot\frac{6435}{32768} = \frac{45045}{524288}$
1 $\frac{1}{4}\cdot\frac{5}{16} = \frac{5}{64}$	$\frac{5}{16}\cdot\frac{63}{256} = \frac{315}{4096}$	$\frac{21}{64}\cdot\frac{429}{2048} = \frac{9009}{131072}$	$\frac{21}{64}\cdot\frac{12155}{65536} = \frac{255255}{4194304}$
$\frac{1}{8}\cdot\frac{35}{128} = \frac{35}{1024}$	$\frac{3}{16}\cdot\frac{231}{1024} = \frac{693}{16384}$	$\frac{7}{32}\cdot\frac{6435}{32768} = \frac{45045}{1048576}$	$\frac{15}{64}\cdot\frac{46189}{262144} = \frac{692835}{16777216}$

$$
\begin{aligned}
-\frac{dV}{f\sigma} = {} & \frac{d\omega}{\sqrt{l^2+r^2+c^2}} + \frac{3}{4}\frac{r^2 d\omega}{(l^2+r^2+c^2)^{\frac{5}{2}}} l^2 + \frac{105}{64}\frac{r^4 d\omega}{(l^2+r^2+c^2)^{\frac{9}{2}}} l^4 + \frac{1155}{256}\frac{r^6 d\omega}{(l^2+r^2+c^2)^{\frac{13}{2}}} l^6 + \frac{225225}{16384}\frac{r^8 d\omega}{(l^2+r^2+c^2)^{\frac{17}{2}}} l^8 + \cdots \\
& + \sin\alpha \left(l \frac{r\, d\omega}{(l^2+r^2+c^2)^{\frac{3}{2}}} \sin u + \frac{15}{8} l^3 \frac{r^3 d\omega}{(l^2+r^2+c^2)^{\frac{7}{2}}} \sin u + \frac{315}{64} l^5 \frac{r^5 d\omega}{(l^2+r^2+c^2)^{\frac{11}{2}}} \sin u + \frac{15015}{1024} l^7 \frac{r^7 d\omega}{(l^2+r^2+c^2)^{\frac{15}{2}}} \sin u + \cdots \right) \\
& + \cos\alpha \left(l \frac{r\, d\omega}{(l^2+r^2+c^2)^{\frac{3}{2}}} \cos u + \cdots \right) \\
& + \sin 2\alpha \left(\frac{3}{4} l^2 \frac{r^2 d\omega}{(l^2+r^2+c^2)^{\frac{5}{2}}} \sin 2u + \frac{35}{16} l^4 \frac{r^4 d\omega}{(l^2+r^2+c^2)^{\frac{9}{2}}} \sin 2u + \frac{3465}{512} l^6 \frac{r^6 d\omega}{(l^2+r^2+c^2)^{\frac{13}{2}}} \sin 2u + \frac{45045}{2048} l^8 \frac{r^8 d\omega}{(l^2+r^2+c^2)^{\frac{17}{2}}} \sin 2u + \cdots \right) \\
& + \cos 2\alpha \left(\frac{3}{4} l^2 \frac{r^2 d\omega}{(l^2+r^2+c^2)^{\frac{5}{2}}} \cos 2u + \cdots \right. \\
& + \sin 3\alpha \left(\frac{5}{8} l^3 \frac{r^3 d\omega}{(l^2+r^2+c^2)^{\frac{7}{2}}} \sin 3u + \frac{315}{128} l^5 \frac{r^5 d\omega}{(l^2+r^2+c^2)^{\frac{11}{2}}} \sin 3u + \frac{9009}{1024} l^7 \frac{r^7 d\omega}{(r^2+l^2+c^2)^{\frac{15}{2}}} \sin 3u + \frac{255255}{8192} l^9 \frac{r^9 d\omega}{(r^2+l^2+c^2)^{\frac{19}{2}}} \sin 3u + \cdots \right) \\
& + \cos 3\alpha \left(\frac{5}{8} l^3 \frac{r^3 d\omega}{(l^2+r^2+c^2)^{\frac{7}{2}}} \cos 3u + \cdots \right) \\
& + \sin 4\alpha \left(\frac{35}{64} l^4 \frac{r^4 d\omega}{(l^2+r^2+c^2)^{\frac{9}{2}}} \sin 4u + \frac{693}{256} l^6 \frac{r^6 d\omega}{(l^2+r^2+c^2)^{\frac{13}{2}}} \sin 4u + \frac{45045}{4096} l^8 \frac{r^8 d\omega}{(l^2+r^2+c^2)^{\frac{17}{2}}} \sin 4u + \frac{692835}{16384} l^{10} \frac{r^{10} d\omega}{(l^2+r^2+c^2)^{\frac{21}{2}}} \sin 4u + \cdots \right. \\
& \left. + \cos 4\alpha \left(\frac{35}{64} l^4 \frac{r^4 d\omega}{(l^2+r^2+c^2)^{\frac{9}{2}}} \cos 4u + \cdots \right. \right\} \\
& \cdots
\end{aligned}
$$

$x^7 = r^7\left(\frac{1}{64}\cos 7\alpha + \frac{7}{64}\cos 5\alpha + \frac{21}{64}\cos 3\alpha + \frac{35}{64}\cos\alpha\right)$

$x^6y = r^7\left(+\frac{1}{64}\sin 7\alpha + \frac{5}{64}\sin 5\alpha + \frac{9}{64}\sin 3\alpha + \frac{5}{64}\sin\alpha\right)$

$x^5y^2 = r^7\left(-\frac{1}{64}\cos 7\alpha - \frac{3}{64}\cos 5\alpha - \frac{1}{64}\cos 3\alpha + \frac{5}{64}\sin\alpha\right)$

$x^4y^3 = r^7\left(-\frac{1}{64}\sin 7\alpha - \frac{1}{64}\sin 5\alpha + \frac{3}{64}\sin 3\alpha + \frac{3}{64}\sin\alpha\right.$

$x^3y^4 = r^7\left(+\frac{1}{64}\cos 7\alpha - \frac{1}{64}\cos 5\alpha - \frac{3}{64}\cos 3\alpha + \frac{3}{64}\cos\alpha\right.$

$x^2y^5 = r^7\left(+\frac{1}{64}\sin 7\alpha - \frac{3}{64}\sin 5\alpha + \frac{1}{64}\sin 3\alpha + \frac{5}{64}\sin\alpha\right.$

$xy^6 = r^7\left(-\frac{1}{64}\cos 7\alpha + \frac{5}{64}\cos 5\alpha - \frac{9}{64}\cos 3\alpha + \frac{5}{64}\cos\alpha\right.$

$y^7 = r^7\left(-\frac{1}{64}\sin 7\alpha + \frac{7}{64}\sin 5\alpha - \frac{21}{64}\sin 3\alpha + \frac{35}{64}\sin\alpha\right)$

$a^2 + b^2 + 2ab$

$a^3 + ab^2 + 2a^2b$

$b^3 \quad ab^2 \quad ba^2$

$x^6y = \sin\alpha\cos^6\alpha = \sin\alpha\cos^2$

$(1-y^2)^3 y = y(1+3y^4-3y^2-y^6)$

$= y^7 + 3y^5 - 3y^3 + y$

$xy^6 = x^7 + 3x^5 - 3x^3 + x$

$= +\frac{1}{64}\sin 7\alpha - \frac{7}{64}\sin 5\alpha + \frac{21}{64}\sin 3\alpha - \frac{35}{64}\sin\alpha$

$+\frac{3}{16}\sin 5\alpha - \frac{15}{16}\sin 3\alpha + \frac{30}{16}\sin\alpha$ 144

$+\frac{3}{4}\sin 3\alpha - \frac{9}{4}\sin\alpha$

$+\sin\alpha$

64 35

120 179

184

$x^6y = +\frac{1}{64}\sin 7\alpha + \frac{5}{64}\sin 5\alpha + \frac{9}{64}\sin 3\alpha + \frac{5}{64}\sin\alpha$

$xy^6 = -\frac{1}{64}\cos 7\alpha - \frac{7}{64}\cos 5\alpha - \frac{21}{64}\cos 3\alpha - \frac{35}{64}\cos\alpha$

$+\frac{3}{16}\cos 5\alpha + \frac{15}{16}\cos 3\alpha + \frac{30}{16}\cos\alpha$

$-\frac{3}{4}\cos 3\alpha - \frac{9}{4}\cos\alpha$

$+\cos\alpha$

$xy^6 = -\frac{1}{64}7\alpha + \frac{5}{64}\cos 5\alpha - \frac{9}{64}\cos 3\alpha + \frac{5}{64}\cos\alpha$

$$x^8 = r^8\left(+\frac{1}{128}\cos 8\alpha + \frac{8}{128}\cos 6\alpha + \frac{28}{128}\cos 4\alpha + \frac{56}{128}\cos 2\alpha + \frac{35}{128}\right)$$

$$x^7y = r^8\left(+\frac{1}{128}\sin 8\alpha + \frac{6}{128}\sin 6\alpha + \frac{14}{128}\sin 4\alpha + \frac{14}{128}\sin 2\alpha\right.$$

$$x^6y^2 = r^8\left(-\frac{1}{128}\cos 8\alpha - \frac{4}{128}\cos 6\alpha - \frac{4}{128}\cos 4\alpha + \frac{4}{128}\cos 2\alpha + \frac{5}{128}\right.$$

$$x^5y^3 = r^8\left(-\frac{1}{128}\sin 8\alpha - \frac{2}{128}\sin 6\alpha + \frac{2}{128}\sin 4\alpha + \frac{6}{128}\sin 2\alpha\right.$$

$$x^4y^4 = r^8\left(+\frac{1}{128}\cos 8\alpha - \frac{4}{128}\cos 4\alpha + \frac{3}{128}\right)$$

$$x^3y^5 = r^8\left(+\frac{1}{128}\sin 8\alpha - \frac{2}{128}\sin 6\alpha - \frac{2}{128}\sin 4\alpha + \frac{6}{128}\sin 2\alpha\right.$$

$$x^2y^6 = r^8\left(-\frac{1}{128}\cos 8\alpha + \frac{4}{128}\cos 6\alpha - \frac{4}{128}\cos 4\alpha - \frac{4}{128}\cos 2\alpha + \frac{5}{128}\right.$$

$$xy^7 = r^8\left(-\frac{1}{128}\sin 8\alpha + \frac{6}{128}\sin 6\alpha - \frac{14}{128}\sin 4\alpha + \frac{14}{128}\sin 2\alpha\right.$$

$$y^8 = r^8\left(+\frac{1}{128}\cos 8\alpha - \frac{8}{128}\cos 6\alpha + \frac{28}{128}\cos 4\alpha - \frac{56}{128}\cos 2\alpha + \frac{35}{128}\right)$$

$$x^7y = \frac{1}{2}\sin 2\alpha\left(\cos^2\alpha\right)^3 = \cancel{\frac{1}{2}\sin 2\alpha\left(\frac{1}{2}\cos 2\alpha + \frac{1}{2}\right)^3}$$

$$= \frac{1}{2}\sin 2\alpha\left(\frac{1}{8}\cos^3 2\alpha + \frac{1}{8} + \frac{3}{8}\cos^2 2\alpha + \frac{3}{8}\cos 2\alpha\right)$$

$$= \frac{1}{32}\sin 4\alpha\cos^2 2\alpha + \frac{1}{16}\sin 2\alpha + \frac{3}{16}\sin 2\alpha\cos^2 2\alpha + \frac{3}{32}\sin 4\alpha$$

$$\cancel{= \frac{1}{32}\sin 4\alpha\left(\cos^2 2\alpha + 3\right) + \frac{1}{16}\sin 2\alpha + \frac{3}{16}\sin 2\alpha\left(\frac{1}{2}\cos 2\alpha + \frac{1}{2}\right)}$$

$$= \frac{1}{32}\sin 4\alpha\left(\frac{1}{2}\cos 4\alpha + \frac{1}{2}\right) + \frac{1}{16}\sin 2\alpha + \frac{3}{16}\sin 2\alpha - \cancel{\frac{3}{16}\sin^3 2\alpha} + \frac{3}{32}\sin 4\alpha$$

$$32 \qquad + \frac{3}{64}\sin 6\alpha - \frac{9}{64}\sin 2\alpha$$

$$x^7y = \frac{1}{128}\sin 8\alpha + \frac{6}{128}\sin 6\alpha + \frac{14}{128}\sin 4\alpha + \frac{14}{128}\sin 2\alpha$$

$$xy^7 = \frac{1}{2}\sin 2\alpha\left(\sin^2\alpha\right)^3 = \frac{1}{2}\sin 2\alpha\left(-\frac{1}{2}\cos 2\alpha + \frac{1}{2}\right)$$

$$= \frac{1}{2}\sin 2\alpha\left(-\frac{1}{8}\cos^3 2\alpha + \frac{1}{8} + \frac{3}{8}\cos^2 2\alpha - \frac{3}{8}\cos 2\alpha\right)$$

$$= -\frac{1}{32}\sin 4\alpha\cos^2 2\alpha + \frac{1}{16}\sin 2\alpha + \frac{3}{16}\sin 2\alpha\cos^2 2\alpha - \frac{3}{32}\sin 4\alpha$$

$$= -\frac{1}{32}\sin 4\alpha\left(\frac{1}{2}\cos 4\alpha + \frac{1}{2}\right) + \frac{1}{16}\sin 2\alpha + \frac{3}{16}\sin 2\alpha + \frac{3}{64}\sin 6\alpha - \frac{2}{64}\sin 2\alpha - \frac{3}{32}\sin 4\alpha$$

$$= -\frac{1}{128}\sin 8\alpha + \frac{6}{128}\sin 6\alpha - \frac{14}{128}\sin 4\alpha + \frac{14}{128}\sin 2\alpha$$

$$x^6 - x^8 = \frac{1}{32}\cos 6\alpha + \frac{6}{32}\cos 4\alpha + \frac{15}{32}\cos 2\alpha + \frac{10}{32}$$
$$-\frac{1}{128}\cos 8\alpha - \frac{8}{128}\cos 6\alpha - \frac{28}{128}\cos 4\alpha - \frac{56}{128}\cos 2\alpha - \frac{35}{128}$$

$$-\frac{1}{128}\cos 8\alpha - \frac{4}{128}\cos 6\alpha - \frac{4}{128}\cos 4\alpha + \frac{4}{128}\cos 2\alpha + \frac{5}{128}$$

$$y^6 - y^8 = -\frac{1}{32}\cos 6\alpha + \frac{6}{32}\cos 4\alpha - \frac{15}{32}\cos 2\alpha + \frac{10}{32}$$
$$-\frac{1}{128}\cos 8\alpha + \frac{8}{128}\cos 6\alpha - \frac{28}{128}\cos 4\alpha + \frac{56}{128}\cos 2\alpha - \frac{35}{128}$$

$$-\frac{1}{128}\cos 8\alpha + \frac{4}{128}\cos 6\alpha - \frac{4}{128}\cos 4\alpha - \frac{4}{128}\cos 2\alpha + \frac{5}{128}$$

$$x^5y^3 = \frac{1}{8}\sin^3 2\alpha\left(\frac{1}{2}\cos 2\alpha + \frac{1}{2}\right) = \frac{1}{32}\sin 4\alpha \sin^2 2\alpha + \frac{1}{16}\sin^3 2\alpha$$

$$\cancel{-\frac{1}{32}\sin 6\alpha} + \frac{3}{32}\sin 2\alpha$$
$$= \frac{1}{32}\sin 4\alpha\left(-\frac{1}{2}\cos 4\alpha + \frac{1}{2}\right) - \frac{1}{64}\sin 6\alpha + \frac{3}{64}\sin 2\alpha$$

$$x^5y^3 = -\frac{1}{128}\sin 8\alpha - \frac{2}{128}\sin 6\alpha + \frac{2}{128}\sin 4\alpha + \frac{6}{128}\sin 2\alpha$$

$\otimes \longrightarrow$

$$x^3y^5 = \frac{1}{8}\sin^3 2\alpha\left(-\frac{1}{2}\cos 2\alpha + \frac{1}{2}\right)$$

$$= -\frac{1}{32}\sin 4\alpha \sin^2 2\alpha + \frac{1}{16}\sin^3 2\alpha$$

$$= -\frac{1}{32}\sin 4\alpha\left(-\frac{1}{2}\cos 4\alpha + \frac{1}{2}\right) - \frac{1}{64}\sin 6\alpha + \frac{3}{64}\sin 2\alpha$$

$$x^3y^5 = +\frac{1}{128}\sin 8\alpha - \frac{2}{128}\sin 6\alpha - \frac{2}{128}\sin 4\alpha + \frac{6}{128}\sin 2\alpha$$

$\otimes \longrightarrow$

$$x^4y^4 = \frac{1}{16}\sin^4 2\alpha = \frac{1}{128}\cos 8\alpha - \frac{4}{128}\cos 4\alpha + \frac{3}{128}$$

$\otimes \longrightarrow$

4α

$$\frac{1}{8}\sin^3 2\alpha = -\frac{1}{32}\sin 6\alpha + \frac{3}{32}\sin 2\alpha$$

$$\frac{1}{2}\sin 2\alpha \sin^4\alpha = \frac{1}{16}\sin 2\alpha\cos 4\alpha - \frac{4}{16}\sin 2\alpha\cos 2\alpha + \frac{3}{16}\sin 2\alpha.$$

$$= \frac{1}{16}\sin 2\alpha\cos^2 2\alpha - \frac{1}{16}\sin^3 2\alpha - \frac{4}{32}\sin 4\alpha + \frac{6}{32}\sin 2\alpha$$

$$= \frac{1}{16}\sin 2\alpha - \frac{1}{8}\sin^3 2\alpha - \frac{4}{32}\sin 4\alpha + \frac{6}{32}\sin 2\alpha$$

$$= \frac{1}{16}\sin 2\alpha + \frac{1}{32}\sin 6\alpha - \frac{3}{32}\sin 2\alpha - \frac{4}{32}\sin 4\alpha + \frac{6}{32}\sin 2\alpha$$

$$= +\frac{1}{32}\sin 6\alpha - \frac{4}{32}\sin 4\alpha + \frac{5}{32}\sin 2\alpha$$

$xy^3 = r^4\cos\alpha\sin^3\alpha$

$\sin^3\alpha = \sin^3$

$\sin^2\alpha = -\frac{1}{2}\cos 2\alpha + \frac{1}{2}$

$\frac{r^4}{2}\sin 2\alpha$

$$\left(-\frac{3}{4}r^4 + 2r^4\cos 2\alpha + \frac{1}{2}r^4\cos 4\alpha\right)\frac{\partial^4 V}{\partial x^3\partial y}$$

$$\left(+\frac{3}{4}r^4 - 2r^4\cos 2\alpha - \frac{1}{2}r^4\cos 4\alpha\right)\frac{\partial^4 V}{\partial x\partial y^3}$$

$$\frac{\partial^4 V}{\partial x^3\partial y}(x^4 - 3x^2y^2) + \frac{\partial^4 V}{\partial x\partial y^3}(3x^2y^2 - y^4)$$

$$\frac{3}{8}\frac{\partial^4 V}{\partial x^3\partial y} - \frac{3}{8}\frac{\partial^4 V}{\partial x\partial y^3} + \frac{15}{8}\frac{\partial^4 V}{\partial x\partial y^3}$$

$$\frac{3}{8}\frac{\partial^4 V}{\partial x^3\partial y} - \frac{3}{8}\frac{\partial^4 V}{\partial x\partial y^3} + \frac{15}{8}\frac{\partial^4 V}{\partial x\partial y^3} - \frac{15}{8}\frac{\partial^4 V}{\partial x^3\partial y}$$

$$-\frac{12}{8}\left(\frac{\partial^4 V}{\partial x^3\partial y} - \frac{\partial^4 V}{\partial x\partial y^3}\right)$$

$$-x^6 = r^6\left(\frac{1}{32}\cos 6\alpha + \frac{6}{32}\cos 4\alpha + \frac{15}{32}\cos 2\alpha + \frac{10}{32}\right)$$

$$-x^5y = r^6\left(\frac{1}{32}\sin 6\alpha + \frac{4}{32}\sin 4\alpha + \frac{5}{32}\sin 2\alpha\right)$$

$$x^4y^2 = r^6\left(-\frac{1}{32}\cos 6\alpha - \frac{2}{32}\cos 4\alpha + \frac{1}{32}\cos 2\alpha + \frac{2}{32}\right)$$

$$x^3y^3 = r^6\left(-\frac{1}{32}\sin 6\alpha + \frac{3}{32}\sin 2\alpha\right)$$

$$x^2y^4 = r^6\left(+\frac{1}{32}\cos 6\alpha - \frac{2}{32}\cos 4\alpha - \frac{1}{32}\cos 2\alpha + \frac{2}{32}\right)$$

$$-x\,y^5 = r^6\left(+\frac{1}{32}\sin 6\alpha - \frac{4}{32}\sin 4\alpha + \frac{5}{32}\sin 2\alpha\right)$$

$$-y^5 = r^6\left(-\frac{1}{32}\cos 6\alpha + \frac{6}{32}\cos 4\alpha - \frac{15}{32}\cos 2\alpha + \frac{10}{32}\right)$$

$$\frac{1}{2}\sin 2\alpha\left(\frac{1}{8}\cos 4\alpha + \frac{4}{8}\cos 2\alpha + \frac{3}{8}\right)$$

$$\sin 2\alpha \cos 4\alpha = \sin 2\alpha\cos^2 2\alpha - \sin^3 2\alpha = \sin 2\alpha - 2\sin^3 2\alpha = \sin 2\alpha + \frac{1}{2}\sin 3\alpha - \frac{3}{8}\sin\alpha$$
$$+\frac{1}{8}\sin 4\alpha + \frac{3}{16}\sin 2\alpha$$

$$\frac{1}{2}\sin 2\alpha + \frac{1}{4}\sin 3\alpha - \frac{3}{16}\sin\alpha + \frac{1}{16}\sin 4\alpha$$
$$+\frac{3}{16}\sin 2\alpha \qquad \frac{1}{16}\sin 4\alpha + \frac{4}{16}\sin 3\alpha + \frac{11}{16}\sin 2\alpha - \frac{1}{16}\sin\alpha$$

$$x^5y = xy\,x^4 = \frac{1}{2}\sin 2\alpha\left(\frac{1}{8}\cos 4\alpha + \frac{4}{8}\cos 2\alpha + \frac{3}{8}\right)$$

$$= \frac{1}{16}\sin 2\alpha\cos 4\alpha + \frac{2}{16}\sin 4\alpha + \frac{3}{16}\sin 2\alpha$$

$$= \frac{1}{16}\sin 2\alpha(\overset{1}{\cos^2 2\alpha} - \overset{2}{\sin^2 2\alpha}) + \frac{2}{16}\sin 4\alpha + \frac{3}{16}\sin 2\alpha.$$

$$= \frac{1}{16}\sin 2\alpha - \frac{2}{16}\sin^3 2\alpha + \frac{2}{16}\sin 4\alpha + \frac{3}{16}\sin 2\alpha$$

$$= \frac{1}{16}\sin 2\alpha + \frac{1}{32}\sin 6\alpha - \frac{3}{32}\sin 2\alpha + \frac{8}{64}\sin 4\alpha + \frac{62}{64}\sin 2\alpha.$$

$$= \frac{1}{32}\sin 6\alpha + \frac{8}{64}\sin 4\alpha + \frac{50}{32}\sin 2\alpha.$$

$$x^4y^2 = x^4 - x^6 = -\frac{1}{32}\cancel{\cos} 6\alpha - \frac{6}{32}\cancel{\cos} 4\alpha - \frac{15}{32}\cancel{\cos} 2\alpha - \frac{10}{32} + \frac{4}{32}\cancel{\cos} 4\alpha + \frac{16}{32}\cancel{\cos 2\alpha} + \frac{12}{32}$$

$$x^2y^4 = y^4 - y^6 = +\frac{1}{32}\cancel{\cos} 6\alpha - \frac{6}{32}\cos 4\alpha + \frac{15}{32}\cos 2\alpha - \frac{10}{32} + \frac{4}{32}\cancel{\cos 4\alpha} - \frac{16}{32}\cos 2\alpha + \frac{12}{32}$$

$x = r\cos\alpha$ $\qquad$ $y = r\sin\alpha$

$xy = \frac{1}{2}r^2\sin 2\alpha$ $\qquad$ $xy = \frac{1}{2}r^2\sin 2\alpha$

$x^2 = r^2\cos^2\alpha = r^2\left(\frac{1}{2} + \frac{1}{2}\cos 2\alpha\right)$ $\qquad$ $y^2 = r^2\sin^2\alpha = r^2\left(\frac{1}{2} - \frac{1}{2}\cos 2\alpha\right)$

$x^3 = r^3\cos^3\alpha = r^3\frac{1}{4}\cos 3\alpha + r^3\frac{3}{4}\cos\alpha$

$x^2y = r^3\cos^2\alpha\sin\alpha = r^3(\sin\alpha - \sin^3\alpha) = r^3\left(+\frac{1}{4}\sin 3\alpha + \frac{1}{4}\sin\alpha\right)$

$xy^2 = r^3\cos\alpha\sin^2\alpha = r^3(\cos\alpha - \cos^3\alpha) = r^3\left(-\frac{1}{4}\cos 3\alpha + \frac{1}{4}\cos\alpha\right)$

$y^3 = r^3\sin^3\alpha = -r^3\frac{1}{4}\sin 3\alpha + r^3\frac{3}{4}\sin\alpha$

$x^4 = r^4\cos^4\alpha \qquad = r^4\frac{1}{8}(\cos 4\alpha + 4\cos 2\alpha + 3)$

$x^3y = r^4\frac{1}{2}\sin 2\alpha\cos^2\alpha \qquad = r^4\frac{1}{2}\sin 2\alpha\left(\frac{1}{2} + \frac{1}{2}\cos 2\alpha\right) = \frac{r^4}{4}\sin 2\alpha + \frac{r^4}{8}\sin 4\alpha$

$x^2y^2 = r^4(\cos^2\alpha - \cos^4\alpha) = r^4\left(\frac{1}{2} + \frac{1}{2}\cos 2\alpha - \frac{1}{8}\cos 4\alpha - \frac{1}{2}\cos 2\alpha - \frac{3}{8}\right) = r^4\left(\frac{1}{8} - \frac{1}{8}\cos 4\alpha\right)$

$xy^3 = r^4\frac{1}{2}\sin 2\alpha\sin^2\alpha = r^4\frac{1}{2}\sin 2\alpha\left(\frac{1}{2} - \frac{1}{2}\cos 2\alpha\right) = \frac{r^4}{4}\sin 2\alpha - \frac{r^4}{8}\sin 4\alpha$

$y^4 = r^4\sin^4\alpha \qquad = r^4\frac{1}{8}(\cos 4\alpha - 4\cos 2\alpha + 3)$

$x^5 = r^5\cos^5\alpha \qquad = \frac{r^5}{16}(\cos 5\alpha + 5\cos 3\alpha + 10\cos\alpha)$

$x^4y = r^5\frac{1}{2}\sin 2\alpha\cos^3\alpha \qquad = r^5\frac{1}{2}\sin 2\alpha\left(\frac{1}{4}\cos 3\alpha + \frac{3}{4}\cos\alpha\right) = r^5\left(\frac{1}{16}\sin 5\alpha + \frac{3}{16}\sin 3\alpha + \frac{2}{16}\sin\alpha\right)$

$x^3y^2 = r^5(\cos^3\alpha - \cos^5\alpha) = \cancel{\ldots} = r^5\left(-\frac{1}{16}\cos 5\alpha - \frac{1}{16}\cos 3\alpha + \frac{2}{16}\cos\alpha\right)$

$x^2y^3 = r^5(\sin^3\alpha - \sin^5\alpha) = \qquad = r^5\left(-\frac{1}{16}\sin 5\alpha + \frac{1}{16}\sin 3\alpha + \frac{2}{16}\sin\alpha\right)$

$xy^4 = r^5\frac{1}{2}\sin 2\alpha(\sin^3\alpha) = r^5\left(\frac{1}{16}\cos 5\alpha - \frac{3}{16}\cos 3\alpha + \frac{2}{16}\cos\alpha\right)$

$y^5 = r^5\sin^5\alpha \qquad = \frac{r^5}{16}(\sin 5\alpha - 5\sin 3\alpha + 10\sin\alpha)$

$x^2y = \sin\alpha\cos^2\alpha = \sin\alpha\cos^2\alpha - \sin^3\alpha\cos^2\alpha$

$= r^5\left(\frac{1}{4}\sin 3\alpha + \frac{1}{4}\sin\alpha\right) - r^5\sin^3\alpha + r^5\sin^5\alpha$

$\frac{1}{16}\sin 5\alpha - \frac{15}{16}\sin 3\alpha + \frac{10}{16}\sin\alpha$

$-\frac{4}{16}\sin 3\alpha + \frac{12}{16}\sin\alpha$

$\sin^4\alpha = \sin^2\alpha(1-\cos = r^5\left(\frac{1}{4}\sin 3\alpha + \frac{1}{4}\sin\alpha + \frac{1}{4}\sin 3\alpha - \frac{3}{4}\sin\alpha + \frac{1}{16}\sin 5\alpha - \frac{5}{16}\sin 3\alpha + \frac{10}{16}\sin\alpha\right.$

$xy^4 = \cos\alpha\sin^4\alpha = \cos^3\alpha\sin^2\alpha - \sin^2\alpha\cos^2\alpha$

$\frac{1}{16}\cos 5\alpha + \frac{5}{16}\cos 3\alpha + \frac{10}{16}\cos\alpha$

$+\frac{4}{16}\cos\alpha + \frac{12}{16}\cos\alpha$

$= r^5\left(-\frac{1}{4}\cos 3\alpha + \frac{1}{4}\cos\alpha\right) - r^5\cos^3\alpha + r^5\cos^5\alpha$

$-\frac{1}{4}\cos 3\alpha + \frac{1}{4}\cos\alpha - \frac{1}{4}\cos 3\alpha - \frac{3}{4}\cos\alpha + \frac{1}{16}\cos 5\alpha + \frac{5}{16}\cos 3\alpha + \frac{10}{16}\cos\alpha$

$$f(x,y) = f(0,y) + \frac{\partial}{\partial x} f(0,y)\frac{x}{1} + \frac{\partial^2}{\partial x^2} f(0,y)\frac{x^2}{1.2} + \cdots$$

$$f(0) + \left(\frac{\partial f}{\partial y}\right)_0 y + \left(\frac{\partial^2 f}{\partial y^2}\right)\frac{y^2}{2!} + \left(\frac{\partial^3 f}{\partial y^3}\right)\frac{y^3}{3!} + \left(\frac{\partial^4 f}{\partial y^4}\right)_0\frac{y^4}{4!} + \left(\frac{\partial^5 f}{\partial y^5}\right)\frac{y^5}{5!} + \cdots$$

$$+ \left(\frac{\partial f}{\partial x}\right)_0 \frac{x\cancel{y}}{1} + \left(\frac{\partial^2 f}{\partial x \partial y}\right)_0 \frac{x}{1}\frac{y}{1} + \left(\frac{\partial^3 f}{\partial x \partial y^2}\right)\frac{x}{1}\frac{y^2}{2!} + \left(\frac{\partial^4 f}{\partial x \partial y^3}\right)\frac{x}{1}\frac{y^3}{3!} + \left(\frac{\partial^5 f}{\partial x \partial y^4}\right)\frac{x}{1}\frac{y^4}{4!}$$

$$+ \left(\frac{\partial^2 f}{\partial x^2}\right)_0 \frac{x^2}{2!} + \left(\frac{\partial^3 f}{\partial x^2 \partial y}\right)_0 \frac{x^2}{2!}\frac{y}{1} + \left(\frac{\partial^4 f}{\partial x^2 \partial y^2}\right)\frac{x^2}{2!}\frac{y^2}{2!} + \left(\frac{\partial^5 f}{\partial x^2 \partial y^3}\right)\frac{x^2}{2!}\frac{y^3}{3!}$$

$$+ \left(\frac{\partial^3 f}{\partial x^3}\right)_0 \frac{x^3}{3!} + \left(\frac{\partial^4 f}{\partial x^3 \partial y}\right)\frac{x^3}{3!}\frac{y}{1} + \left(\frac{\partial^5 f}{\partial x^3 \partial y^2}\right)_0 \frac{x^3}{3!}\frac{y^2}{1.2}$$

$$+ \left(\frac{\partial^4 f}{\partial x^4}\right)_0 \frac{x^4}{4!} + \left(\frac{\partial^5 f}{\partial x^4 \partial y}\right)\frac{x^4}{4!}\cdot\frac{y}{1}$$

$$+ \left(\frac{\partial^5 f}{\partial x^5}\right)\frac{x^5}{5!}$$

$$f_0 + \left(\frac{\partial f}{\partial x}\right)_0 \frac{x}{1} + \left(\frac{\partial f}{\partial y}\right)_0 \frac{y}{1}$$

$$\frac{\partial f}{\partial x} = \left(\frac{\partial f}{\partial x}\right)_0 + \left(\frac{\partial^2 f}{\partial x \partial y}\right)_0 \frac{y}{1} + \left(\frac{\partial^3 f}{\partial x \partial y^2}\right)_0 \frac{y^2}{1.2} + \left(\frac{\partial^4 f}{\partial x \partial y^3}\right)\frac{y^3}{1.2.3} + \left(\frac{\partial^5 f}{\partial x \partial y^4}\right)\frac{y^4}{1.2.3.4} + \cdots$$

$$+ \left(\frac{\partial^2 f}{\partial x^2}\right)_0 \frac{x}{1} + \left(\frac{\partial^3 f}{\partial x^2 \partial y}\right)_0 \frac{xy}{1} + \frac{\partial^4 f}{\partial x^2 \partial y^2}\frac{xy^2}{2!} + \left(\frac{\partial^5 f}{\partial x^2 \partial y^3}\right)_0 x\frac{y^3}{3!} + \cdots$$

$$+ \left(\frac{\partial^3 f}{\partial x^3}\right)\frac{x^2}{2!} + \left(\frac{\partial^4 f}{\partial x^3 \partial y}\right)_0 \frac{x^2}{2!}\frac{y}{1} + \left(\frac{\partial^5 f}{\partial x^3 \partial y^2}\right)\frac{x^2}{2!}\frac{y^2}{2!} + \cdots$$

$$+ \left(\frac{\partial^4 f}{\partial x^4}\right)_0 \frac{x^3}{3!} + \left(\frac{\partial^5 f}{\partial x^4 \partial y}\right)\frac{x^3}{3!}\frac{y}{1}$$

$$+ \left(\frac{\partial^5 f}{\partial x^5}\right)\frac{x^4}{4!}$$

$$-\sin^4\alpha + 3\sin^2\alpha\cos^2\alpha$$

$$\sin^2\alpha\,(-\sin^2\alpha + 3\cos^2\alpha)$$

$$\sin^2\alpha\,(\cos 2\alpha + 2\cos^2\alpha)$$

$$\mathcal{F} = x\frac{\partial V}{\partial y} - y\frac{\partial V}{\partial x} \qquad x = r\cos\alpha \qquad y = r\sin\alpha$$

$$y\frac{\partial V}{\partial x} = \left(\frac{\partial V}{\partial x}\right)_0 y + \left(\frac{\partial^2 V}{\partial x_0^2}\right) xy + \left(\frac{\partial^2 V}{\partial x\partial y}\right)_0 y^2 + \frac{1}{2!}\left\{\left(\frac{\partial^3 V}{\partial x^3}\right)_0 x^2 y + 2\left(\frac{\partial^3 V}{\partial x^2\partial y}\right) xy^2 + \left(\frac{\partial^3 V}{\partial x\partial y^2}\right)_0 y^3\right\}$$

$$+\frac{1}{3!}\left\{\left(\frac{\partial^4 V}{\partial x^4}\right) x^3 y + 3\left(\frac{\partial^4 V}{\partial x^3\partial y}\right)_0 x^2y^2 + 3\left(\frac{\partial^4 V}{\partial x^2\partial y^2}\right)_0 xy^3 + \left(\frac{\partial^4 V}{\partial x\partial y^3}\right)_0 y^4\right\}$$

$$x\frac{\partial V}{\partial y} = \left(\frac{\partial V}{\partial y}\right)_0 x + \left(\frac{\partial^2 V}{\partial x\partial y}\right)_0 x^2 + \left(\frac{\partial^2 V}{\partial y^2}\right)_0 yx + \frac{1}{2!}\left\{\left(\frac{\partial^3 V}{\partial x^2\partial y}\right)_0 x^3 + 2\left(\frac{\partial^3 V}{\partial x\partial y^2}\right)_0 x^2 y + \left(\frac{\partial^3 V}{\partial y^3}\right) y^2 x\right\}$$

$$+\frac{1}{3!}\left\{\left(\frac{\partial^4 V}{\partial x^3\partial y}\right) x^4 + 3\left(\frac{\partial^4 V}{\partial x^2\partial y^2}\right)_0 x^3 y + 3\left(\frac{\partial^4 V}{\partial x\partial y^3}\right) x^2y^2 + \left(\frac{\partial^4 V}{\partial y^4}\right)_0 y^3 x\right\}$$

$$\mathcal{F} = \left(\frac{\partial V}{\partial y}\right)_0 r\cos\alpha - \left(\frac{\partial V}{\partial x}\right)_0 r\sin\alpha + \left(\left(\frac{\partial^2 V}{\partial y^2}\right)_0 - \left(\frac{\partial^2 V}{\partial x^2}\right)_0\right) r^2 \frac{1}{2}\sin 2\alpha + \left(\frac{\partial^2 V}{\partial x\partial y}\right)_0 r^2\cos 2\alpha$$

$$x^2 y = r^3\cos^2\alpha\sin\alpha = r^3\sin\alpha - r^3\sin^3\alpha = r^3\sin\alpha - \frac{3}{4}r^3\sin\alpha + \frac{1}{4}r^3\sin 3\alpha$$
$$= \frac{1}{4}r^3\sin\alpha + \frac{1}{4}r^3\sin 3\alpha.$$

$$xy^2 = r^3\cos\alpha\sin^2\alpha = r^3\cos\alpha - r^3\cos^3\alpha = r^3\cos\alpha - \frac{3}{4}r^3\cos\alpha - \frac{1}{4}r^3\cos 3\alpha$$
$$= \frac{1}{4}r^3\cos\alpha - \frac{1}{4}r^3\cos 3\alpha$$

$$y^3 = r^3\sin^3\alpha = r^3\frac{3}{4}\sin\alpha - r^3\frac{1}{4}\sin 3\alpha$$

$$x^3 = r^3\cos^3\alpha = r^3\frac{3}{4}\cos\alpha + \frac{1}{4}r^3\cos 3\alpha$$

$$x^3 y = r^4\cos^3\alpha\sin\alpha = r^4\frac{3}{4}\cos\alpha\sin\alpha + r^4\cos 3\alpha\sin\alpha = \frac{r^4}{2}\cos^2\alpha\sin 2\alpha = \frac{r^4}{4}(1+\cos 2\alpha)\sin 2\alpha = \frac{r^4}{4}\sin 2\alpha + \frac{r^4}{8}\sin 4\alpha$$

$$x^2y^2 = r^4\sin^2\alpha\cos^2\alpha = r^4 - r^4\cos^4\alpha = r^4 - \frac{3}{8}r^4 - \frac{r^4}{2}\cos 2\alpha - \frac{1}{8}r^4\cos 4\alpha = \frac{5}{8}r^4 - \frac{1}{2}r^4\cos 2\alpha - \frac{1}{8}r^4\cos 4\alpha$$

$$xy^3 = r^4\cos\alpha\sin^3\alpha = r^4\frac{3}{4}\cos\alpha\sin\alpha - \frac{1}{4}r^4\sin 3\alpha\cos\alpha = \frac{r^4}{2}\sin^2\alpha\sin 2\alpha = \frac{r^4}{4}(1-\cos 2\alpha)\sin 2\alpha = \frac{r^4}{4}\sin 2\alpha - \frac{r^4}{8}\sin 4\alpha$$

$$y^4 = r^4\sin^4\alpha = \frac{3}{8}r^4 - \frac{1}{2}r^4\cos 2\alpha + \frac{1}{8}r^4\cos 4\alpha$$

$$x^4 = r^4\cos^4\alpha = \frac{3}{8}r^4 + \frac{1}{2}r^4\cos 2\alpha + \frac{1}{8}r^4\cos 4\alpha$$

$$\frac{1}{f_6}\frac{\partial^6 V}{\partial x^6} = -\frac{2bc}{N^2 M^{\frac{3}{2}}}\left[126a^5 + 110a^3(b^2+c^2) + a(21(b^2+c^2)^2 - 6b^2c^2)\right]$$ jó

$$+\frac{4bc}{N^3 M^{\frac{3}{2}}}\left[204a^9 + 376a^7(b^2+c^2) + a^5(247(b^2+c^2)^2 - 36b^2c^2) + a^3(b^2+c^2)(69(b^2+c^2)^2 - 44b^2c^2) + a(b^2+c^2)^2(6(b^2+c^2)^2 - 9b^2c^2)\right]$$ jó

$$+\frac{3bc}{N^2 M^{\frac{5}{2}}}\left[90a^7 + 121a^5(b^2+c^2) + 7a^3(7(b^2+c^2)^2 - 2b^2c^2) + 3a(b^2+c^2)(2(b^2+c^2)^2 - 3b^2c^2)\right]$$ jó

$$-\frac{8bc}{N^4 M^{\frac{3}{2}}}\left[72a^{13} + 204a^{11}(b^2+c^2) + a^9(234(b^2+c^2)^2 - 24b^2c^2) + a^7(b^2+c^2)(141(b^2+c^2)^2 - 60b^2c^2) + a^5(b^2+c^2)^2(45(b^2+c^2)^2 - 42b^2c^2) + a^3(b^2+c^2)^3(6(b^2+c^2)^2 - 9b^2c^2)\right]$$ jó

$$-\frac{6bc}{N^3 M^{\frac{5}{2}}}\left[48a^{11} + 112a^9(b^2+c^2) + a^7(100(b^2+c^2)^2 - 16b^2c^2) \cancel{+a^5(b^2+c^2)(44(b^2+c^2)^2 - 32b^2c^2)} + a^3(b^2+c^2)^2(8(b^2+c^2)^2 - 9b^2c^2)\right]$$ jó

$$a^5(b^2+c^2)(44(b^2+c^2)^2 - 32b^2c^2)$$

$$-\frac{15bc}{N^2 M^{\frac{7}{2}}}\left[6a^9 + 11a^7(b^2+c^2) + a^5(7(b^2+c^2)^2 - 2b^2c^2) + a^3(b^2+c^2)(2(b^2+c^2)^2 - 3b^2c^2)\right]$$ jó

$$\frac{1}{\gamma\sigma}\frac{\partial^6 V}{\partial x^6} = -\frac{abc}{N^2 M^{\frac{3}{2}}}\left[252a^4 + 220a^2(b^2+c^2) + 42(b^2+c^2)^2 - 12b^2c^2\right]$$ 0 jó

$$+\frac{4abc}{N^3 M^{\frac{3}{2}}}\left[120a^8 + 224a^6(b^2+c^2) + a^4\left(150(b^2+c^2)^2 - 24b^2c^2\right) + a^2(b^2+c^2)\left(44(b^2+c^2)^2 - 32b^2c^2\right) + (b^2+c^2)^2\left(4(b^2+c^2)^2 - 6b^2c^2\right)\right]$$ jó

$$+\frac{3abc}{N^2 M^{\frac{5}{2}}}\left[48a^6 + 66a^4(b^2+c^2) + 4a^2\left(7(b^2+c^2)^2 - 2b^2c^2\right) + 2(b^2+c^2)\left(2(b^2+c^2)^2 - 3b^2c^2\right)\right]$$ jó

$$+\frac{abc}{N^3 M^{\frac{3}{2}}}\left[336a^8 + 608a^6(b^2+c^2) + a^4\left(388(b^2+c^2)^2 - 48b^2c^2\right) + a^2(b^2+c^2)\left(100(b^2+c^2)^2 - 48b^2c^2\right) + 4(b^2+c^2)^2\left[2(b^2+c^2)^2 - 3b^2c^2\right]\right]$$ jó

$$+\frac{abc}{N^2 M^{\frac{5}{2}}}\left[126a^6 + 165a^4(b^2+c^2) + 9a^2\left(7(b^2+c^2)^2 - 2b^2c^2\right) + 3(b^2+c^2)\left(2(b^2+c^2)^2 - 3b^2c^2\right)\right]$$ jó

$$-\frac{abc}{N^4 M^{\frac{3}{2}}}\left[576a^{12} + 1632a^{10}(b^2+c^2) + a^8\left(1872(b^2+c^2)^2 - 192b^2c^2\right) + a^6(b^2+c^2)\left(1128(b^2+c^2)^2 - 480b^2c^2\right) + a^4(b^2+c^2)^2\left(360(b^2+c^2)^2 - 336b^2c^2\right) + a^2(b^2+c^2)^3\left(48(b^2+c^2)^2 - 72b^2c^2\right)\right]$$ jó

$$-\frac{abc}{N^3 M^{\frac{5}{2}}}\left[144a^{10} + 336a^8(b^2+c^2) + a^6\left(300(b^2+c^2)^2 - 48b^2c^2\right) + a^4(b^2+c^2)\left(132(b^2+c^2)^2 - 96b^2c^2\right) + a^2(b^2+c^2)^2\left(24(b^2+c^2)^2 - 36b^2c^2\right)\right]$$ jó

$$-\frac{abc}{N^3 M^{\frac{5}{2}}}\left[144a^{10} + 336a^8(b^2+c^2) + a^6\left(300(b^2+c^2)^2 - 48b^2c^2\right) + a^4(b^2+c^2)\left(132(b^2+c^2)^2 - 96b^2c^2\right) + a^2(b^2+c^2)^2\left(24(b^2+c^2)^2 - 36b^2c^2\right)\right]$$ jó

$$-\frac{15abc}{N^2 M^{\frac{7}{2}}}\left[6a^8 + 11a^6(b^2+c^2) + a^4\left(7(b^2+c^2)^2 - 2b^2c^2\right) + a^2(b^2+c^2)\left(2(b^2+c^2)^2 - 3b^2c^2\right)\right]$$

$$\frac{1}{2}\frac{1}{f\sigma}\frac{\partial^5 u}{\partial x^4 \partial y} = \Bigg[-\frac{ac}{N^2 M^{\frac{3}{2}}}\left[14b^6+5b^4(7a^2+3c^2)+3b^2(11a^4+12a^2c^2+3c^4)+(6a^6+11a^4c^2+7a^2c^4+2c^6)\right]$$

$$+\frac{4ac(a^2+c^2)}{N^3 M^{\frac{3}{2}}}\left[2b^8+b^6(7a^2+3c^2)+b^4(11a^4+12a^2c^2+3c^4)+b^2(6a^6+11a^4c^2+7a^2c^4+2c^6)\right]$$

jó

$$+\frac{3ac}{N^2 M^{\frac{5}{2}}}\left[2b^8+b^6(7a^2+3c^2)+b^4(11a^4+12a^2c^2+3c^4)+b^2(6a^6+11a^4c^2+7a^2c^4+2c^6)\right]\Bigg]_{\substack{a+x\\b+y}} - \Big[\quad\Big]_{\substack{a+x\\b-y}} + \Big[\quad\Big]_{\substack{a-x\\b+y}} - \Big[\quad\Big]_{\substack{a-x\\b-y}}$$

$$\frac{1}{2}\frac{1}{f\sigma}\frac{\partial^6 u}{\partial x^4 \partial y^2} = \Bigg[-\frac{abc}{N^2 M^{\frac{3}{2}}}\left[84b^4+20b^2(7a^2+3c^2)+(66a^4+72a^2c^2+18c^4)\right]$$

$$+\frac{abc}{N^3 M^{\frac{5}{2}}}\left[4(a^2+c^2)M+3N\right]\left\{30b^6+11b^4(7a^2+3c^2)+7b^2(11a^4+12a^2c^2+3c^4)+(18a^6+33a^4c^2+21a^2c^4+6c^6)\right\}$$

jó

$$-\frac{4abc(a^2+c^2)}{N^4 M^{\frac{5}{2}}}\left[2b^8+b^6(7a^2+3c^2)+b^4(11a^4+12a^2c^2+3c^4)+b^2(6a^6+11a^4c^2+7a^2c^4+2c^6)\right]\left(6(a^2+c^2)M+3N\right)$$

$$-\frac{3abc}{N^3 M^{\frac{7}{2}}}\left[2b^8+b^6(7a^2+3c^2)+b^4(11a^4+12a^2c^2+3c^4)+b^2(6a^6+11a^4c^2+7a^2c^4+2c^6)\right]\left(4(a^2+c^2)M+5N\right)\Bigg]_{\substack{a+x\\b+y}}$$

$$+\Big[\quad\Big]_{\substack{a+x\\b-y}} + \Big[\quad\Big]_{\substack{a-x\\b+y}} + \Big[\quad\Big]_{\substack{a-x\\b-y}}$$

$$\frac{1}{2}\frac{1}{f\sigma}\frac{\partial^3 u}{\partial x^3} = + \frac{(b-y)c}{[(a+x)^2+(b-y)^2][(a+x)^2+c^2]}\,\frac{2(a+x)^2+(b-y)^2+c^2}{\sqrt{(a+x)^2+(b-y)^2+c^2}} - \frac{(b-y)c}{[(a-x)^2+(b-y)^2][(a-x)^2+c^2]}\,\frac{2(a-x)^2+(b-y)^2+c^2}{\sqrt{(a-x)^2+(b-y)^2+c^2}} +$$

$$+ \frac{(b+y)c}{[(a+x)^2+(b+y)^2][(a+x)^2+c^2]}\,\frac{2(a+x)^2+(b+y)^2+c^2}{\sqrt{(a+x)^2+(b+y)^2+c^2}} - \frac{(b+y)c}{[(a-x)^2+(b+y)^2][(a-x)^2+c^2]}\,\frac{2(a-x)^2+(b+y)^2+c^2}{\sqrt{(a-x)^2+(b+y)^2+c^2}}\,.$$

$$\frac{1}{2}\frac{1}{f\sigma}\frac{\partial^4 u}{\partial x^4} = - \frac{2(a+x)(b-y)c}{[(a+x)^2+(b-y)^2]^2[(a+x)^2+c^2]^2}\,\frac{[2(a+x)^2+(b-y)^2+c^2]^2}{\sqrt{(a+x)^2+(b-y)^2+c^2}} + \frac{(a+x)(b-y)c}{[(a+x)^2+(b-y)^2][(a+x)^2+c^2]}\,\frac{2(a+x)^2+3(b-y)^2+3c^2}{[(a+x)^2+(b-y)^2+c^2]^{\frac{3}{2}}} +$$

$$- \frac{2(a-x)(b-y)c}{[(a-x)^2+(b-y)^2]^2[(a-x)^2+c^2]^2}\,\frac{[2(a-x)^2+(b-y)^2+c^2]^2}{\sqrt{(a-x)^2+(b-y)^2+c^2}} + \frac{(a-x)(b-y)c}{[(a-x)^2+(b-y)^2][(a-x)^2+c^2]}\,\frac{2(a-x)^2+3(b-y)^2+3c^2}{[(a-x)^2+(b-y)^2+c^2]^{\frac{3}{2}}} +$$

$$- \frac{2(a+x)(b+y)c}{[(a+x)^2+(b+y)^2]^2[(a+x)^2+c^2]^2}\,\frac{[2(a+x)^2+(b+y)^2+c^2]^2}{\sqrt{(a+x)^2+(b+y)^2+c^2}} + \frac{(a+x)(b+y)c}{[(a+x)^2+(b+y)^2][(a+x)^2+c^2]}\,\frac{2(a+x)^2+3(b+y)^2+3c^2}{[(a+x)^2+(b+y)^2+c^2]^{\frac{3}{2}}} +$$

$$- \frac{2(a-x)(b+y)c}{[(a-x)^2+(b+y)^2]^2[(a-x)^2+c^2]^2}\,\frac{[2(a-x)^2+(b+y)^2+c^2]^2}{\sqrt{(a-x)^2+(b+y)^2+c^2}} + \frac{(a-x)(b+y)c}{[(a-x)^2+(b+y)^2][(a-x)^2+c^2]}\,\frac{2(a-x)^2+3(b+y)^2+3c^2}{[(a-x)^2+(b+y)^2+c^2]^{\frac{3}{2}}}\,.$$

vzj: $\frac{1}{2}\frac{1}{f\sigma}\frac{\partial^4 u}{\partial x^4} = \left[-\frac{2abc(2a^2+b^2+c^2)^2}{N^2\sqrt{M}} + \frac{abc(2a^2+3b^2+3c^2)}{N M^{\frac{3}{2}}}\right]_{\substack{a+x\\ b-y}} + \Big[\quad\Big]_{\substack{a-x\\ b-y}} + \Big[\quad\Big]_{\substack{a+x\\ b+y}} + \Big[\quad\Big]_{\substack{a-x\\ b+y}}$

$N = a^4 + a^2(b^2+c^2) + b^2c^2 = (a^2+b^2)(a^2+c^2)$ $\qquad M = a^2+b^2+c^2$

$$N = a^4 + a^2(b^2+c^2) + b^2c^2 = (a^2+b^2)(a^2+c^2) \qquad M = a^2+b^2+c^2$$

+) [illegible]

$$\frac{1}{2}\frac{1}{\sqrt{\sigma}}\frac{\partial^5 U}{\partial x^5} = \Bigg[\frac{1}{a}\frac{1}{2}\frac{1}{\sqrt{\sigma}}\left[\frac{\partial^4 U}{\partial x^4}\right]_{\substack{a+x\\b+y}} + \frac{8bc}{N^3\sqrt{M}}\left(8a^8+12a^6(b^2+c^2)+6a^4(b^2+c^2)^2+a^2(b^2+c^2)^3\right) - \frac{3bc}{NM^{\frac{5}{2}}}\left(2a^4+3a^2(b^2+c^2)\right) - \frac{4bc}{N^2M^{\frac{3}{2}}}\left(7a^6+13a^4(b^2+c^2)+a^2\left(5(b^2+c^2)^2-\cancel{4}\,b^2c^2\right)\right)\Bigg]_{\substack{a+x\\b-y}}^{+)}$$

$$-\Big[\quad\Big]_{\substack{a-x\\b-y}} + \Big[\quad\Big]_{\substack{a+x\\b+y}} - \Big[\quad\Big]_{\substack{a-x\\b+y}}$$

$$\Big]_{\substack{a-x\\b-y}} \qquad \Big]_{\substack{a+x\\b+y}}$$

$$\begin{aligned}
\frac{1}{2}\frac{1}{\sqrt{\sigma}}\frac{\partial^6 U}{\partial x^6} = \Bigg[&-\frac{1}{a^2}\frac{1}{2}\frac{1}{\sqrt{\sigma}}\left[\frac{\partial^4 U}{\partial x^4}\right]_{\substack{a+x\\b+y}} + \frac{1}{a}\frac{1}{2}\frac{1}{\sqrt{\sigma}}\left[\frac{\partial^5 U}{\partial x^5}\right]_{\substack{a+x\\b+y}} + \frac{8bc}{N^3\sqrt{M}}\left(64a^7+72a^5(b^2+c^2)+24a^3(b^2+c^2)^2+2a(b^2+c^2)^3\right) \\
&- \frac{8bc}{N^4M^{\frac{3}{2}}}\left(8a^8+12a^6(b^2+c^2)+6a^4(b^2+c^2)^2+a^2(b^2+c^2)^3\right)\left(3M\frac{\partial N}{\partial x}^{+)} + \frac{1}{2}N\frac{\partial M}{\partial x}^{+)}\right) \\
&- \frac{3bc}{NM^{\frac{5}{2}}}\left(8a^3+6a(b^2+c^2)\right) + \frac{3bc}{N^2M^{7/2}}\left(2a^4+3a^2(b^2+c^2)\right)\left(M\frac{\partial N}{\partial x}^{+)} + \frac{5}{2}N\frac{\partial M}{\partial x}^{+)}\right) \\
&- \frac{4bc}{N^2M^{\frac{3}{2}}}\left(42a^5+52a^3(b^2+c^2)+2a\left(5(b^2+c^2)^2-\cancel{4}\,b^2c^2\right)\right) \\
&+ \frac{4bc}{N^3M^{\frac{5}{2}}}\left(7a^6+13a^4(b^2+c^2)+a^2\left(5(b^2+c^2)^2-\cancel{4}\,b^2c^2\right)\right)\left(2M\frac{\partial N}{\partial x}^{+)} + \frac{3}{2}N\frac{\partial M}{\partial x}^{+)}\right)\Bigg]_{\substack{a+x\\b-y}}
\end{aligned}$$

$$\Big]_{\substack{a-x\\b+y}}$$

+) [illegible]

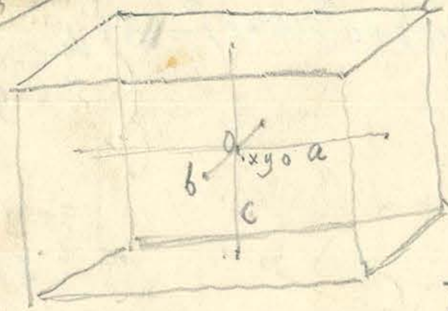
$$\frac{\partial N}{\partial x} = 4a^3 + 2a(b^2+c^2)$$

$$\frac{\partial M}{\partial x} = 2a$$

$$+\Big[\quad\Big]_{\substack{a-x\\b-y}} + \Big[\quad\Big]_{\substack{a+x\\b+y}} + \Big[\quad\Big]_{\substack{a-x\\b+y}}.$$

Paralelepiped 1915

Az egész paralelepipedre vonatkozó Potentiál = U



$$\frac{1}{2}\frac{1}{f\sigma}\frac{\partial U}{\partial x} = \left[-a \operatorname{arctg}\frac{bc}{a\sqrt{a^2+b^2+c^2}} - b \log\frac{\sqrt{a^2+b^2}\,(c+\sqrt{c^2+b^2})}{b\,(c+\sqrt{a^2+b^2+c^2})} - c \log\frac{\sqrt{a^2+c^2}}{c}\cdot\frac{(b+\sqrt{b^2+c^2})}{b+\sqrt{a^2+b^2+c^2}}\right]_{\substack{a+x\\b+y}} - \Big[\ \Big]_{\substack{a-x\\b+y}} + \Big[\ \Big]_{\substack{a+x\\b-y}} - \Big[\ \Big]_{\substack{a-x\\b-y}}$$

$$\frac{1}{2}\frac{1}{f\sigma}\frac{\partial U}{\partial x^2} = \left[-\operatorname{arctg}\frac{bc}{a\sqrt{a^2+b^2+c^2}}\right]_{\substack{a+x\\b+y}} + \Big[\ \Big]_{\substack{a-x\\b+y}} + \Big[\ \Big]_{\substack{a+x\\b-y}} + \Big[\ \Big]_{\substack{a-x\\b-y}}$$

$$\frac{1}{2}\frac{1}{f\sigma}\frac{\partial^3 U}{\partial x^2\partial y} = \left[-\frac{ac(a^2+c^2)}{N\sqrt{M}}\right]_{\substack{a+x\\b+y}} + \Big[\ \Big]_{\substack{a-x\\b+y}} - \Big[\ \Big]_{\substack{a+x\\b-y}} - \Big[\ \Big]_{\substack{a-x\\b-y}}$$

$$\frac{1}{2}\frac{1}{f\sigma}\frac{\partial^4 U}{\partial x^2\partial y^2} = \left[+\frac{abc(a^2+c^2)^2}{N^2M^{\frac{3}{2}}}(3b^2+3a^2+2c^2)\right]_{\substack{a+x\\b+y}} + \Big[\ \Big]_{\substack{a-x\\b+y}} + \Big[\ \Big]_{\substack{a+x\\b-y}} + \Big[\ \Big]_{\substack{a-x\\b-y}}$$

vagy $$\frac{1}{2}\frac{1}{f\sigma}\frac{\partial^4 U}{\partial x^2\partial y^2} = \left[\frac{bc}{N^2M^{\frac{3}{2}}}\left(3a^7 + a^5(8c^2+3b^2) + a^3(7c^4+6c^2b^2) + a(2c^6+3b^2c^4)\right)\right]_{\substack{a+x\\b+y}} + \Big|\ \Big|_{\substack{a+x\\b-y}} + \Big|\ \Big|_{\substack{a-x\\b+y}} + \Big|\ \Big|_{\substack{a-x\\b-y}}$$

$$\frac{1}{2}\frac{1}{f\sigma}\frac{\partial^5 U}{\partial x^3\partial y^2} = \left[\begin{aligned}&\frac{bc}{N^2M^{\frac{3}{2}}}\left(21a^6+5a^4(8c^2+3b^2)+3a^2(7c^4+6b^2c^2)+(2c^6+3b^2c^4)\right)\\&-\frac{3bc}{N^2M^{\frac{5}{2}}}\left(3a^8+a^6(8c^2+3b^2)+a^4(7c^4+6c^2b^2)+a^2(2c^6+3b^2c^4)\right)\\&-\frac{4bc}{N^3M^{\frac{3}{2}}}\left(6a^{10}+a^8(9b^2+19c^2)+a^6(3b^4+23b^2c^2+22c^4)+a^4(6b^4c^2+19b^2c^4+11c^6)+a^2(3b^4c^4+5b^2c^6+2c^8)\right)\end{aligned}\right]_{\substack{a+x\\b+y}}$$ jó

$$- \Big[\ \Big]_{\substack{a-x\\b+y}} + \Big[\ \Big]_{\substack{a+x\\b-y}} - \Big[\ \Big]_{\substack{a-x\\b-y}}$$

$a = 2.\varrho \quad b = 1.\varrho \quad c = 2.\varrho$

Ad Pendelungsprei 1915

$\frac{1}{f\sigma}\frac{\partial^2 V}{\partial x^2} = -1{,}410574 \qquad \frac{1}{f\sigma}\frac{\partial^2 V}{\partial y^2} = -8{,}105581 \qquad \frac{1}{f\sigma}\left(\frac{\partial^2 V}{\partial x^2} - \frac{\partial^2 U}{\partial y^2}\right) = +6{,}695007$

$\frac{1}{f\sigma}\frac{\partial^4 V}{\partial x^4} = -0{,}570504\frac{1}{\varrho^2} \qquad \frac{1}{f\sigma}\frac{\partial^4 V}{\partial y^4} = -1{,}557752\frac{1}{\varrho^2} \qquad \frac{1}{f\sigma}\left(\frac{\partial^4 V}{\partial x^4} - \frac{\partial^4 V}{\partial y^4}\right) = +0{,}987248\frac{1}{\varrho^2}$

$\frac{1}{f\sigma}\frac{\partial^4 V}{\partial x^2\partial y^2} = +0{,}034820\frac{1}{\varrho^2}$

$-0{,}578288$ régi

$\frac{1}{f\sigma}\frac{\partial^6 V}{\partial x^6} = -0{,}589440\frac{1}{\varrho^4} \qquad \frac{1}{f\sigma}\frac{\partial^6 V}{\partial y^6} = +2{,}172792\frac{1}{\varrho^4} \qquad \frac{1}{f\sigma}\left(\frac{\partial^6 V}{\partial x^6} - \frac{\partial^6 V}{\partial y^6}\right) = -2{,}762232$

új

$\frac{1}{f\sigma}\frac{\partial^6 V}{\partial x^4\partial y^2} = \cancel{+0{,}544656}\ +444520 \qquad \frac{1}{f\sigma}\frac{\partial^6 V}{\partial x^2\partial y^4} = \cancel{-0{,}474424}\ \cancel{-0{,}375504}\ -0{,}374424 \mid -0{,}374392$

$\frac{1}{2}\frac{1}{f\sigma}\left[\frac{\partial^5 V}{\partial x^5}\right]_{\substack{a+x\\b+y}} = +0{,}067138$

$\frac{1}{f\sigma}\cdot\frac{\partial^4 V}{\partial x^3\partial y} = +0{,}616992 \qquad \frac{1}{f\sigma}\frac{\partial^4 V}{\partial x\partial y^3} = -1{,}136240$

$$F = -\sin 2\alpha\, f\sigma\left(3{,}3475035\, r^2 + 0{,}0411353\frac{1}{\varrho^2}r^4 - 0{,}0025304\frac{1}{\varrho^4}r^6 + 0{,}0000591\frac{1}{\varrho^6}r^8 + \dots\right)$$

$$-\sin 4\alpha\, f\sigma\left(-0{,}0486912\frac{1}{\varrho^2}r^4 + 0{,}0012841\frac{1}{\varrho^4}r^6 - 0{,}0000591\frac{1}{\varrho^6}r^8 + \dots\right)$$

$$-\sin 6\alpha\, f\sigma\left(-0{,}0039182\frac{1}{\varrho^4}r^6 + 0{,}0000079\frac{1}{\varrho^6}r^8 + \dots\right)$$

$$-\sin 8\alpha\, f\sigma\left(-0{,}0001789\frac{1}{\varrho^6}r^8 + \dots\right)$$

$\frac{1}{f\sigma}\frac{\partial^8 V}{\partial x^8} = -0{,}925688 \qquad \frac{1}{f\sigma}\frac{\partial^8 V}{\partial y^8} = -3{,}413312$

$\frac{1}{f\sigma}\frac{\partial^8 V}{\partial x^6\partial y^2} = +0{,}949640 \qquad \frac{1}{f\sigma}\frac{\partial^8 V}{\partial y^6\partial x^2} = +6{,}832464$

$\frac{1}{f\sigma}\frac{\partial^8 V}{\partial x^4\partial y^4} = -0{,}874296$

$a = 4e \qquad b = 1e \qquad c = 1{,}5e$

$\frac{1}{p\sigma}\frac{\partial^2 u}{\partial x^2} = -0{,}682114$ $\qquad \frac{1}{p\sigma}\frac{\partial^2 u}{\partial y^2} = -7{,}515233$ $\qquad \frac{1}{p\sigma}\left(\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2}\right) = +6{,}833119$

$\frac{1}{p\sigma}\frac{\partial^2 u}{\partial x'^2} = -0{,}502184$ $\qquad \frac{1}{p\sigma}\frac{\partial^2 u}{\partial y'^2} = -2{,}3387552$ $\qquad \frac{1}{p\sigma}\left(\frac{\partial^2 u}{\partial x'^2} - \frac{\partial^2 u}{\partial y'^2}\right) = +1{,}836568$

$a = 3e \qquad b = 1{,}e \qquad c = 3.e.$

$\frac{1}{p\sigma}\frac{\partial^2 u}{\partial x^2} = -1{,}804089$ $\qquad \frac{1}{p\sigma}\frac{\partial^2 u}{\partial y^2} = -8{,}958038$ $\qquad \frac{1}{p\sigma}\left(\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2}\right) = 7{,}153946$

$\frac{1}{p\sigma}\frac{\partial^2 u}{\partial x'^2} = -0{,}567553$ $\qquad \frac{1}{p\sigma}\frac{\partial^2 u}{\partial y'^2} = -0{,}834592$ $\qquad \frac{1}{p\sigma}\left(\frac{\partial^2 u}{\partial x'^2} - \frac{\partial^2 u}{\partial y'^2}\right) = +0{,}267039$

$a=5 \qquad b=1 \qquad c=4$

$\cancel{\frac{1}{\sigma}\frac{\partial^2 u}{\partial x^2} = -0{,}486416} \qquad \cancel{\frac{1}{\sigma}\frac{\partial^2 u}{\partial y^2} = -0{,}621828} \qquad \cancel{\frac{1}{\sigma}\left(\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2}\right) = -0{,}135408}$

$a_1 = 1 \quad b = 1 \quad c = 2$

$$\frac{1}{f\sigma}\frac{\partial^4 V}{\partial x^4} = -4{,}550608$$

$$\frac{1}{f\sigma}\frac{\partial^4 V}{\partial x^2 \partial y^2} = +3{,}870212$$

$$\frac{1}{f\sigma}\frac{\partial^6 V}{\partial x^6} = +0{,}943408$$

$$\frac{1}{f\sigma}\frac{\partial^6 V}{\partial x^4 \partial y^2} = +0{,}985216$$

$$\frac{1}{f\sigma}\frac{\partial^8 V}{\partial x^8} = +121{,}634480$$

$$\frac{1}{f\sigma}\frac{\partial^8 V}{\partial x^6 \partial y^2} = -99{,}156720$$

$$\frac{1}{f\sigma}\frac{\partial^8 V}{\partial x^4 \partial y^4} = +120{,}535280$$

$$\frac{F}{f\sigma} = -\sin 4\alpha\left\{-0{,}6658977\frac{1}{c^2}r^4 - 0{,}0082972\frac{1}{c^4}r^6 - 0{,}0036608\frac{1}{c^6}r^8) \cdots\right\}$$
$$-\sin 8\alpha\left\{+0{,}0220634\frac{1}{c^6}r^8 + \cdots\right\}$$

Oszlopos bemélyedésre mint a fűrészben. $a=3 \quad b=1 \quad c=2.$

$$\frac{F}{f\sigma} = -\sin 2\alpha\left(3{,}347504\cdot r^2 + 0{,}041135\frac{1}{c^2}r^4 - 0{,}002530\frac{1}{c^4}r^6 + 0{,}000059\frac{1}{c^6}r^8 + \cdots\right)$$
$$-\sin 4\alpha\left(+0{,}617207\frac{1}{c^2}r^4 + 0{,}009581\frac{1}{c^4}r^6 + 0{,}003605\frac{1}{c^6}r^8 + \cdots\right)$$
$$-\sin 6\alpha\left(-0{,}003918\frac{1}{c^4}r^6 + 0{,}000008\frac{1}{c^6}r^8 + \right)$$
$$-\sin 8\alpha\left(-0{,}022242\frac{1}{c^6}r^8 + \cdots\right)$$

$$\frac{1}{\sqrt{\sigma}}\frac{\partial^6 v}{\partial x^6}=\frac{1}{\mathcal{N}\mathcal{M}^{\frac{3}{2}}}\left\{-2bc\left[126a^5+110a^3(b^2+c^2)+3a\left(7(b^2+c^2)^2-26b^2c^2\right)\right]\right\},$$

$$+\frac{4bc}{\mathcal{N}^3\mathcal{M}^{\frac{3}{2}}}\left[204a^9+376a^7(b^2+c^2)+a^5\left(247(b^2+c^2)^2-366b^2c^2\right)+a^3(b^2+c^2)\left(69(b^2+c^2)^2-446b^2c^2\right)+3a(b^2+c^2)^2\left(2(b^2+c^2)^2-3b^2c^2\right)\right]$$

$$+\frac{36c}{\mathcal{N}^2\mathcal{M}^{\frac{5}{2}}}\left[90a^7+121a^5(b^2+c^2)+7a^3\left(7(b^2+c^2)^2-26b^2c^2\right)+3a(b^2+c^2)\left(2(b^2+c^2)^2-3b^2c^2\right)\right]$$

$$-\frac{8bc}{\mathcal{N}^4\mathcal{M}^{\frac{3}{2}}}\left[72a^{13}+204a^{11}(b^2+c^2)+6a^9\left(39(b^2+c^2)^2-46b^2c^2\right)+3a^7(b^2+c^2)\left(47(b^2+c^2)^2-20b^2c^2\right)+3a^5(b^2+c^2)^2\left(15(b^2+c^2)^2-14b^2c^2\right)+3a^3(b^2+c^2)^3\left(2(b^2+c^2)^2-3b^2c^2\right)\right]$$

$$-\frac{6bc}{\mathcal{N}^3\mathcal{M}^{\frac{5}{2}}}\left[48a^{11}+112a^9(b^2+c^2)+4a^7\left(25(b^2+c^2)^2-46b^2c^2\right)+a^5\left(44(b^2+c^2)^2-36b^2c^2\right)+4a^3(b^2+c^2)\left(2(b^2+c^2)^2-3b^2c^2\right)\right]$$

$$-\frac{15bc}{\mathcal{N}^2\mathcal{M}^{\frac{7}{2}}}\left[6a^9+11a^7(b^2+c^2)+a^5\left(7(b^2+c^2)^2-26b^2c^2\right)+a^3(b^2+c^2)\left(2(b^2+c^2)^2-3b^2c^2\right)\right]$$

$$\mathcal{B}=\mathcal{N}^n\mathcal{M}^{\frac{m}{2}} \qquad \frac{1}{\mathcal{B}^2}\frac{\partial\mathcal{B}}{\partial x}=\frac{a}{\mathcal{N}^{n+1}\mathcal{M}^{\frac{m}{2}+1}}\left(\mathcal{N}m+\mathcal{M}n(4a^2+2b^2+c^2)\right)$$

$$\frac{1}{\mathcal{B}^2}\frac{\partial^2\mathcal{B}}{\partial x^2}=\frac{1}{\mathcal{N}^{n+2}\mathcal{M}^{\frac{m}{2}+2}}\left[nm\,\mathcal{N}\mathcal{M}\left(8a^4+4a^2(b^2+c^2)\right)+m(m-2)\mathcal{N}^2a^2+m\mathcal{N}^2\mathcal{M}+n(n-1)\mathcal{M}^2\left(4a^3+2a(b^2+c^2)\right)^2+2n\mathcal{N}\mathcal{M}^2\left(6a^2+(b^2+c^2)\right)\right]$$

$$\frac{1}{\mathcal{B}^3}\left(\frac{\partial\mathcal{B}}{\partial x}\right)^2=\frac{a^2}{\mathcal{N}^{n+2}\mathcal{M}^{\frac{m}{2}+2}}\left(\mathcal{N}m+\mathcal{M}n(4a^2+2(b^2+c^2))\right)^2$$

$$U=\frac{A}{B} \qquad \frac{\partial U}{\partial x}=\frac{1}{B}\frac{\partial A}{\partial x}-\frac{A}{B^2}\frac{\partial B}{\partial x}$$

$$\frac{\partial^2 U}{\partial x^2}=+\frac{1}{B}\frac{\partial^2 A}{\partial x^2}-\frac{2}{B^2}\frac{\partial A}{\partial x}\frac{\partial B}{\partial x}-\frac{A}{B^2}\frac{\partial^2 B}{\partial x^2}+2\frac{A}{B^3}\left(\frac{\partial B}{\partial x}\right)^2$$

$$\frac{\partial^2 U}{\partial x^2}=+\frac{1}{\mathcal{N}^n\mathcal{M}^{\frac{m}{2}}}\frac{\partial^2 A}{\partial x^2}-\frac{2a}{\mathcal{N}^{n+1}\mathcal{M}^{\frac{m}{2}+1}}\frac{\partial A}{\partial x}\left(\mathcal{N}m+\mathcal{M}n(4a^2+2(b^2+c^2))\right)-A\frac{1}{\mathcal{N}^{n+2}\mathcal{M}^{\frac{m}{2}+2}}\left[\quad\right]+\frac{2a^2A}{\mathcal{N}^{n+2}\mathcal{M}^{\frac{m}{2}+2}}\left(\quad\right)^2$$

2)

$$\frac{1}{2}\frac{1}{\sqrt{5}}\frac{\partial^4 u}{\partial x^4} = -\frac{bc}{N^2\sqrt{M}}\left[8a^5+8a^3b^2+2ab^4+8a^3c^2+4abc^2+2ac^4\right]+\frac{bc}{NM^{\frac{3}{2}}}(2a^3+(3b^2+3c^2)a)$$

$$\frac{1}{2}\frac{1}{\sqrt{5}}\frac{\partial^4 u}{\partial x^4} = -\frac{bc}{N^2M^{\frac{3}{2}}}\left[+6a^7+11a^5(b^2+c^2)+a^3(7(b^2+c^2)^2-2b^2c^2)+a(2(b^2+c^2)^3-3b^2c^2(b^2+c^2))\right]$$

$$\frac{1}{2}\frac{1}{\sqrt{5}}\frac{\partial^5 u}{\partial x^5} = -\frac{bc}{N^2M^{\frac{3}{2}}}\left[42a^6+55a^4(b^2+c^2)+3a^2(7(b^2+c^2)^2-2b^2c^2)+(b^2+c^2)(2(b^2+c^2)^2-3b^2c^2)\right]$$

$$\cancel{+\frac{bc}{N^3M^{\frac{3}{2}}}\left[6a^7+11a^5(b^2+c^2)+a^3(7(b^2+c^2)^2-2b^2c^2)+a(b^2+c^2)(2(b^2+c^2)^2-3b^2c^2)\right](8a^3+4a(b^2+c^2))}$$

$$+\frac{4bc}{N^3M^{\frac{3}{2}}}\left[12a^{10}+28a^8(b^2+c^2)+a^6(25(b^2+c^2)^2-4b^2c^2)+a^4(b^2+c^2)(11(b^2+c^2)^2-8b^2c^2)+a^2(b^2+c^2)^2(2(b^2+c^2)^2-3b^2c^2)\right]$$

$$+\frac{3bc}{N^2M^{\frac{5}{2}}}\left[6a^8+11a^6(b^2+c^2)+a^4(7(b^2+c^2)^2-2b^2c^2)+a^2(b^2+c^2)(2(b^2+c^2)^2-3b^2c^2)\right]$$

jó!

$$\frac{\partial}{\partial x}\frac{1}{N^2M^{\frac{3}{2}}} = -\frac{1}{N^4M^3}\left[2NM^{\frac{3}{2}}\frac{\partial N}{\partial x}+\frac{3}{2}N^2\sqrt{M}\frac{\partial M}{\partial x}\right]$$

$$\frac{\partial N}{\partial x} = 4a^3+2a(b^2+c^2) \qquad \frac{\partial M}{\partial x} = 2a$$

$$\frac{\partial}{\partial x}\frac{1}{N^2M^{\frac{3}{2}}} = -\frac{4}{N^3M^{\frac{3}{2}}}(8a^3+4a(b^2+c^2)) - \frac{3a}{N^2M^{\frac{5}{2}}}$$

$$\frac{1}{2}\frac{1}{\sqrt{5}}\frac{\partial^6 u}{\partial x^6} = -\frac{abc}{N^2M^{\frac{3}{2}}}\left[252a^4+220a^2(b^2+c^2)+42(b^2+c^2)^2-12b^2c^2\right]$$

$$+\frac{4abc}{N^3M^{\frac{3}{2}}}\left[120a^8+224a^6(b^2+c^2)+a^4(150(b^2+c^2)^2-24b^2c^2)+a^2(b^2+c^2)(44(b^2+c^2)^2-32b^2c^2)+(b^2+c^2)^2(4(b^2+c^2)^2-6b^2c^2)\right]$$

$$+\frac{3abc}{N^2M^{\frac{5}{2}}}\left[48a^6+66a^4(b^2+c^2)+4a^2(7(b^2+c^2)^2-2b^2c^2)+2(b^2+c^2)(2(b^2+c^2)^2-3b^2c^2)\right]$$

$$+\frac{abc}{N^3M^{\frac{5}{2}}}\left[42a^6+55a^4(b^2+c^2)+3a^2(7(b^2+c^2)^2-2b^2c^2)+(b^2+c^2)(2(b^2+c^2)^2-3b^2c^2)\right]\left[(8a^2+4(b^2+c^2))M+3N\right]$$

$$-\frac{4abc}{N^4M^{\frac{5}{2}}}\left[12a^{10}+28a^8(b^2+c^2)+a^6(25(b^2+c^2)^2-4b^2c^2)+a^4(b^2+c^2)(11(b^2+c^2)^2-8b^2c^2)+a^2(b^2+c^2)^2(2(b^2+c^2)^2-3b^2c^2)\right]\left[(12a^2+6(b^2+c^2))M+3N\right]$$

$$-\frac{3abc}{N^3M^{\frac{7}{2}}}\left[6a^8+11a^6(b^2+c^2)+a^4(7(b^2+c^2)^2-2b^2c^2)+a^2(b^2+c^2)(2(b^2+c^2)^2-3b^2c^2)\right]\left[(8a^2+4(b^2+c^2))M+5N\right]$$

jó!

$$N = (a^2+b^2)(a^2+c^2)$$

$$M = a^2+b^2+c^2$$

$$\frac{1}{2}\frac{1}{f\sigma}\frac{\partial^6 y}{\partial x^4 \partial y^2} = \frac{abc}{N^2 M^{\frac{3}{2}}}\left(126a^4 + 20a^2(8c^2+3b^4) + (42c^4+36b^2c^2)\right)$$

$$-\frac{3abc}{N^2 M^{\frac{5}{2}}}\left(24a^6 + 6a^4(8c^2+3b^2) + 4a^2(7c^4+6c^2b^2) + (4c^6+6b^2c^4)\right)$$

$$-\frac{4abc}{N^3 M^{\frac{3}{2}}}\left(60a^8 + 8a^6(9b^2+19c^2) + 6a^4(3b^4+23b^2c^2+22c^4) + 4a^2(6b^4c^2+19b^2c^4+11c^6) + (6b^4c^4+10b^2c^6+4c^8)\right)$$

$$-\frac{abc}{N^3 M^{\frac{5}{2}}}\left[\left(8a^2+4(b^2+c^2)\right)M+3N\right]\left\{21a^6 + 5a^4(8c^2+3b^2) + 3a^2(7c^4+6b^2c^2) + (2c^6+3b^2c^4)\right\}$$

$$+\frac{3abc}{N^3 M^{7/2}}\left[\left(8a^2+4(b^2+c^2)\right)M+5N\right]\left\{3a^8 + a^6(8c^2+3b^2) + a^4(7c^4+6c^2b^2) + a^2(2c^6+3b^2c^4)\right\}$$

$$+\frac{4abc}{N^4 M^{\frac{5}{2}}}\left[\left(12a^2+6(b^2+c^2)\right)M+3N\right]\left\{6a^{10} + a^8(9b^2+19c^2) + a^6(3b^4+23b^2c^2+22c^4) + a^4(6b^4c^2+19b^2c^4+11c^6) + a^2(3b^4c^4+5b^2c^6+2c^8)\right\}$$

Jó

a másika kényelmesebb.

$$bc\left(a^5+2a^3c^2+ac^4\right)\left(\right.$$

$$\frac{bc}{N^2M^{\frac{3}{2}}}\left(3a^7+a^5(8c^2+3b^2)+a^3(7c^4+6c^2b^2)+a(2c^6+3b^2c^4)\right)$$

$$bc\left[\begin{array}{l}3a^7+6a^5c^2+3a^3c^4\\ \quad 2a^5c^2+4a^3c^4+2ac^6\\ \quad 3a^5b^2+6a^3c^2b^2+3ab^2c^4\end{array}\right.$$

$$\frac{\partial}{\partial x}\frac{1}{N^2M^{\frac{3}{2}}}=-\frac{1}{N^4M^3}\left(2NM^{\frac{3}{2}}\frac{\partial N}{\partial x}+N^2\frac{3}{2}\sqrt{M}\frac{\partial M}{\partial x}\right)$$

$$=-\frac{1}{N^3M^{\frac{5}{2}}}\left(2M(4a^3+2a(b^2+c^2))+3Na\right)$$

$$\frac{1}{2}\frac{1}{\sqrt{10}}\frac{\partial^5 U}{\partial x^3\partial y^2}=\frac{bc}{N^2M^{\frac{3}{2}}}\left(21a^6+5a^4(8c^2+3b^2)+3a^2(7c^4+6c^2b^2)+(2c^6+3b^2c^4)\right)$$

$$-\frac{bc}{N^3M^{\frac{5}{2}}}\left(3a^7+a^5(8c^2+3b^2)+a^3(7c^4+6c^2b^2)+a(2c^6+3b^2c^4)\right)\left(2M(4a^3+2a(b^2+c^2))+3Na\right)$$

$$\left(8a^3M+4a(b^2+c^2)M+3aN\right)$$

$$-\frac{8bc}{N^3M^{\frac{5}{2}}}\left(3a^{10}+a^8(8c^2+3b^2)+a^6(7c^4+6c^2b^2)+a^4(2c^6+3b^2c^4)\right)$$

$$-\frac{4bc}{N^3M^{\frac{5}{2}}}\left(\quad+3a^8(b^2+c^2)+a^6(8c^2+3b^2)(b^2+c^2)+a^4(7c^4+6c^2b^2)(b^2+c^2)+a^2(2c^6+3b^2c^4)(b^2+c^2)\right)$$

$$-\frac{3bc}{N^2M^{\frac{5}{2}}}\left(3a^8+a^6(8c^2+3b^2)+a^4(7c^4+6c^2b^2)+a^2(2c^6+3b^2c^4)\right)$$

$$-\frac{4bc}{N^3M^{\frac{5}{2}}}\left(6a^{10}+a^8(9b^2+19c^2)+a^6(3b^4+23b^2c^2+22c^4)+a^4(6b^4c^2+19b^2c^4+11c^6)+a^2(3b^4c^4+5b^2c^6+2c^8)\right)$$

$+12c^2b^2+14c^4$
$+3b^4+8c^2b^2+8c^4$
$+3b^2c^2$
$23 \quad 22$
$\cancel{+6b^2c^4+4c^6}$

$16c^2+6b^2$
$3c^2 \quad 3b^2$

$+6b^2c^4+4c^6$
$+6b^4c^2+7b^2c^4+7c^6$
$+6b^2c^4$
$19 \quad 11$

$+3b^4c^4+2b^2c^6+2c^8$
$+3b^2c^6$

$$\left(\frac{1}{4}\frac{\partial^2 V}{\partial x^2}\right)_{x=0} = -2\,\mathrm{arc\,tg}\,\frac{bc}{a\sqrt{a^2+b^2+c^2}}$$

$$\left(\frac{1}{4}\frac{\partial^3 V}{\partial x^3}\right)_{x=0} = 0$$

$$\left(\frac{1}{4}\frac{\partial^4 V}{\partial x^4}\right)_{x=0} = -\frac{2abc}{\left[b^2c^2+a^2(a^2+b^2+c^2)\right]}\frac{1}{\sqrt{a^2+b^2+c^2}}\left\{2\frac{(2a^2+b^2+c^2)^2}{b^2c^2+a^2(a^2+b^2+c^2)} - \frac{2a^2+3b^2+3c^2}{a^2+b^2+c^2}\right\}$$

$$\left(\frac{1}{4}\frac{\partial^3 V}{\partial x^2\partial y}\right)_{y=0} = 0$$

$$\left(\frac{1}{4}\frac{\partial^4 V}{\partial x^2\partial y^2}\right)_{y=0} = +\frac{2abc\,(a^2+c^2)}{\left[a^2(a^2+b^2+c^2)+b^2c^2\right]^2\left\{a^2+b^2+c^2\right\}^{\frac{3}{2}}}\left(3a^2(a^2+b^2+c^2)+c^2(2a^2+3b^2+2c^2)\right)$$

$$\frac{1}{4}\frac{\partial^2 V}{\partial x^2} = -\operatorname{arc\,tg}\frac{bc}{(a+x)\sqrt{(a+x)^2+b^2+c^2}} - \operatorname{arc\,tg}\frac{bc}{(a-x)\sqrt{(a-x)^2+b^2+c^2}}$$

$$\frac{1}{4}\frac{\partial^3 V}{\partial x^3} = +\frac{bc}{b^2c^2+(a+x)^2\left[(a+x)^2+b^2+c^2\right]}\,\frac{2(a+x)^2+b^2+c^2}{\sqrt{(a+x)^2+b^2+c^2}} - \frac{bc}{b^2c^2+(a-x)^2\left[(a-x)^2+b^2+c^2\right]}\,\frac{2(a-x)^2+b^2+c^2}{\sqrt{(a-x)^2+b^2+c^2}}$$

$$\frac{1}{4}\frac{\partial^4 V}{\partial x^4} = -\frac{2(a+x)bc}{\left[b^2c^2+(a+x)^2\left((a+x)^2+b^2+c^2\right)\right]^2}\,\frac{\left[2(a+x)^2+b^2+c^2\right]^2}{\sqrt{(a+x)^2+b^2+c^2}} + \frac{(a+x)bc}{b^2c^2+(a+x)^2\left((a+x)^2+b^2+c^2\right)}\,\frac{2(a+x)^2+3b^2+3c^2}{\left[(a+x)^2+b^2+c^2\right]^{\frac{3}{2}}}$$

$$-\frac{2(a-x)bc}{\left[b^2c^2+(a-x)^2\left\{(a-x)^2+b^2+c^2\right\}\right]^2}\,\frac{\left[2(a-x)^2+b^2+c^2\right]^2}{\sqrt{(a-x)^2+b^2+c^2}} + \frac{(a-x)bc}{b^2c^2+(a-x)^2\left((a-x)^2+b^2+c^2\right)}\,\frac{2(a-x)^2+3b^2+3c^2}{\left[(a-x)^2+b^2+c^2\right]^{\frac{3}{2}}}$$

$$\frac{1}{4}\frac{\partial^3 V}{\partial x^2\partial y} = -\frac{ac(a^2+c^2)}{\left[a^2\left(a^2+(b+y)^2+c^2\right)+(b+y)^2c^2\right]\sqrt{a^2+(b+y)^2+c^2}} + \frac{ac(a^2+c^2)}{\left[a^2\left(a^2+(b-y)^2+c^2\right)+(b-y)^2c^2\right]\sqrt{a^2+(b-y)^2+c^2}}$$

$$\frac{1}{4}\frac{\partial^4 V}{\partial x^2\partial y^2} = +\frac{ac(b+y)(a^2+c^2)}{\left[a^2\left(a^2+(b+y)^2+c^2\right)+(b+y)^2c^2\right]^2\left\{a^2+(b+y)^2+c^2\right\}^{\frac{3}{2}}}\left[3a^2\left(a^2+(b+y)^2+c^2\right)+c^2\left(2a^2+3(b+y)^2+2c^2\right)\right]$$

$$+\frac{ac(b-y)(a^2+c^2)}{\left[a^2\left(a^2+(b-y)^2+c^2\right)+(b-y)^2c^2\right]^2\left\{a^2+(b-y)^2+c^2\right\}^{\frac{3}{2}}}\left[3a^2\left(a^2+(b-y)^2+c^2\right)+c^2\left(2a^2+3(b-y)^2+2c^2\right)\right]$$